FINAL COURSE STUDY MATERIAL

PAPER 5

ADVANCED MANAGEMENT ACCOUNTING

MODULE – 1

BOARD OF STUDIES
THE INSTITUTE OF CHARTERED ACCOUNTANTS OF INDIA

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SYLLABUS

PAPER 5: ADVANCED MANAGEMENT ACCOUNTING

(One paper – Three hours – 100 marks)

Level of Knowledge: Advanced knowledge

Objective:

To apply various management accounting techniques to all types of organizations for planning, decision making and control purposes in practical situations.

To develop ability to apply quantitative techniques to business problems

1. Cost Management

(a) Developments in the business environment; just in time; manufacturing resources planning; (MRP); automated manufacturing; synchronous manufacturing and back flush systems to reflect the importance of accurate bills of material and routings; world class manufacturing; total quality management.

(b) Activity based approaches to management and cost analysis

(c) Analysis of common costs in manufacturing and service industry

(d) Techniques for profit improvement, cost reduction, and value analysis

(e) Throughput accounting

(f) Target costing; cost ascertainment and pricing of products and services

(g) Life cycle costing

2. Cost Volume Profit Analysis

(a) Relevant cost

(b) Product sales pricing and mix

(c) Limiting factors

(d) Multiple scarce resource problems

(e) Decisions about alternatives such as make or buy, selection of products, etc.

(f) Shut down and divestment.

3. Pricing Decisions

(a) Pricing of a finished product

(b) Theory of price

(c) Pricing policy

(d) Principles of product pricing

(e) New product pricing

(f) Pricing strategies

(g) Pricing of services

(h) Pareto analysis

4. Budgets and Budgetary Control

The budget manual, Preparation and monitoring procedures, Budget variances, Flexible budgets, Preparation of functional budget for operating and non-operating functions, Cash budgets, Capital expenditure budget, Master budget, Principal budget factors.

5. Standard Costing and Variance Analysis

Types of standards and sources of standard cost information; evolution of standards, continuous -improvement; keeping standards meaningful and relevant; variance analysis; disposal of variances.

(a) Investigation and interpretation of variances and their inter relationship

(b) Behavioural considerations.

6. Transfer pricing

(a) Objectives of transfer pricing

(b) Methods of transfer pricing

(c) Conflict between a division and a company

(d) Multi-national transfer pricing.

7. Cost Management in Service Sector

8. Uniform Costing and Inter firm comparison

9. Profitability analysis - Product wise / segment wise / customer wise

10. Financial Decision Modeling

(a) Linear Programming

(b) Network analysis - PERT/CPM, resource allocation and resource leveling

(c) Transportation problems

(d) Assignment problems

(e) Simulation

(f) Learning Curve Theory

A WORD ABOUT STUDY MATERIAL

The Institute of Chartered Accountants of India develops the course curriculum for its students and undertakes the periodic review of the course keeping in mind the developments in different subjects world wide and the objective of equipping the students with necessary knowledge and skill to serve the needs of Indian industry. The essence of successful management is decision making with the help of relevant information. Management Accounting influence decision making capacity and capabilities of a manager. The changes in business process across the globe and the continuous research work have evolved various advanced tools and techniques in the field of management accounting. The Institute has brought the modern techniques like Just in Time (JIT), Total Quality Management (TQM), Life Cycle Costing, Value Analysis, Throughput Accounting etc in the syllabus of Advanced Management Accounting. Equal importance has also been given in traditional tools of management accounting like Standard Costing, Budgeting, CVP Analysis etc which have great role to play in controlling and managing costs as well as decision making.

The Board of Studies which is instrumental in imparting theoretical education for the students of Chartered Accountancy Course develops the Study Materials of all subjects with the objective of developing the clear understanding of the concept of different topics covered in the subject among the students. The Study Material on Advanced Management Accounting covers sixteen chapters and topics included in each chapter are explained in details with explanation, examples and illustrations. As Management Accounting builds on various cross functional areas, comprehensive understanding of the subject is possible if only one covers all the topics of management accounting. A real life problem relates a number of topics of management accounting which are closely linked and its solution asks for clear understanding of all related topics. Thus, the students are advised to go through the whole study material and are expected to supplement their studies by referring to the recommended books of the subject in order to equip themselves with necessary professional knowledge of the subject. If required, they are also advised to brush up their knowledge of related topics of Intermediate (IPC) level.

The book attempts to elaborate and explain the conceptual basis of management accounting and also covers all applicable areas in depth.

The Board of Studies has also developed Practice Manual of the subject to provide an effective guidance material by providing clarification / solution to very important topics / issues, both theoretical and practical, of different chapters. Moreover, it will serve as Revision Help book towards preparing for Final Examination of the Institute and help the students in identifying the gaps in the preparation of the examination and developing plan to make it up. It will also provide standard of solutions to the questions which will act as a bench mark towards developing the skill of students on framing standard answer to a question. For any further clarification/guidance, students are requested to send their queries at *bosnoida@icai.in*,

Happy Reading and Best Wishes!

SIGNIFICANT CHANGES IN THE REVISED EDITION

Chapter No.	Name of the Chapter	Section/Sub-Sections Where in Major Additions / Deletions *have been done*	Page Numbers
3.	Pricing Decisions	3.2.1 Pricing Model	3.3
		3.2.2 Pricing under Different Market Structure	3.4
		3.4 Principles of Product Pricing	3.9
		3.5 New Product Pricing	3.11
		3.9 Summary	3.31
9.	Profitability Analysis - Product Wise / Segment Wise / Customer Wise	Name of the chapter has been changed from 'Cost sheet, Profitability analysis and Reporting' to 'Profitability Analysis- Product wise/ Segment wise/ Customer wise.'	9.1
		9.1 Profitability Analysis	9.1
		9.1.1 Operating Profit Analysis	9.1
		Illustration-1	9.4
		9.2 Profitability Analysis - Product Wise	9.5
		Illustration- 2	9.9
		9.3 Profitability Analysis - Segment Wise	9.12
15.	Simulation	15.2.1 Models of Simulation	15.2
		15.2.2 Stages of Simulation Process	15.2

STUDY PLAN – KEY TO EFFECTIVE LEARNING

Introduction

Main focus of 'Advanced Management Accounting' is on the application of 'Cost Management' techniques and 'Financial Decision Modeling' tools in various types of decisions making at all levels of management. Application of 'Advanced Management Accounting' helps to understand the ways and means to maximize revenue by reducing cost without affecting the essential qualities of products (decisions on what to produce, at what price, how to manage cost to maximize profitability, quality management etc.) and imposing control by classifying divisions as responsibility centres, allocating budgets and performance evaluation by setting various standards and variance analysis. The subject of 'Advanced Management Accounting' focuses on developing of knowledge required for analysis of quantitative and qualitative information in order to help the management in decision making. The students are suggested to understand the concept of each topic, use of relevant tools and techniques involved in the analysis of various problem situations covered under the 'Syllabus'. Students should keep in mind that the problem related to 'Management Accounting' in practical situation may often involve various issues together. Students are required to develop a comprehensive understanding of handling multiple issues involved in a problem which are closely linked and take all the factors into consideration (related to that problem) while evolving a rational solution. The basic objective of 'Advanced Management Accounting' subject is to apply various 'Management Accounting' techniques to all types of organizations for planning, decision making & control purpose in practical situation and to develop ability to apply 'Quantitative Techniques' to business problems.

Suggestive Approach to Study

'Study Material' of 'Advanced Management Accounting' has been developed to explain different concepts, tools and techniques related to 'Management Accounting' with examples and illustrations. The students are suggested to go through the 'Study Material' and conceptualize the topics given in the 'Syllabus' and understand depth of knowledge required for achieving success in the examination. In 'Advanced Management Accounting' examination, emphasis is on testing comprehension, self expression and ability to apply knowledge in divergent situations. Success in examinations considerably depends on style of preparation which can be achieved thorough practice, vision and objectivity. Before appearing for the examination, students need to make a comprehensive study plan. Study plan should be developed by keeping adequate time margin for study revision. Students must envisage the whole exercise of preparation before starting the actual work. The time span available till examination may be broken into four stages i.e.

(i) Initial Planning Stage (ii) In-Depth Study Stage	Students should go through the 'Study Material' for conceptual clarity & understanding and while doing so they should never hesitate to refer some good books so that no doubt can creeps into their mind. Make necessary notes for peculiar treatments and important key terms whenever come across. After grasping all the concepts and techniques; the next step is application of these concepts and techniques in solving different varieties of numerical from the 'Practice Manual'.
(iii) Revision Stage and (iv) Examination Stage	In these two stages students have to concentrate on work done to improve their confidence. Students are advised to solve the question given in the 'Revision Test Paper (RTP)' independently without referring to the answer first. To build confidence, pressure tolerant practice under examination condition will definitely help to manage time and answer framing. Take part in 'Mock Tests' arranged by the institute. Solving 'Mock Test' papers under examination condition is a good idea. Continuous practice under examination condition (Mock Test) will help students to approach examination with greater degree of confidence.

Chapter Specific

Chapter 1 to 9 emphasizes the role of *'Management Accounting'* in decision making, particularly in providing information and analysis to support strategic management activity. The focus is on evaluating existing competitive strategies, developing new strategies, and monitoring and assessing progress towards chosen strategies.

Chapter-1, Developments in the Business Environment

This chapter introduces students to the area of modern concepts in 'Cost Management'. It discusses recent developments in business environment and advanced management accounting techniques such as Total Quality Management (TQM), Activity Based Costing (ABC), Target Costing, Life Cycle Costing, Value Chain Analysis, Cost Control and Cost Reduction, Computer-Aided Manufacturing, Just in Time (JIT), Manufacturing Resources Planning, Synchronous Manufacturing, Business Process Re-engineering and Theory of Constraints. While going through these topics students need to link their study with other chapters or previous studies at Intermediate (IPC) level. Some topic wise useful techniques are given as under:

Total Quality Management: This topic is based on continuous effort of management in maintaining quality of a product; believe in the product and improvement in the product. In this topic students need to get conversant with various techniques of 'Quality Management'. The concept of 'Six Sigma' also shall be thoroughly understood by the students.

Activity Based Costing: As said earlier students need to inter-connect their study with other chapters or topics. ABC is connected with 'Absorption Costing' studied at Intermediate (IPC) level under the chapter 'Overheads'. ABC is also used as a tool for 'Decision Making', so their learning in this topic may be tested in succeeding chapters of 'Decision Making' and 'Costing for Service Sectors'. Students should be able to use of 'Direct' and 'Activity-Based Cost' methods in tracing costs to 'Cost Objects', such as customers or distribution channels, and the comparison of such costs with appropriate revenues to establish 'tiered' contribution levels, as in the activity-based cost hierarchy.

Target Costing: Every organization is driven by a corporate strategy which fulfills the mission and goals of an organization. In doing so organizations complying with its long term goal, it fixes its desired profit without losing its market share. In this topic students shall understand how an organization maintains its selling price with variable cost targets. This topic also requires *application of decision making techniques* used in the succeeding topics.

Life Cycle Costing, Value Chain Analysis, Cost control and Cost Reduction, Business process re-engineering, Theory of Constraints: In these topics students are required to identify the factors which have significant implications on product manufacturing and in product's cost.

Just in Time (JIT), Computer-aided manufacturing, Manufacturing Resource Planning and Synchronous Manufacturing: These topics are related with 'Inventory Control' and 'Production Management Techniques' to reduce or control costs.

This chapter is very important from the students perspective. Generally, students pay less attention to theory based chapters and the theoretical concepts of underlying different topics. But it is very important that students have thoroughly studied the theoretical aspects of the subject so that theoretical aspects help them in understanding the concepts and logic behind the mathematical workings and formulae while solving problems related to that particular concept.

Chapter-2, Decision Making Using Cost Concepts and CVP Analysis; Chapter-3, Pricing Decisions

In these two chapters students have to study Different Cost Concepts, Application of Cost Concepts in Decision Making, Cost-Volume-Profit (CVP) Analysis and Pricing Decision. 'Management Accounting' is fundamental in strategic planning. 'Managerial Accounting' information provides data-driven input to the decisions, which can improve decision-making over the long term. For example- Should a company shut down a division, Should it make or buy a product, Should it export or not, Should it accept an offer? Students should understand the concepts, need and importance of 'Marginal Costing' in decision making. *Students should also have understanding of area of 'Financial Decision Modeling' and application of the same in predicting product/service costs.* Clarity of concepts and self expression is essential for success.

Chapter-4, Budget & Budgetary Control

This chapter basically tries to impart students the concept of 'Budgeting'. 'Budgeting' and 'Financial Statement Projections' are just a few examples of how managerial accounting information is used to provide information to help management to guide the future of a company. By focusing on this data, one can make decisions that aim for continuous improvement and are justifiable based on intelligent analysis of the company data. Students are required to learn the difference between various types of budgets and process of preparation of budgets.

Chapter-5, Standard Costing

This chapter examines the functional-based standard costing systems in managing costs, improving planning and control, and facilitating decision making and product costing. This chapter has very important concepts of standard costing like computation of variances, control through variance analysis, accounting and reporting of variances. Classification of variances and interrelationship could be understood from the chart given in the 'Study Material'. *Students should be versed with variance analysis under marginal costing and absorption costing with concept of reconciliation of actual data and be familiar with the application of learning curve in standard costing.* This chapter requires lots of practice. 'Study Material' is very helpful for clear understanding of the concept. Students should do thorough practice to avoid computational errors.

Chapter-6, Costing of Service Sector

This chapter introduces students to various costing systems in the service sectors, the different types of cost behavior and their uses for decision making and planning via CVP analysis. It is important for the students to know the concept of relevant costing in relation to pricing decisions, joint cost and service department cost allocations.

Chapter-7, Transfer Pricing

This chapter covers concepts of 'Transfer Pricing'. 'Transfer Pricing' are used to evaluate the goods and services exchanged between profit centers of a decentralized firm. Students should be able to analyze the situation *when a division operating at capacity*. Students should also be versed with concept of 'Multinational Transfer Pricing'. Thorough practice of the problems is required for better understanding of transfer pricing concept.

Chapter-8, Uniform Costing

This chapter basically tries to impart about 'Uniform Costing'. It is a system of cost accounting to be used by the members of the industry. It involves adoption of same costing principles, practices and procedures by the individual members of the industry for inter-firm comparison. This is very important theoretical chapter.

Chapter-9, Profitability Analysis- Product wise/ Segment wise/ Customer wise.

This chapter enables students to understand and analyse the factors responsible for the variation in the profitability of a company with regard to budgeted or previous year's figures. This chapter requires the application of Standard Costing techniques to determine the variances in the profitability. In addition to this application of Activity Based Costing (ABC) will be required to determine profitability product wise/ segment wise/ customer wise. To measure the overall performance of an organisation 'Balanced Scorecard' is prepared. Balance Scorecard assesses the overall performance of an organisation by taking both financial and non-financial factors into account.

Financial Decision Modeling (Chapter 10-16) has become an essential tool in business applications. Modeling and analysis play major roles in abstract representation of business systems and data analysis and the subsequent generation of relevant information for making more accurate decisions. It consists of mathematical techniques that are increasingly used in decision making process such as Linear Programming, Transportation, Simulation, Network Analysis, Assignment and Learning Curve. 'Syllabus' covers applications of *quantitative techniques* for solving problems in manufacturing and service organizations. Key problem areas include marketing, production, logistics, procurement, and finance etc.

Chapter-10, Linear Programming

'Linear Programming' is a mathematical tool for determining the optimum allocation of resources and obtaining a particular objective. Students should be able to solve complex situations involving multiple constraints by various methods.

Chapter-11, Transportation Problem

This chapter deals with a special class of 'Linear Programming' problem in which the objective is to *'transport'* a single commodity from several *'sources'* to different *'destinations'* at a minimum total cost. Students should be versed with treatment of unbalanced problem. Students should also learn different methods for finding initial basic feasible solution.

Chapter-12, Assignment Problem

This chapter deals with assigning sources so that the total cost for performing all jobs is minimum. Students should be able to crack scenario of multiple solutions, unbalanced problem and prohibited assignments.

Chapter-13, Critical Path Analysis; Chapter-14, Program Evaluation and Review Technique

Both 'Critical Path Analysis' and 'Program Evaluation and Review Technique' are 'Management Accounting' techniques for planning and control of large complex projects. Both are techniques to network analysis wherein a network is prepared to analyze interrelationships between different activities of a project. Students should be familiar with concept of Resource Leveling, Smoothing and Crashing related to the networking analysis.

Chapter-15, Simulation

It is important for the students to understand the application of 'Simulation' techniques in managerial accounting practice for financial forecasting, analyzing capital investment, inventory analysis, production planning, and strategic enterprise management.

Chapter-16, Learning Curve Theory

The principle underlying learning curves is generally well understood -'if we perform tasks of a repetitive nature, the time we take to complete subsequent tasks reduces until it can reduce no more'. This is relevant to 'Management Accounting' in the two key areas of 'Cost Estimation' and 'Standard Costing'. Students should try to link this chapter with the concept of the 'Management Accounting' and try to understand *application of the same in predicting product/service costs*.

CONTENTS

DETAILED CONTENTS: MODULE – 1

CHAPTER 3 – PRICING DECISIONS

1

Developments in the Business Environment

LEARNING OBJECTIVES

After studying this area you should be able to:

- Explain the meaning of total quality management (TQM)
- Contributions in the field of TQM by Deming
- Know the 6 C's of TQM and Six Sigma
- Identify features of the TQM philosophy
- Describe tools for identifying and solving quality problems
- Understand Activity Based Costing, Activity Based Management and Activity Based Budgeting
- Understand difference between Activity Based Costing and Traditional Costing
- Describe how activity measures are chosen when using the ABC approach
- Describe the ABC cost hierarchy
- Explain the conceptual distinction between activities, drivers and activity measures
- Compute product cost in ABC problems
- Discuss CAM-I's involvement in developing and implementing ABC concepts and techniques
- Estimate target costs and describe the processes of target costing that lead to cost reduction and enhanced customer value
- Analyse life cycle costs and revenues and understand how to use life cycle management to reduce costs
- Identify opportunities for cost reduction by undertaking value analysis
- Explain difference between Cost Control and Cost Reduction
- Understand Manufacturing Resources Planning (MRP I&II)
- Describe a just-in-time (JIT) production system
- Identify the major features of a JIT production system
- Understand key JIT operating procedures and methods
- Understand the concept ofbusiness process re-engineering, Computer-aided manufacturing, and Synchronous manufacturing
- Undertake analyses using the theory of constraints and throughput accounting, to manage costs and time

1.1 Impact of changing Environment on Management Accounting

Since the time of industrialisation, cost and management reporting has always been the responsibility of either cost accountant or financial accountants or both. Apart from the statutory balance sheet, profit and loss account and the cash flow statements, the financial accountants of companies would provide other detailed reports to the management using the same set of historical data. However allocation and apportionment of expenses to cost centres and finally their absorption on the finished product continued to be the responsibility of the costing professionals. Many companies adapted the integrated model to combine the costing and the accounting functions and get real time information, which would be of greater use than the historical data provided by financial accounts.

With the advent of financial audit and its increasing importance ever since, product costing systems have increasingly concentrated on the production portion of the value chain as shown below,

RESEARCH ⇨ DEVELOPMENT ⇨ PRODUCTION ⇨ MARKETING ⇨ DISTRIBUTION ⇨ CUSTOMER SUPPORT

This is understandable since during the first half of the nineteenth century and perhaps till a couple of decades later, manufacturing costs accounted for the bulk of total costs incurred by the industry. The reason being the lack of competitive markets resulting in less advertising and distribution costs coupled with very little marketing and customer support. Manufactures worked in a monopolistic or a near monopolistic environment with products having long product life cycles and so did not require incurring large quantum of expenditure on functional areas like Research, Development etc. With most of the money being expended on the production function, reports provided by financial accountants for inventory valuation purposes gave enough information to the management about the majority of expenses being incurred by the company. The other costs incurred in the other than production functions of the value chain were considered discretionary and since the total quantum of such costs would not be huge, frequently they were excluded from decision-making purposes.

Manufacturing costs computed then were typically characterised by simplistic assumptions, with the use of 'blanket' overhead rates and simple labour overhead recovery bases being the common practice. In case of a relatively refined system, manufacturing overheads were segregated into fixed and variable. Whereas variable overheads could be identified with the production pattern with ease, the fixed overheads needed to be imputed over the products. This used to be done by identifying appropriate cost centres and overhead absorption rates. Fixed manufacturing overheads were initially allocated over the cost centres and then finally absorbed over the output at the rates, which were pre-established.The overhead rates were established considering the maximum output, which could be achieved by the specific cost centre as compared to the budgeted costs, which would be incurred for that level of activity. The result was that in case a company did not produce to potential, certain amount of these fixed overheads would not be absorbed over the products and hence remains unabsorbed. Such overheads were subsequently charged to the Profit and Loss Account and also provided the management with information about the productivity of the workers on the shop floor. However, Product Costing done on the basis of imputing fixed costs gives approximate results and is only useful in case the product has a long life cycle in the market. In the present competitive scenario, where innovation is the rule of the day, product life cycles have shortened and the competition has increased amongst companies at an unprecedented level. Such a scenario requires companies to produce in small batches as per customers requirements (implying higher raw material costs due to smaller purchases than before) , deliver quickly and efficiently (higher incidence of cost on the customer support and distribution functions of the value chain)and most importantly be prepared for product obsolescence. Hence, traditional costing may not be appropriate today as what it was when the market conditions were different.

The above mentioned issues in the changed industrial environment have resulted in new concepts of cost management in companies e.g. Total Quality Management, Just in Time, Activity Based Costing, Target Costing, Back flush Costing etc. These concepts have been imbibed by the Japanese, US and the other western economies with favourable results. Today, many companies in India have adapted such systems in order to remain competitive in the modern day environment in which production is highly automated and frequently, computer aided manufacturing resorted to.

1.2 Total Quality Management

1.2.1 It is too often viewed as a technique whose usefulness is confined to manufacturing processes. However, TQM also assumes potentially greater importance as a tool for improved efficiency in service sector. By focusing on the management accounting function, we will devise a process through which quality improvement methods might be used to highlight problem areas and facilitate their solution. An initial understanding of the difference between the three major 'quality' terms, **quality control, quality assurance** and **quality management** is essential to the short- medium- and long-term focus of business.

Quality: It is a measure of goodness to understand how a product meets its specifications. ISO 8402-1986 standard defines quality as *"the totality of features and characteristics of a product or service that bears its ability to satisfy stated or implied needs."*

When the expression "quality" is used, we usually think terms of an excellent product or service that fulfills or exceeds our expectations. These expectations are based on the intended use and the selling price. When a product surpasses our expectations we consider that quality. Thus, it is somewhat of an intangible based on perception Quality has nine important dimensions demonstrated in the table below. These dimensions are somewhat independent; therefore, a product can be excellent in one dimension and average or poor in another. Very few, if any, products excel in all nine dimensions. For example, the Japanese were cited for high quality cars in the 1970s based only on the dimensions of reliability, conformance, and aesthetics. Therefore, quality products can be determined by using a few of the dimensions of quality.

Dimension	Meaning and Example
Performance	Primary product characteristic, such as the brightness of the picture
Features	Secondary characteristic, added features, such as remote control
Conformance	Meeting specifications or industry standards, workmanship
Reliability	Consistency of performance over time, average time for the unit to fail
Durability	Useful life, includes repair
Service	Resolution of problems and complaints, ease of repair
Response	Human-to- human interface, such as the courtesy of the dealer
Aesthetics	Sensory characteristics, such as exterior finish
Reputation	Past performance and other intangibles, such as being ranked first

Quality Cost: Cost of performing the activities to check failure in meeting the quality specification. The "cost of quality" isn't the price of creating a quality product or service. It's the cost of not creating a quality product or service.Every time work is redone, the cost of quality increases. Obvious examples include:

- The reworking of a manufactured item.
- The retesting of an assembly.
- The rebuilding of a tool.
- The correction of a bank statement.
- The reworking of a service, such as the reprocessing of a loan operation or the replacement of a food order in a restaurant.

In short, any cost that would not have been expended if quality were perfect contributes to the cost of quality.

Quality costs are the total of the cost incurred by;

- Investing in the prevention of nonconformance to requirements.
- Appraising a product or service for conformance to requirements.
- Failing to meet requirements, which can be internal failure or external failure

Prevention costs	Appraisal Costs	Internal Failure Costs	External Failure Costs
QualityEngineering	Inspection	Scrap	Revenue loss
Quality training	Product acceptance	Rework	Warranties
Quality Audits	Packaging inspection	Re-inspection	Discount due to defects
Design Review	Field testing	Re-testing	Product liability
Quality circles etc	Continuing supplier verification etc	Repair etc	Warranty etc

Quality Control (QC): It is concerned with the *past*, and deals with data obtained from previous production which allow action to be taken to stop the production of defective units.

Quality Assurance (QA): It deals with the *present*, and concerns the putting in place of systems to prevent defects from occurring.

Quality Management (QM): It is concerned with the *future*, and manages people in a process of continuous improvement to the products and services offered by the organisation.

Thus, while section of the QA is responsible for systems which prevent departures from budgeted costs and corrective mechanisms to prevent future departures from budgeted costs. QM uses the skills and participation of the workforce to reduce the costs of production of goods and services. It becomes TQM when it embraces the whole organisation.

Total Quality Management (TQM): TQM is a management approach for an organization, centered on quality, based on the participation of all its members and aiming at long-term success through customer satisfaction, and benefits to all members of the organization and to society.

CIMA defines 'Total Quality Management' as *"Integrated and comprehensive system of planning and controlling all business functions so that products or services are produced which meet or exceed customer expectations. TQM is a philosophy of business behaviour, embracing principles such as employee involvement, continuous improvement at all levels andddocus, as well as being a collection of related techniques aimed at improving quality such as full documentation of activities, clear goal-setting and performance measurement from the customer perspective."*

TQM is composed of three paradigms:

- Total: Organization wide
- Quality: With its usual Definitions, with all its complexities
- Management: The system of managing with steps like Plan, Organise, Control, Lead, Staff, etc.

Thus, Total Quality Management (TQM) is a management strategy aimed at embedding awareness of quality in all organizational processes. TQM requires that the company maintain this quality standard in all aspects of its business. This requires ensuring that things are done right the first time and that defects and waste are eliminated from operations.

TQM is a comprehensive management system which:

- Focuses on meeting owner's/customer's needs, by providing quality services at a reasonable cost.
- Focuses on continuous improvement.
- Recognizes role of everyone in the organization.
- Views organization as an internal system with a common aim.
- Focuses on the way tasks are accomplished.
- Emphasizes teamwork

1.2.2 Operationalising TQM

Following are the universal Total Quality Management beliefs:

- Owner/customer satisfaction is the measure of quality
- Everyone is an owner/customer.
- Quality improvement must be continuous.
- Analysis of the processes is the key to quality improvement.
- Measurement, a skilled use of analytical tools, and employee involvement are critical sources of quality improvement ideas and innovations
- Sustained total quality management is not possible without active, visible, consistent, and enabling leadership by managers at all levels
- It is essential to continuously improve the quality of products and services that we provide to our owners/customers.

In order to make the concept of total quality management operationalising,followingchart outlines a systematic process for the examination of a number of fundamental questions. The focus is on the accounting function with the objective of implementing a process which will lead to the adoption of new strategies, the solving of problems and the elimination of identifiable deficiencies. The first four stages of this procedure are conducted internally within the management accounting team. They comprise a situation audit of current practice embracing corporate culture, product and customers.

The Process of reviewing the management accounting function

Stage 1

Who is the customer?

⇩

Stage 2

What does the customer expect from us?

⇩

Stage 3

What are the customer's decision-making requirements?

⇩

Stage 4

What problem areas do we perceive in the decision-making process?

⇩

Stage 5

How do we compare with other organisations?

What can we gain from bench- marking?

⇩

Stage 6

What does the customer think?

⇩

Stage 7

Identification of improvement opportunities

⇩

Stage 8

Quality improvement process

⇩ ⇩ ⇩

New strategies Elimination of deficiencies Solutions

Stage 1: Who is the customer?

A team approach was adopted to generate priorities in the identification of customers and critical issues in the provision of decision-support information. This provided a structured, group decision-making process for reaching consensus through the assignment of ranked priorities together with an environment conducive to the development of creative suggestions. The nominal group technique discussed earlier was employed.The ranking or perceived customer importance reveals the priority customers for management accounting services as:

- Manager;
- Engineers; and
- Leading hands.

Stage 2: What does the customer expect from us?

Managers having been identified as the priority group in receipt of accounting output, a second brainstorming session were used to generate a comprehensive list of their perceived expectations from the accounting function. Multi-voting was again used to identify the relative importance of these expectations, providing a ranking of 12 accounting functions:

- Compliance with procedures;
- Focus on problems;
- Performance reviews;
- Provision of budget information;
- Assessment of proposals;
- Payment of salaries;
- Tax advice;
- Management processes advice;
- Information forecasting;
- Commercial training;

- Information-processing skills; and
- Professional advice.

Stage 3: What are the customer's decision-making requirements?

Brainstorming revealed a list of 18 processes perceived to be major elements of the serviceprovided by management accountants:

- Pay people (wages and salaries);
- Pay accounts (vendors and contractors);
- Keep the books of account;
- Budget;
- Forecast;
- Audit;
- Conduct business-impact analyses;
- Manageauthorisation procedures;
- Issue guidelines;
- Maintain a library of procedures;
- Analyse performance;
- Managelicenses;
- Contribute to meetings;
- Manage property;
- Carry out strategic planning;
- Train others;
- Evaluate insurance requirements; and
- Produce ad hoc reports;

Combining management perceptions of customer expectations and the importance of the various functions, we find four processes clearly ranked as the key areas of importance to managers:

- Performance analysis;
- Ad hoc reporting;
- Strategic planning; and
- Contribution to meetings.

This series of steps, therefore, establishes managers as the priority customers for management accounting reporting and procedures, while performance analysis is the priority consideration in their use of management accounting information.

Typically, management accountants focus on the analysis of total performance in cost centres, using cost-per-unit comparisons and calculations of variance to generate plans. Where the focus is on quality improvement, the overriding need is to stay close to the customers and follow their suggestions. In this way, a decision-support system can be developed, incorporating both financial and non-financial information, which provides a flexible reporting system meeting user requirements.

In order to do this properly, we need to know:

- The nature of the decisions being made;
- The nature of the decision-making process; and
- The degree to which information requirements are being met.

A survey of users is required to provide this information, but critical issues can be identified and prioritised in advance, in order to refine the necessary survey questions.

Stage 4: What problem areas do we perceive in the decision-making process?

Once again using brainstorming and multi-voting, the team ranked the characteristics of an accounting information system thought most desirable from a decision-making point of view, as follows:

- **Relevance:** A targeted decision-making process.
- **Congruence:** Consistency with the long-term strategy of the business.
- **Comprehensibility:** Systems should be readily understandable and, therefore, readilyusable, by customers.
- **Linkage to non-financial indicators:** Systems need to reflect the monetary impact of physical parameters.
- **Timelines:** Systems should be on-time and on-line.

These characteristics were perceived as being areas of weakness where the greatest impact could be achieved through the implementation of improvements. It is instructive to consider some of the actual situations that might be associated with improvements in these areas.

- **Lack of relevance:**If line managers ignore most of the data reported to them by traditional cost accounting systems and treat head office cost analysis with disdain, they may prefer to perform their own specific cost investigations to determine the cause of deviations from plan, seeing management accounting reports as irrelevant and technically unrealistic. These informal systems may incorporate superior information which would be of benefit to all and which would be better incorporated within a global management information system.

The solution: Develop formal and informal reporting mechanism targeted to the needs of the user.

- **Lack of comprehensibility:**If management accountants believe that they prepare detailed financial reports for their managers to enable them to report to the managing director at the monthly board meeting, and the managing director declares that he or she is cognizant with all the relevant reported material for informal sources well in advance of the meeting, then clearly the customer for existing management accounting reports is not the managing director.

  Where such reports do not embrace the full extent of information generators, and fail to target a designated customer, there is room for a distinct improvement in the service offered. This may derive from more timely reporting, the provision of non-financial indicators, new performance measures, or a complete reformatting of the reporting process.

  The solution: Generate accounting information systems of a format and content suitable to meet user requirements.

- **Absence of a link to non-financial indicators:** The focus of management accounting must move beyond summary, financial measures of manufacturing operations if it is to maintain its central evaluation and control role. If a corporate goal of rapid internal growth is being pursued through a strategy of introducing automated production processes requiring less direct labour, then products using automated machinery intensely will be under-cost if direct labour hours are used to allocate manufacturing overhead costs for products. A more flexible allocation procedure should be adopted incorporating non-financial indicators, such as inspection and set-up times, in order to provide a 'fairer' distribution. In the absence of a 'right' answer, corporate strategy might serve to provide more guidance. Perseverance with an allocation on the basis of direct labourpenalises those products reliant on manual operations and provides an incentive to automate, consistent with the corporate strategy.

  The solution: Generate a concise group of non-financial indicators which reflect the overall performance of the company.

- **Lack of timeliness:** Suppose that the management accounting team prides itself on producing its monthly operating report on the eighth working day of the following month. An unexpected equipment failure means that it is unable to meet its accustomed deadline until the fifteenth working day. The team receives no complaints or enquiries during the interim on timeliness. The following month it produces, but does not distribute, the report. There is no response from the customer. The team continues this practice for the next three months until an internal memo indicates that the customer no longer wishes to receive the report – it is now surplus to requirements. In this case, the relevance of the whole reporting process is questionable and a close look at the distribution list of any given report, if not the existence of the report itself, is advisable.

  The solution: Generate reports in a form and time-envelope which meets the needs of the target customer.

Stage 5: How do we compare with other organisations? What can we gain from benchmarking?

Detailed and systematic internal deliberations allow the accounting team to develop a clear idea of their own strengths and weaknesses and of the areas of most significant deficiency. The benchmarking exercise at stage 5 of the TQM review process allows us to see how other similar companies are coping with similar problems and opportunities.

Stage 6: What does the customer think?

Respondents to the survey were encouraged to talk freely about their attitudes towards accounting information services, within a semi-structured outline covering:

- Nature of decisions made;
- Use made of existing formal reports;
- Preferred format (graphical, tabular or narrative) for formal reporting;
- Other information sources employed;
- Information, currently unavailable, which would aid decision-making; and
- Non-financial indicators used in performance appraisal.

However, formal reports were generally perceived as having four positive features. They were seen as useful in:

- Highlighting and reinforcing the existence of large variances, especially when closeto the budget setting period;
- Reporting unanticipated items associated with unexpected and late accruals, end of month '₹ adjustments', and misallocations to inappropriate accounts;
- Providing information which might change priorities, and
- Communicating a degree of analysis not available through on-line systems.

However, a number of criticisms of content were widespread. The reports were considered to:

- Place too much emphasis on the reporting of unfavourable variances constituting insignificantly small monetary amounts rather than focusing on an explanation of large expenditures actually incurred;
- Expend too much energy chasing inconsequential items representing minor out-of-budget fluctuations, rather than focusing on wrongly trended items (even where in-budget);
- Show an unrealistic concern with comparison of actual versus budgeted outcomes where unfavourable variances were in fact inevitable and symptomatic of inflexible budgeting and time shifts; and
- Report too many items for their own sake rather than to satisfy particular objectives or meet the requirements of particular individuals.

Unsatisfied needs embraced three major areas:

Ease of access to labour information to facilitate:

- The quantification and explanation of severe downturns in maintenance productivity;
- The distinction between normal and overtime hours on maintenance jobs, replacinginadequate composite hourly rates;
- Accounting for non-productive hours per worker resulting from the adoption of a more participatory style of management;

Predictive models concerning:

- Early warning of massive deteriorations;
- Forecasts of monthly maintenance expenditures;
- Relationships between breakdown and scheduled maintenance expenditures;
- The impact of performance of safety training;
- Probability-based analysis of risk to facilitate the management of maintenance expenditure; and

Trend information, ideally weekly and on-line, covering:

- Downtime and cost of breakdowns;
- Operating supplies;
- Maintenance materials;
- Purchased services; and
- Statistical process control.

Stage 7 &Step 8: The Identification of improvement opportunity and implementation of Quality Improvement Process.

The outcomes of the customer survey, benchmarking and internal analysis, provides the raw material for stage 7 and 8 of the review process: the identification of improvement opportunities and the implementation of a formal improvement process.Table 1 depicts the framework for the six-step analysis, identified by the acronym ‘PRAISE’.

The successful adoption of this sequence of steps demands discipline and commitment. The goal of quality improvement is paramount and guides the actions of the change team throughout the process.

Table 1: The PRAISE six-step quality improvement process

Step	*Activity*	*Elements*
1.	**Problem identification**	• Areas of customer dissatisfaction • Absence of competitive advantage • Complacency regarding present arrangements
2.	**Ranking**	Prioritise problems and opportunities by • Perceived importance, and • Ease of measurement and solution
3.	**Analysis**	• Ask 'Why?' to identify possible causes • Keep asking 'Why?' to move beyond the symptoms and to avoid jumping to premature conclusions • Ask 'What?' to consider potential implications • Ask 'How much?' to quantify cause and effect
4.	**Innovation**	Use creative thinking to generatepotential solutions • Barriers to implementation • Available enablers, and • People whole co-operation must be sought
5.	**Solution**	• Implement the preferred solution • Take appropriate action to bring about required changes • Reinforce with training and documentation back-up.
6.	**Evaluation**	• Monitor the effectiveness of actions Establish and interpret performance indicators to track progress towards objectives • Identify the potential for further improvementsand return to step 1

Table 2: Difficulties experienced at each step

Step	Activity	Difficulties	Remedies
1.	**Problem Identification**	• Effects of a problem are apparent but problem themselves are difficult to identify • Problem may be identifiable, but it is difficult to identify a measurable improvement opportunity	• Participative approaches like brainstorming,multi-voting,panel discussion • Quantification and precise definition of problem
2.	**Ranking**	• Difference in perception of individuals in ranking • Difference in preferences based on functions e.g. production, finance, marketing etc • Lack of consensus between individuals	• Participative approach • Subordination of individual to group interest
3.	**Analysis**	• Adoption of ad hoc approaches and quick fix solutions	• Lateral thinking brainstorming
4.	**Innovation**	• Lack of creativity or expertise • Inability to operationalise ideas, i.e. convert thoughts into action points	• Systematic evaluation of all aspects of each strategy
5.	**Solution**	• Resistance from middle managers	• Effective internal communication • Training of personnel and managers • Participative approach
6.	**Evaluation**	• Problem in implementation • Lack of measurable data for comparison of expectations with actual	• Effective control system to track actual feedback system

1.2.3 Contributions in the field of TQM by Deming

W. Edwards Deming:W. Edwards Deming is often referred to as the “father of quality control.” He was a statistics professor at New York University in the 1940s. AfterWorld War II he assisted many Japanese companies in improving quality. The Japanese regarded him so highly that in 1951 they established the Deming Prize, an annualaward given to organisations that demonstrate outstanding quality. It was almost 30 yearslater that American businesses began adopting Deming’s philosophy.A number of elements of Deming’s philosophy depart from traditional notions of quality. The first is the role management should play in a company’s quality improvement effort. Historically, poor quality was blamed on workers — on their lack of productivity, laziness, or carelessness. However, Deming pointed out that only 15 percent of quality problems are actually due to worker error. The remaining 85 percent are caused by processes and systems, including poor management. Deming said that it is up to management to correct system problems and create an environment that promotes quality and enables workers to achieve their full potential. He believed that managers should drive out any fear employees have of identifying quality problems, and that numerical quotas should be eliminated. Proper methods should be taught and detecting and eliminating poor quality should be everyone’s responsibility.

Deming outlined his philosophy on quality in his famous “14 Points.” These points are principles that help guide companies in achieving quality improvement. The principles are founded on the idea that upper management must develop a commitment to quality and provide a system to support this commitment that involves all employees and supplier’Deming stressed that quality improvements cannot happen without organizational change that comes from upper management.

Deming “14 points”

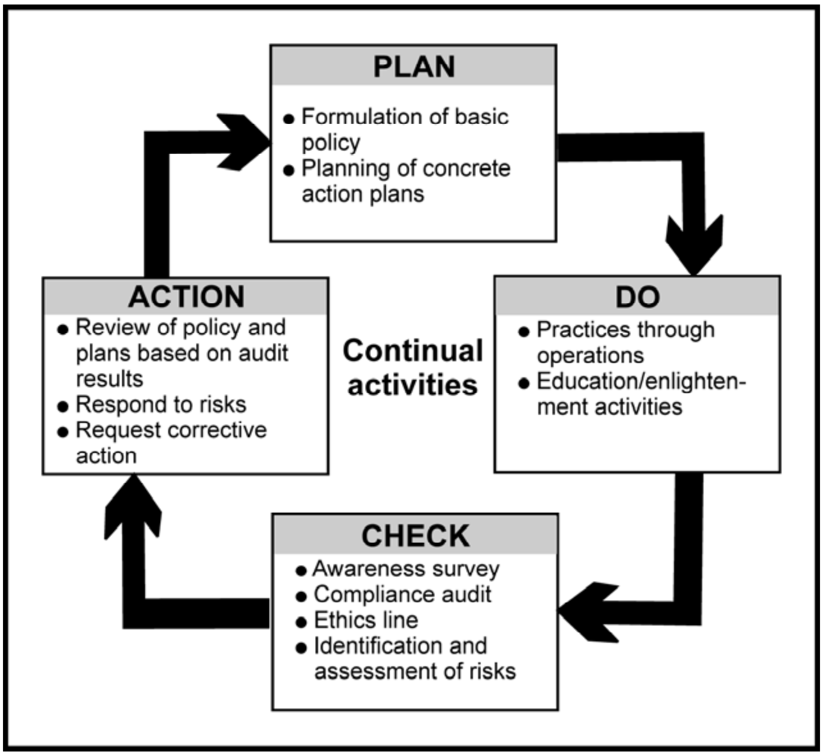
1.	"Create constancy of purpose towards improvement". Replace short-term reaction with long-term planning.	2	."Adopt the new philosophy". The implication is that management should actually adopt his philosophy, rather than merely expect the workforce to do so.
3.	"Cease dependence on inspection". If variation is reduced, there is no need to inspect manufactured items for defects, because there won’t be any.	4.	"Move towards a single supplier for any one item." Multiple suppliers mean variation between feedstock.
5.	"Improve constantly and forever". Constantly strive to reduce variation.	6.	"Institute training on the job". If people are inadequately trained, they will not all work the same way, and this will introduce variation.
7.	"Institute leadership". Deming makes a distinction between leadership and mere supervision. The latter is quota and target-based.	8.	"Drive out fear". Deming sees management by fear as counter-productive in the long term, because it prevents workers from acting in the organisation’s best interests.

9. "Break down barriers between departments". Another idea central to TQM is the concept of the 'internal customer', that each department serves not the management, but the other departments that use its outputs.	10. "Eliminate slogans". Another central TQM idea is that it's not people who make most mistakes - it's the process they are working within. Harassing the workforce without improving the processes they use is counter-productive.
11. "Eliminate management by objectives". Deming saw production targets as encouraging the delivery of poor-quality goods.	12. "Remove barriers to pride of workmanship". Many of the other problems outlined reduce worker satisfaction.
13. "Institute education and self-improvement".	14. "The transformation is everyone's job".

1.2.4 The Plan–Do–Study–Act Cycle

The plan – do – study – act (PDSA) cycle describes the activities a company needs to perform in order to incorporate continuous improvement in its operation. This cycle, is also referred to as the *Shewhartcycle or the Deming wheel*. The circular nature of this cycle shows that continuous improvement is a never-ending process. Let's look at the specific steps in the cycle.

- **Plan:**The first step in the PDSA cycle is to plan. Managers must evaluate the current process and make plans based on any problems they find. They need to document all current procedures, collect data, and identify problems. This information should then be studied and used to develop a plan for improvement as well as specific measures to evaluate performance.
- **Do:** The next step in the cycle is implementing the plan (do). During the implementation process managers should document all changes made and collect data for evaluation.
- **Study/Check:** The third step is to study the data collected in the previous phase. The data are evaluated to see whether the plan is achieving the goals established in the plan phase.
- **Act:** The last phase of the cycle is to act on the basis of the results of the first three phases. The best way to accomplish this is to communicate the results to other members in the company and then implement the new procedure if it has been successful. Note that this is a cycle; the next step is to plan again. After we have acted, we need to continue evaluating the process, planning, and repeating the cycle again.



1.2.5 Six Sigma

Continuous improvement can be brought into the organisational culture by introducing continuously changing planned targets. One such target can be six-sigma accuracy. The sigma accuracy means the process is 99.999998% accurate. That is the process will/can produce only 0.002 defects per million. This is the structural meaning of six-sigma. In quality practice, six-sigma means 3.4 parts per million.

Six sigma is the statistical measure used to ensure quality of products and services. The six sigma academy has developed a break through strategy consisting of measure, analyze, improve and control, that allows companies to make exceptional bottom-line improvements.

In addition to the material and labour savings, which flow directly to the bottom line, a company engaged in six sigma can expect to see:

- Improved customer satisfaction
- Reduction cycle time
- Increasedproductivity
- Reduction in total defect
- Improved process flow

Six sigma Capability Chart

Sigma	Parts per million
Six sigma	3.4 defects per million
Five sigma	233 defects per million
Four sigma	6,120 defects per million
Three sigma	66,807 defects per million
Two sigma	3,08,537 defects per million
One sigma	6,90,000 defects per million

1.2.6 Six C's of TQM

The Six Cs for successful implementation of a Total Quality Management (TQM) process is depicted as follows:

- **Commitment:**If a TQM culture is to be developed, so that quality improvement becomes a normal part of everyone's job, a clear commitment, from the top must be provided. Without this all else fails. It is not sufficient to delegate 'quality' issues to a single person since this will not provide an environment for changing attitudes and breaking down the barriers to quality improvement. Such expectations must be made clear, together with the support and training necessary to their achievement.
- **Culture:**Training lies at the centre of effecting a change in culture and attitudes. Management accountants, too often associate 'creativity' with'creative accounting' and associated negative perceptions. This must be changed to encourage individual contributions and to make 'quality' a normal part of everyone's job.

- **Continuous improvement:**Recognition that TQM is a 'process'not a 'programme'necessitates that we are committed in the long term to the never-ending search for ways to do the job better. There will always be room for improvement, however small.
- **Co-operation:**The application of Total Employee Involvement (TEI) principles is paramount. The on-the-job experience of all employees must be fully utilised and their involvement and co-operation sought in the development of improvement strategies and associated performance measures.
- **Customerfocus:**The needs of the customer are the major driving thrust; not just the external customer (in receipt of the final product or service) but the internal customer's (colleagues who receive and supply goods, services or information). Perfect service with zero defects in all that is acceptable at either internal or external levels. Too frequently, in practice, TQM implementations focus entirely on the external customer to the exclusion of internal relationships; they will not survive in the short term unless they foster the mutual respect necessary to preserve morale and employee participation.
- **Control:**Documentation, procedures and awareness of current best practice are essential if TQM implementation is to function appropriately. The need for control mechanisms is frequently overlooked, in practice, in the euphoria of customer service and employee empowerment. Unless procedures are in place improvements cannot be monitored and measured nor deficiencies corrected.

Difficulties will undoubtedly be experienced in the implementation of quality improvement and it is worthwhile expounding procedure that might be adopted to minimise them in detail.

1.2.7 Overcoming Total Quality Paralysis

Little attention has so far been paid to the practical problems of overcoming the inertia of organisations and the reluctance of some individuals to adopt the new tools of management accounting. This section argues for a systematic approach to overcome the apparent paralysis besetting many companies in implementation of a quality policy.

A quality improvement process like the PRAISE system restricts the adoption of sub–optimum quick-fix solutions and increases the participants'awareness of barriers to change. However, it does not overcome completely some of the behavioural difficulties associated with individual motivation and group dynamics. The problem is not one of an awareness of the usefulness of TQM but rather the ability to do something about it – the inertia associated with total quality paralysis. Some fundamental requirements in getting started are:

- A clear commitment, from the top, to TQM ideals. Without this, everything else fails. It is not sufficient to delegate 'quality'issues to a single person, since this will not provide an appropriate environment for changing attitudes and behaviour and breaking down the barriers to quality improvement. The aim is to develop a TQM culture so that quality improvement becomes a normal part of everyone's job. This expectation must be made clear, and whatever support and training is necessary to its achievement must be provided.

- Managers must be provided with the skills, tools and techniques to pursue systematic improvement. Training should be practical, avoiding unnecessary abstractions and keeping management jargon to a minimum. It may even be necessary to avoid the acronym 'TQM'itself, because of the barriers associated with buzzwords, reverting to reference instead to the phrase 'quality improvement processes.
- The general awareness of improvement opportunities must be improved through the creation of a database documenting the status quo and covering those things that the organisation currently does well, as well as its deficiencies. Such a database should contain answers to questions like these:
  - ✓ Where do we make errors?
  - ✓ Where do we create waste?
  - ✓ What should we do that we currently make no attempt to do?

Ideally; the quality improvement process should be a vehicle for positive and constructive movement within an organisation. We must, however, be aware of the destructive potential of the process. Failure to observe the fundamental principles associated with the 'four Ps'of quality improvement may so severely damage motivation that the organisation is unable to recover fully. Those four Ps are:

- **People:** It will quickly become apparent that some individuals are not ideallysuited to the participatory process. Lack of enthusiasm will be apparent from a generally negative approach and a tendency to have pre-arranged meeting which coincide with the meetings of TQM teams where these individuals are charged with the responsibility for driving group success then progress will be slow or negligible. Quality improvement teams may have to be abandoned largely for associated reasons before they are allowed to grind to a halt.
- **Process:** It is essential to approach problem-solving practically and to regard the formal process as a system designed to prevent participants from jumping to conclusions. As such it will provide a means to facilitate the generation of alternatives while ensuring that important discussion stages are not omitted.
- **Problem:** Experience suggests that the least successful groups are those approaching problems that are deemed to be too large to provide meaningful solutions within a finite time period. Problems need to be approached in bite-sized chunks, with teams tackling solvable problems with a direct economic impact, allowing for immediate feedback together with recognition of the contribution made by individual participants. For example, while 'communications'and 'morale'are frequently cited as key problem areas, they are too broad to provide successful quality improvement targets. Smaller aspects of these issues must be identified.
- **Preparation:**Courses on creative thinking and statistical processes are needed in order to give participants a greater appreciation of the diversity of the process. This training

must quickly be extended beyond the immediate accounting circle to include employees at supervisory levels and below who are involved at the data input stage.

A three-point action plan for the choice of projects and the implementation process is as follows:

- **Bite-sized chunks.** It is tempting to seek a large cherry to pluck, but big improvement opportunities are inevitably complex and require extensive inter-departmental co-operation. The choice of a relatively small problem in the first instance provides a greater chance of success.
- **A solvable problem.** The problem selected should not be trivial, but it should be one with a potential impact and a clear improvement opportunity. Measurable progress towards implementation should be accomplished within three or four months (or less if possible) in order to maintain the motivation of participants and advertise the success of the improvement process itself.
- **Recognition of participants.** The successful projects and team members should receive appropriate recognition throughout the enterprise. Prominent individuals should be rewarded for their efforts both as personal recognition and as encouragement to others. The precise nature of the reward may be recognition itself, although in some situations material, but usually non-monetary, prizes may also be appropriate.

The implementation of TQM processes can provide long-lasting benefits as long as the achievement of quality goals is not in conflict with other objectives. This might be the case, where, for instance.

- Bonuses are based on the volume of output alone; or
- Retrenchments result from the increased efficiency associated with the qualityimprovement process.

By overcoming the initial obstacles, a TQM process can provide us with an additional tool to improve competitiveness and ensure long-term survival.

1.2.8 Control: The Missing Link of TQM

The fundamental principles of TQM focus on a process of continuous improvement which enhances the satisfaction of customer requirements by changing the attitude of the workforce. The reduction of waste is made implicit in each worker's task. This suggests the elimination of all non-value-adding processes, processes which include all control functions – monitoring, inspecting, progress chasing, even auditing – which would now be replaced by self-auditing as part of the change in corporate culture. Such extreme expectations are unrealistic. A control function, properly defined, is essential and can contribute to the achievement of TQM objectives.

The development of TQM provides a vehicle for the accounting function to achieve control, continuous improvement and maximum efficiency by ensuring that all of the processes carried out by that function are both in control and capable. Such movements will have a dramatic effect on the accounting function and may well redefine the audit function.

The basic requirement of accounting control is that a process is capable of meeting customer requirements, whether they are those of the directors, the shareholders, or the law. Techniques which have historically been used to achieve this control include procedures and audit, but these have major flaws. If we are not appropriately focused, it is possible that the process is *never* going to be capable of meeting customer requirements, no matter how complex the levels of audit or procedure adopted. Further, there will be no focus for the documentation of flaws and their subsequent reversal.

Qualitative and non-financial data, vital for control, may not be subject to the same strict standards of measurement as financial and technical data. Their role in the quality programme may, therefore, be underestimated.

Documentation of the *activities* to be performed in the accounting function is an essential first step in identifying the dimensions of processes and the interrelationships between tasks; details (given below) eight basic processes which may be identified in the accounting function, each covering multiple activities and crossing task boundaries.

Dimensions of the accounting function

Process	**Activity**
Planning	Strategic planning Operating planning Forecasts
Book-keeping	Costing Inventory accounting Project accounting Fixed capital Maintenance system
Discharging liabilities	Payroll Accounts receivable Accounts payable Cashier Contracts administration

Reporting	Corporate reporting Statutory reporting Management reporting
Business support	Project or opportunity evaluation Cost improvement Tax advice or guidance Operating centres
Corporate services	Tax Insurance Legal
Functional administration	Technology management Personnel management Non-accounting procedures TQM
Controllership	Agreements Accounting guidelines Accounting procedures Accounting policy and standards Internal audits External audits

A narrow control function is apparent in each process, but this is effectively just the checking or audit component of controllership. The controllership function interacts with the TQM process to impact upon the other six dimensions to provide timely and relevant information to decision-makers and to monitor compliance with corporate expectations where policies, procedures, ethical behaviour and professional conduct are concerned.

The quality manual is usually the major document controlling the implementation of the quality process. It defines the basic philosophy of the organisation, the structure and responsibilities of managers and departments and the relationship between them. It also contains the methods to be used to ensure quality, including the composition of teams, and the audit procedures to be adopted.

The definition of the process, inputs and outputs gives a framework for the writing of procedures and standard methods while also providing a focus for improvement opportunities. Underpinning both is a control and audit process, defining the way that the system is to be checked.

For every process within the accounting organisation, a policy and procedure is established in accordance with industry best practice and communicated throughout the organisation. Its objective is to satisfy customer requirements and to identify improvement opportunities which allow the continuous extension of the customer service provided.

The writing of procedures and standard methods is a fundamental step in pursuing excellence of process. Procedures are concerned with the properties of the system that we are trying to influence (controlled parameters). Standard working methods are concerned with the process variables that are being manipulated in order to influence the system (control points). Thus, if we want to control the water level in a bath, the level is the controlled parameter, and the tap and plug are the control points.

By providing a sound control environment, which supports business decisions with appropriate measurement and analysis, the controllership function pursues complete customer satisfaction. The aim is to achieve acknowledged industry leadership for excellence of process, personnel and service. Underpinning this aim is an audit process that ensures that all of the above are in place and operating. The audit process is partly external, but largely internal, consisting of a control check system that monitors the critical processes of the system. Depending on the breakdown consequences and risk of failure, additional control points can be introduced into the process chain.Thus, the system allows not only for control, but also for continuous improvement. The monitoring of the data around a process will allow modifications which make it in control and capable. As changes or improvements are made they are documented and the system updated so that everyone uses the *current* best method.

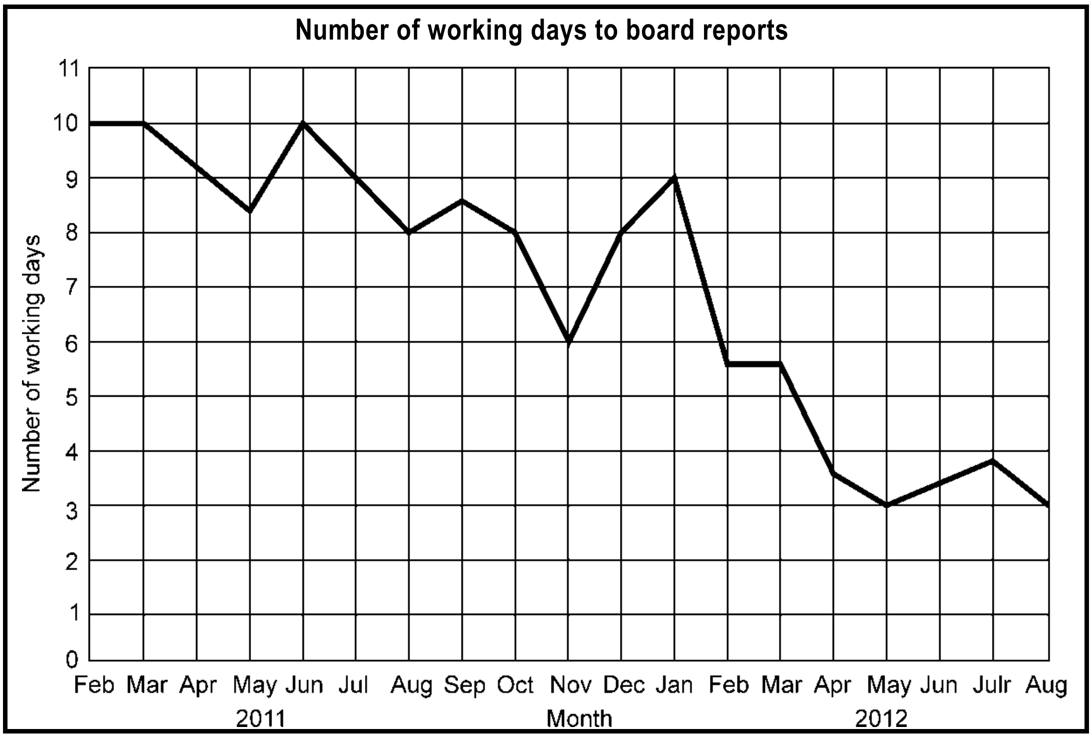
The clear definition and documentation of procedures facilitates job flexibility, making control easier and increasing the level of productivity in the accounting department. Thus, a good control system facilitates continuous improvement by focusing on customer needs, identifying priorities, and relating processes to one another. Variation and inaccuracy is caused by poor control and incompatible systems. A quality system is therefore essential to reduce these problems.

The application of the PRAISE quality improvement process to the timeliness problem provides an excellent example of service improvement, one which observes the fundamental quality principles of waste elimination and doing things right the first time.

Traditionally, a consolidated profit figure has been produced by midday on the fifth working day of the month. Ideally, month-end closing would always be completed on the first working day of the new month, providing more relevant information for decision-making at board level and allowing more efficient use of accounting resources.

By identifying the barriers which prevent the generation of on-time data, a procedure can be implemented to generate a substantial reduction in the completion time for the early-closing process. Careful documentation of the network of tasks allows performance information (embracing financial cost data, technical and non-financial data) to be available at the beginning of the second working day, allowing a full executive performance review to take place before the end of that day. By focusing on further small improvements in procedure, completion might eventually approach the first-day ideal.

Documentation of key data on processes is the first, and arguably the most important, step in the procedure. By charting processes for each activity, establishing time barriers, constraints, priorities, degrees of difficulty and expected improvement times, a critical database is established. Small, dedicated problem-solving teams are charged with developing solutions for task improvements, with the success of the process demonstrated by the dramatic daily improvement apparent at month end illustrated below.



Significant further improvements are also likely to follow:

- The elimination of double handling and manual data delays in day-to-day operations;
- The acceptance of the quality process for problem-solving; and
- The highlighting of opportunities for interdisciplinary teamwork.

The reasons for the success of the improvement process in the area of timeliness are firmly grounded in the principles of TQM, embracing total employee involvement and process measurement. These principles include:

- The clear exposition of the benefits of a project;
- The involvement of all customers and contributors;
- The elimination of non-relevant data;
- An understanding of the needs of the whole process;
- The use of graphical and pictorial techniques to achieve understanding;
- The establishment of performance specifications and targets;
- The use of errors to prompt continuous improvement; and
- The use of statistics to tell people how well they are doing.

The basic requirements of controllership are a practical reality and provide a springboard for the provision of accurate, timely data to manage and enhance a business. Control features are, therefore, essential constituents of the TQM process, facilitating the successful implementation of customer-focused improvements.

The quality improvement process should be a vehicle for positive and constructive movement within an organisation but we must also be aware of the destructive potential of the process. Failure to observe the fundamental principles of quality improvement may destroy motivation irrecoverably.

1.2.9 Criticisms of Total Quality Management

Some authors, notably Carlzon (1987), Albrecht (1985) and Albrecht and Zemke (1988) have criticised the direction that TQM implementations have tended to take in practice, in particular.

- the focus on documentation of process and ill-measurable outcomes;
- the emphasis on quality assurance rather than improvement; and
- an internal focus which is at odds with the alleged customer orientation.

Carlzon has revived the customer focus with an emphasis on total employee involvement (TEI) culminating in the empowerment of the 'front-line' of customer service troops. The main features of his empowerment thrust havebeen:

- loyalty to the vision of the company through the pursuit of tough, visible goals;
- recognition of satisfied customers and motivated employees as the true assets of a company;
- delegation of decision-making to the point of responsibility by eliminating hierarchical tiers ofauthority to allow direct and speedy response to customer needs; and
- decentralisation of management to make best use of the creative energy of the workforce.

Albrecht suggest that TQM may not be appropriate for service based industries, because the standards-based approach of ‘industry best practice’ignores the culture of organisations. He recommends a move towards TQS (total quality service), which is more customer oriented and creates an environment to promote enthusiasm and commitment. Albrecht suggests that poor service is associated with sloppy procedures, errors, inaccuracies and oversights and poor co-ordination, all of which represents improvement opportunities which can be achieved through tighter controls.

1.2.10 Concept of ‘TQM’followed by ‘Tata Steel, General Electric Company and Motorola Inc’

Tata Steel has maintained the confidence to improve performance globally even in the face of a challenging economic climate in which the steel industry happens to be severely affected. One factor that contributes to this confidence is the Company’s adherence to Total Quality Management (TQM) to achieve its goals. Since the formal incorporation of TQM for Business Excellence in the late 1980’s Tata Steel has adopted a number of improvement initiatives popular around the world. At Tata Steel’s European operations, Continuous Improvement activities are focused on providing Business Units with the ability to drive business through Lean Management, a common strategy deployment process, training of CI coaches and knowledge sharing through operations. Tata Steel maintains a systematic approach towards improving productivity and enhancing quality while reducing cost at the same time. The Singapore operations concentrated on yield improvement, reduction in power consumption and a significant bottom line benefit. The Xiamen operations have also adopted measures to reduce vulnerability caused by price fluctuations.

With the Company’s better understanding of TQM and the Theory of Constraints (TOC) on the Deming Application Prize journey, its customer focus and market orientation have undergone a sea-change. Tata Steel has initiated a culture of value creation with customers and suppliers. Specific approaches focus on the ‘needs’of the customer as opposed to ‘wants’. Programmes include those on Customer Value Management, Retail Value Management, and Solution for Sales and Supplier Value Management. The Company emphasises effective daily work management practices, a clean and safe work environment and consistency and stability of processes as important factors in sustaining development and growth.

In the face of high raw-material price volatility and an overall trend of rapidly increasing prices, in 2009-2010 the procurement Division of Tata Steel India focused its efforts on keeping these trends in check by leveraging long-term contracts and relationships, and on minimising risk by hedging and through various other strategic sourcing tools, including innovations and improvement initiatives using Total Quality Management precepts.

Tata Steel is the first integrated steel company in the world, outside of Japan, to win the Deming Application Prize. The steel giant won the 2008 prize for achieving distinctive performance improvements through the application of total quality management (TQM).

General Electric Company and Motorola Inc.: Today’s customers demand and expect high quality. Companies that do not make quality a priority put risk on long-run survival. World-class organizations such as General Electric and Motorola attribute their success to having one of the best quality management programs in the world. These companies were some of the first to implement a quality program called, Six Sigma, where the level of defects is reduced to approximately 3.4 parts per million. To achieve this, everyone in the company is trained in quality. For example, individuals highly trained in quality improvement principles and techniques receive a designation called "Black Belt." The full-time job of Black Belts is to identify and solve quality problems. In fact, Motorola was one of the first companies to win the prestigious Malcolm Baldrige National Quality Award in 1988, due to its high focus on quality. Both GE and Motorola have had a primary goal to achieve total customer satisfaction. To this end, the efforts of these organizations have included eliminating almost all defects from products, processes, and transactions. Both companies consider quality to be the critical factor that has resulted in significant increases in sales and market share, as well as cost savings in the range of millions of dollars.

1.2.11 Illustrations

Illustration 1

Galaxy Ltd has a dedicated set of production facilities for an auto component – coded X pertaining to the gearbox of its leading car – GX2.With a vendor park set up in the vicinity of the parent manufacturing plant, the Just – in – Time system ensures that no stock of materials; work in progress or finished goods are held.

At the beginning of the year 2009, the planned information relating to the production of component X through the dedicated facilities is as follows:

(i) Each unit of component X has input materials; 5 units of materials A at ₹ 20 per unit and 4 units of materials B at ₹ 10 per unit.

(ii) Variable cost per unit of component X (excluding materials) is ₹ 25 per unit worked on.

(iii) Fixed costs of the dedicated facilities for the period: ₹ 250,000.

(iv) It is anticipated that 7.5% of the units of X worked on in the process will be defective and will be scrapped.

It is estimated that customers will require replacement (free of charge) of faulty units of component X at the rate of 1 % of the quantity invoiced to them in fulfillment of orders.

Galaxy Ltd. is pursuing a TQM philosophy. Consequently all losses will be treated as abnormal in recognition of a zero defect policy and will be valued at variable cost of production.

Actual statistics for each of the years 2009-2011 for component X are shown given below–

	2009	*2010*	*2011*
Worked on in the process (units)	*6,005*	*7,500*	*7,000*
Invoiced to customers (units)	*5,500*	*6,500*	*6,500*

Total costs:			
Materials A and B (₹)	*840,700*	*1,050,000*	*980,000*
Variable costs of production (₹) (Excluding materials costs)	*150,125*	*187,500*	*175,000*
Fixed costs (₹)	*287,500*	*262,000*	*290,000*

No changes have occurred from the planned price levels from materials, variable overhead or fixed overhead costs.

Actual free replacements of component X to customers were 250 units and 40 units in years 2010 and 2011 respectively.

Galaxy Ltd. authorized additional expenditure during the year 2010 and 2011 as follows:

2010: Equipment accuracy checks of ₹ 10,000 and staff training of ₹ 5,000.

2011: Equipment accuracy checks of ₹ 10,000 plus ₹ 15,000 of inspection costs; also staff training costs of ₹ 5,000 plus ₹ 3,000 on extra planned maintenance of equipment.

Required:

(a) Analyse the figures given above in table to check whether in the year 2009 actual results were achieved at the planned level in respect of (i) quantities and losses and (ii) units cost levels for material and variable costs.

(b) Use your analysis from (a) in order to calculate the value of the internal and external failure costs for year 2009

(c) Prepare an analysis for the years 2010 and 2011 which provide reconciliation between the number of components invoiced to customers with those worked–on in the production process.The analysis should show the change from the planned quantity of process losses and changes from the planned quantity of replacement of faulty components in customer hands;

(d) Prepare a cost analysis for the years 2010 and 2011 which shows actual internal failure costs, external failure costs, appraisal costs and prevention costs;

(e) Prepare a report, which explains the meaning and inter – relationship of figures given above in table and in the analysis in (a), (b), (c) & (d).The report should also give examples of each cost type and comment on their use in the monitoring and progressing of the TQM policy being pursued by Galaxy Ltd.

[Note: Ignore fractions in case of units]

Solution

(a) (i)

	units
Components worked on in the process	6,005
Less: Planned defective units (7.5% of 6,005)	450

Less: Replacements to customers (1% of 5,555)	55
Components invoiced to customers	5,500

Therefore actual results agree with planned results

(ii) Planned component cost = (5 units x ₹ 20 for materials A) + (4 units x ₹ 10 for material B) + ₹ 25 variable cost = ₹ 165

Comparing with the data in the table:

Materials = ₹ 840,700/6,005 units = ₹140

Variable overhead = ₹ 150,125/6,005 units = ₹25

(b) Internal failure costs = ₹ 74,250 (450 units x ₹ 165)

External failure costs = ₹ 9,075 (55 units x ₹ 165)

(c)

	2010 (units)	**2011 (units)**
Components invoiced to customers	6,500	6,500
Planned replacement (1%)	65	65
Unplanned replacement (Total-Planned)	185	–25
	(250–65)	(40–65)
Components delivered to customers [A]	6,750	6,540
Components worked on in the process [B]	7,500	7,000
Total Process defects [C = B–A]	750	460
Planned process defects (7.5% of worked on In the process) [D]	562	525
Unplanned defects (balancing figure)[C–D]	188	– 65

(d)

	2010 (₹)	**2011 (₹)**
Internal failure costs	123,750 (750 units x ₹ 165)	75,900 (460 units x ₹ 165)
External failure costs	41,250	6,600
	{(65+185) units × ₹ 165}	{(65-25) units x ₹ 165}
Appraisal costs	10, 000	25,000
Prevention costs	5,000	8,000

(e) The following points should be included in the report:

1. Insufficient detail is provided in the statistics shown in the table thus resulting in the need to for an improvement in reporting.

2. The information presented in (c) indicates that free replacements to customers were 185 greater than planned in the year 2010 but 25 less than planned in the

year 2011.In contrast, the in process defects were 188 more than planned (approximately 33%) in the year 2010 and 65 less than plan (approximately 12%) in the year 2011.

	2009 (₹)	2010 (₹)	Change w.r.t previous period (₹)	2011 (₹)	Change w.r.t previous period (₹)
Internal Failure Costs	74,250	123,750	49,500	75,900	(-)47,850
External FailureCosts	9,075	41,250	32,175	6,600	(-)34,650
Total	83,325	165,000	81,675	82,500	(-)82,500

3. Both Internal failure and External failure costs have increased substantially in the year 2010 but decreased significantly in the year 2011.

4. The additional failure cost w.r.t the year 2009 was ₹ 81,675 in the year 2010 and cost savings w.r.t. year 2010 were ₹ 82,500 in the year 2011

   The above savings should be compared against the investment of Equipment accuracy checks of ₹ 10,000 and staff training of ₹ 5,000 in the year 2010 and investment of Equipment accuracy checks of ₹10,000 plus ₹ 15,000 of inspection costs; also staff training costs of ₹ 5,000 plus ₹ 3,000 on extra planned maintenance of equipment in the year 2011. It can be seen that the costs exceed the savings in the year 2010 but the savings exceeded the costs in the year 2011.There has also been an increase in both internal and external failure costs from the year 2009 to 2010.Investigations should be made relating to the likely time lag from incurring prevention/ appraisal costs and their subsequent benefits. Also it seems that expenditure on inspection of ₹ 15,000 and expenditure on extra planned maintenanceof ₹ 3,000 in the year 2011 has yielded major results. This should be thoroughly analysed and be adopted as a successful tool to reduce failure cost. Reduced failures will also improve the brand equity and customer satisfaction of the product.

Illustration2

The budget estimates of a company using sophisticated high speed machines based on a normal working of 50,000 machine hours during 2012 are as under:

	(₹ lakhs)
Sales (1,00,000 units)	*100*
Raw Materials	*20*
Direct Wages	*20*
Factory Overheads—Variable	*10*
Fixed	*10*

Selling and Distribution Overheads— Variable	*5*
Fixed	*5*
Administration Overheads—Fixed	*10*
Total Costs	*80*
Profit	*20*

Since the demand for the company product is high the possibilities of increasing the production are explored by the budget committee. The Technical Director stated that maintenance has not been given due importance in the budget and that if preventive maintenance is introduced, the breakdown repair costs and the hours lost due to breakdown can be reduced and consequently production can be increased.

In support of this, he presented the following data, showing how injection of more and more funds on preventive maintenance will bring down the break-down repair costs and reduce or eliminate stoppages due to breakdown :–

Proposed Expenditure on Preventive Maintenance ₹	***Expenditure Estimated to be Incurred on Breakdown Repairs*** ₹	***Machine Hours Saved*** ₹
19,200	*1,92,000*	*Nil*
38,400	*1,53,600*	*800*
76,800	*1,15,200*	*1,600*
1,53,600	*76,800*	*2,400*
3,07,200	*57,600*	*3,200*
6,14,400	*—*	*4,000*

Using the different cost and contribution concept, advice the management upto what level breakdown hours can be reduced to increase production and maximise profits of the company consistent with minimum costs.

Solution

Workings:

	(₹ in lakhs)
Contribution per unit and per hour:	
Sales (1,00,000 units)	100
Raw materials	20
Direct wages	20
Factory overheads (Variable)	10
Selling & distribution overheads (Variable)	5
Total variable costs	55
Contribution	45

Contribution per unit : $\frac{₹\,45,00,000}{1,00,000}$ = ₹ 45	
Machine hours planned	50,000
Production units	1,00,000

In one machine hour 2 units will be produced

Hence contribution per hour : ₹45 x 2= ₹90.

Statement showing differential cost and incremental contribution at different levels of machine hours saved.

Machine hours saved	---	800	1,600	2,400	3,200	4,000
Estimated breakdown Repair costs (₹)	1,92,000	1,53,600	1,15,200	76,800	57,600	---
Differential savings in Breakdown repair Costs (₹)	---	38,400	38,400	38,400	19,200	57,600
Incremental contribution (₹) (see note below)	---	72,000	72,000	72,000	72,000	72,000
Total differential Savings ₹ (A)	---	1,10,400	1,10,400	1,10,400	91,200	1,29,600
Expenditure on preventive maintenance	19,200	38,400	76,800	1,53,600	3,07,200	6,14,400
Differential expenditure on Preventive Maintenance (₹) (B)	---	19,200	38,400	76,800	1,53,600	3,07,200
Incremental profit (₹) (A–B)	---	91,200	72,000	33,600	(–) 62,400	(–) 1,77,600

Note: Incremental contribution is calculated by multiplying differential hours saved by contribution per hour i.e. 800 × ₹90 = ₹72,000

Recommendation: It may be observed from the above table that savings in machine hours upto 2,400 hours yields incremental profit. Beyond this level, the differential maintenance costs exceed the differential savings. Therefore, the management is advised to reduce the level of breakdown hours upto 1,600(4,000-2,400) or save 2,400 breakdown hours to increase production. At his level, the company will be able to maximise profits consistent with minimum costs.

Illustration3

ABC Road Carriers is a transporting company that transports goods from one place to another. It measures quality of service in terms of :

(i) Time required to transport goods

(ii) On–time delivery

(iii) Number of lost or damaged cartons.

To improve its business prospects and performance the company is seriously considering to install a scheduling and tracking system, which involves an annual outlay of ₹ 1,25,000. The company furnishes the following information about its present and anticipated future performance:

	Current	***Expected***
On–time delivery	*85%*	*95%*
Variable costs per carton lost or damaged	*₹ 55*	*₹ 55*
Fixed costs per carton lost or damaged	*₹ 45*	*₹ 45*
Number of cartons lost or damaged	*2,500*	*1,200*

The company expects that each half per cent point increase in on–time performance will result in revenue increase of ₹ 9,000 per annum. Contribution margin of 45% is required. Should ABC Road Carriers acquire and install the new system? Also calculate additional amount of revenue required if benefits from new system is equal to cost & Contribution margin is 47.5%.

Solution

Should ABC Road Carriers acquire and install the new system?

		₹
Additional Costs of the new scheduling & tracking systemp.a.		1,25,000
Additional Revenue from Improvement in on-time performance (₹9,000 x 10%/0.5%)		1,80,000
Contribution from Additional Annual Revenue (45% x ₹ 1,80,000)…	(A)	81,000
Cost Saving in respect of Cartons[(2,500-1,200) x ₹55]…	(B)	71,500
Total Benefits (A+B)		1,52,500

As Expected Benefits are more than the costs. Accordingly company should install the new system.

Calculation of additional amount of revenue required if benefits from new system are equal to cost & Contribution margin is 47.5%:

		₹
Costs of the new scheduling & tracking system	(A)	1,25,000
Cost Saving in respect of Cartons	(B)	71,500
Contribution Margin	(A – B)	53,500
Contribution Margin %		47.5
Corresponding Additional Revenue		1,12,632

1.3 Activity Based Costing, Activity Based Management and Activity Based Budgeting

1.3.1 Activity Based Costing: Contrary to what might be imagined, many organisations do not wish to know how much it costs to make a product with precise accuracy. This is because pricing is based on what the market will bear, competitors' moves, etc. Others however fix their price on cost and need to be able to determine it with reasonable accuracy. The latter organisations have been greatly benefitted from the development of activity based costing (ABC), which is a modern absorption costing method,

It should not be assumed that all traditional absorption costing systems are not accurate enough to give adequate information for pricing purposes or other, long-run management decision purposes. Some traditional systems treat overheads in a detailed way and relate them to service cost centres as well as production cost centres. The service centre overheads are then spread over the production cost centres before absorption rates are calculated. The main cause of inaccuracy is in the calculation of the overhead rate itself, which is usually based on direct labour hours or machine hours'These rates assume that products that take longer to make, generate more overheads. Thus traditional cost system over-cost high volume products and under-cost low volume products.Factors prompting the development of ABC system include:

- Growing overhead costs because of increasingly automated production
- Increasing market competition which necessitated more accurate product costs.
- Increasing product diversity to secure economies of scope & increased marketshare.
- Decreasing costs of information processing because of continual improvementsandincreasing application of information technology

Activity Based Costing is an accounting methodology that assigns costs to activities rather than products or services. This enables resources & overhead costs to be more accurately assigned to products & services that consume them.

CIMA defines'Activity Based Costing'as *"An approach to the costing and monitoring of activities which involves tracing resource consumption and costing final outputs. Resources are assigned to activities, and activities to cost objects based on consumption estimates. The latter utilise cost drivers to attach activity costs to outputs."*

ABC is a technique which involves identification of cost with each cost driving activity and making it as the basis for apportionment of costs over different cost objects/ jobs/ products/ customers or services.

ABC assigns cost to activities based on their use of resources. It then assigns cost to cost objects, such as products or customers, based on their use of activities. ABC can track the flow of activities in organization by creating a link between the activity (resource consumption) and the cost object.

The flow is characterized through four core areas:Cost object, Cost driver, Resource Cost driver&Activity Cost driver'In order to understand how ABC operates it is necessary to understand the meaning of above terms.

- **Activity -**Activity, here, refers to an event that incurs cost.
- **A Cost Object–**It is an item for which cost measurement is required e.g. a product or a customer.
- **A Cost Driver–**It is any factor or activity that causes a change in the cost of an activity. It has a direct cause and effect relationship with the resources consumed. There are two categories of cost driver:
- **A Resource Cost Driver–**It is a measure of the quantity of resources consumed by an activity. It is used to assign the cost of a resource to an activity or cost pool.
- **An Activity Cost Driver–**It is a measure of the frequency and intensity of demand, placed on activities by cost objects. It is used to assign activity costs to cost objects.

The cost driver for business functions viz., Research & Development and Customer Service are as below:

Business functions :	**Cost Driver**
Research and Development	• Number of research projects • Personnel hours on a project
Design of products, services and procedures	• Number of products in design • Number of parts per product • Number of engineering hours
Customer Service	• Number of service calls • Number of products serviced • Hours spent on servicing products
Marketing	• Number of advertisements

	• Number of sales personnel • Sales revenue
Distribution	• Number of units distributed • Number of customers

In traditional costing overheads are first related to cost centres (Production & Service Centres) and then to cost objects, i.e., products. In ABC overheads are related to activities or grouped into cost pools (depending on the terminology preferred). Then they are related to the cost objects, e.g., products. The two processes are, therefore, very similar, but the first stage is different as ABC uses activities instead of functional departments (cost centres). The problem with functional departments is that they tend to include a series of different activities, which incur a number of different costs that behave in different ways. Activities also tend to run across functions; for instance, procurement of materials often includes raising a requisition note in a manufacturing department or stores. It is not raised in the purchasing department where most procurement costs are incurred. Therefore, ABC gives a more realistic picture of the way in which costs behave.

As with traditional absorption costing ABC rates are calculated in advance, normally for a year ahead, and so the same rates are used for a year at a time. The advantage of this is that any seasonal variations will be spread giving an average cost. If this was not done and actual rates were used the absorption rates would vary monthly. This would mean that when output was high the overhead rate would be low and vice versa; if pricing were based on cost the prices quoted would be higher when the business was slack.

1.3.2 Stages in Activity Based Costing

The different stages in activity based costing are listed below:

(1) Identification of the activities that may take place in an organisation.The first stage is to identify the major activities in the organization. There can be machine related activities, direct labour related activities and various support activities such as ordering, receiving, material parts handling etc. Usually the number of cost centres that a traditional overhead system uses is quite small, say up to fifteen. In ABC the number of activities will be much more, say 200; the exact number will depend on how the management subdivides the organisation's activities. It is possible to break the organisation down into many very small activities. But if ABC is to be acceptable as practical system it is necessary to use larger groupings, so that, say, 40 activities may be used in practice. The activities may be listed as follows:-

- Production schedule changes
- Customer liaison
- Purchasing
- Production process set up

- Quality control
- Material handling
- Maintenance

(2) Assigning costs to cost pool for each activity both support and primary activities, that caused them. This creates 'cost pools'or 'cost buckets'. This will be done using resource cost drivers that reflect causality.

(3) Support activities are then spread across the primary activities on some suitable base, which reflects the use of the support activity. The base is the cost driver that is the measure of how the support activities are used.

(4) Determine the cost drivers for each activity that will be used to relate the overheads collected in the cost pools to the cost objects/products. A cost Driver is a variable, which determines the work volume or work load of a particular activity. This is based on the factor that drives the consumption of the activity. The question to ask is – what causes the activity to incur costs? In production scheduling, for example, the driver will probably be the number of batches ordered. Some questions which are aimed at bringing out the cost drivers are given below:-

- What is the number of staff working on a particular activity?
- Why is overtime worked?
- Why does the idle time occur?
- What is it that determines the amount of time spent on a particular activity?
- The end result of this type of questioning will be a typical set of cost drivers for each sub-activity.

(5) Assigning the costs of activities to products according to product demand for activities:This step involves tracing the cost of the activities to products according to products demand for these activities during the production process. This requires calculating cost driver rate for each activity, just as an overhead absorption rate would be calculated in the traditional system.

$$\text{Activity cost driver rate} = \frac{\text{Total Cost Activity}}{\text{Activity Driver}}$$

The activity driver rate can be used to cost products, as in traditional absorption costing, but it can also cost other cost objects such as customers/customer segments and distribution channels. The possibility of costing objects other than products is part of the benefit of ABC. The activity cost driver rates will be multiplied by the different amounts of each activity that each product/other cost object consumes.

Let us take a small example to understand the steps stated above:

Assume that a company makes widgets and the management decides to install an ABC system. The management decides that all overhead costs will have only three cost drivers viz.

Direct labour hours, Machine hours and number of purchase orders and the general ledger of the company shows the following overhead costs –

General Ledger	Amount (₹)
Payroll taxes	1,000
Machine maintenance	500
Purchasing Dept. labour	4,000
Fringe benefits	2,000
Purchasing Dept. Supplies	250
Equipment depreciation	750
Electricity	1,250
Unemployment insurance	1,500
Total	**11,250**

So, which overheads do you think are driven by direct labour hours?

The answer is	— Payroll taxes	– ₹ 1,000
	—Fringe benefits	– ₹ 2,000
	— Unemployment insurance	– ₹ 1,500
	—Total	– ₹ 4,500

Similarly, overheads driven by machine hours include Machine maintenance, depreciation and Electricity totaling ₹ 2,500 and finally overheads driven by number of purchase orders include purchasing department labour and purchasing department supplies totaling ₹ 4,250.

Now, overhead rate is calculated by the formula total cost in the activity pool / Base, base being the total number of labour hours, machine hours and total number of purchase orders in the given case.

Assume that the total number of labour hours be 1,000 hours, machine hours be 250 hours and total purchase orders be 100 orders.

So, the ABC rates would be

₹ 4,500/ 1,000 =₹ 4.50 per labour hour

₹ 2,500/ 250 =₹ 10 per machine hour

₹ 4,250/ 100 = ₹ 42.50 per purchase order.

Now, let’s allocate the overheads between two widgets A and B the details of which are given below

Particulars	Widget A	Widget B
Labour hours	400	600
Machine Hours	100	150
Purchase Orders	50	50

So, total overhead costs applied to widget A = (400x4.50) + (100x10) + (50x42.50) = ₹ 4,925

And total overheads applied to widget B = (600x4.50) + (150x10) + (50x42.50) = ₹ 6,325

So total overheads = ₹ 4,925 + ₹ 6,325 = ₹ 11,250.

Generally, in the traditional costing method, overheads are applied on the basis of direct labour hours (total 1,000 labour hours in the given case). So, in that case the overhead absorption rate would be – ₹ 11,250/1,000 = ₹ 11.25 per hour and the total overheads applied to Widget A would have been = 400x11.25 = ₹ 4,500 and to Widget B = 600x11.25 = ₹ 6,750.

Hence Widget A would have been undervalued and Widget B overvalued by ₹ 425.

Example of cost drivers for different activity pools in a production department can be explained below:

Activity Cost Pools	Related Cost Drivers
Ordering and Receiving Materials cost	Number of purchase orders
Setting up machines costs	Number of set-ups
Machining costs	Machine hours
Assembling costs	Number of parts
Inspecting and testing costs	Number of tests
Painting costs	Number of parts
Supervising Costs	Direct labour hours

1.3.3 Hierarchy in Activity-Based Costing

Activities basically fall into four different categories, known as the manufacturing cost hierarchy. These categories are generally accepted today but were first identified by Cooper (1990). The categories of activities help to determine the type of activity cost driver required.

The categories of activities are:

1. **Unit Level Activities:** The costs of some activities (mainly primary activities) are strongly correlated to the number of units produced. For example, the use of indirect materials/consumables tends to increase in proportion to the number of units produced. Another example of a unit level activity is the inspection or testing of every item produced, if this was deemed necessary or, perhaps more likely, every 100th item produced.

2. **Batch Level Activities:** The cost of some activities (mainly manufacturing support activities) is driven by the number of batches of units produced. Examples of this are:

- Material ordering–where an order is placed for every batch of production
- Machine set-up costs–where machines need resetting between each differentbatch of production.
- Inspection of products–where the first item in every batch is inspected rather than every 100th item quoted above.

3. Product Level Activities: The costs of some activities (often once only activities) are driven by the creation of a new product line and its maintenance, for example, designing the product, producing parts specifications and keeping technical drawings of products up to date. Advertising costs fall into this category if individual products are advertised rather than the company's name.

4. Facility Level Activities: Some costs cannot be related to a particular product line; instead they are related to maintaining the buildings and facilities. Examples are the maintenance of buildings, plant security, business rates, etc. Also included in this category are salaries, such as the production manager's. Advertising campaigns that promote the organisationwould also be included.

The first and last categories above are the same as those in traditional absorption costing and so if an organisation costs are mainly made up of these two categories ABC, will not improve the overhead analysis greatly. But if the organisation's costs fall mainly in the second and third categories an ABC analysis will provide a different and more accurate analysis.

1.3.4 ABC Vs Absorption Costing

The points of differences between activity based costing and traditional absorption costing can be enumerated below:

Activity Based Costing	Traditional Absorption Costing
1. Overheads are related to activities and grouped into activity cost pools.	1. Overheads are related to cost centers/departments.
2. Activities are classified as – (i) Unit Level, (ii) Batch Level, (iii) Product Level and (iv) Facility Level activities.	2. Only (i) Unit Level (Variable) and (ii) Facility Level (Fixed) activities are identified.
3. Costs are related to activities and hence are more realistic.	3. Costs are related to cost centers and hence not realistic of cost behaviour.
4. Activity–wise cost drivers are determined.	4. Time (Hours) are assumed to be the only cost driver governing costs in all departments.
5.Activity–wise recovery rates are determined and there is no concept of a single overhead recovery rate.	5. Either multiple overhead recovery rate (for each department) or a single overhead recovery rate may be determined for absorbing overheads.
6. Cost are assigned to cost objects , e.g. customers, products, services, departments, etc	6. Costs are assigned to Cost Units i.e. to products, or jobs or hours.
7. Essential activities can be simplified and unnecessary activities can be eliminated. Thus the corresponding	7. Cost Centers/ departments cannot be eliminated. Hence not suitable for cost control.

costs are also reduced/ minimized. Hence ABC aids cost control.	

1.3.5 Purposes and benefits of Activity Based Costing

Initially companies switched from traditional absorption costing to ABC in order to produce more accurate cost information for products, as shown above. The managers in some of these companies were surprised by the information revealed which indicated inordinately high cost.,This led them to adjust their pricing policies and to developdifferent product strategies, as they found that previously high volume, long production run products had been over-costed and low volume, short production run products under-costed. ABC is particularly needed by organisations for product costing where:

- Production overheads are high in relation to direct costs
- There is a great diversity in the product range
- Products use very different amounts of the overhead resources
- Consumption of overhead resources is not primarily driven by volume.

But if ABC is only considered to be a more detailed and accurate overhead absorption costing system many organisations may decide to do without it. Advocates of using ABC for an accurate overhead apportionment usually compare the ABC technique with the most basic traditional absorption costing system where one blanket overhead rate is applied.

The **main advantages** of using Activity Based Costing are:

- More accurate costing of products/services, customers, SKUs, distribution channels
- Better understanding overhead
- Utilizes unit cost rather than just total cost
- Integrates well with Six Sigma and other continuous improvement programs
- Makes visible waste and non-value added
- Supports performance management and scorecards
- Enables costing of processes, supply chains, and value streams
- Activity Based Costing mirrors way work is done
- Facilitates benchmarking

1.3.6 Activity Based Cost system installation and operation

The motives for pursuing an ABC implementation, or at least of investigating its feasibility, must be established at the outset. Most commonly these will be:

- To improve product costing especially in those situations where existing methods undercost some products and overcost others; or

- To identify non-value-adding activities in the production process that might be a suitable focus for attention or elimination.

In practice, the former is the most quoted goal, even though the latter may be more appropriate. This is especially so for firms which are highly labour intensive and which do not have a great diversity of products in their range, and where allocation of overhead based on direct labour hours may already function efficiently.

Direct costs, like materials and direct labour, are easily assigned directly to products. Some indirect costs, particularly those selling costs which are product specific (e.g., advertising), may be directly assigned to the product too. The remaining indirect costs are those which are problematical and provide the focus for ABC, with resource costs indirectly assigned to the cost object via cost pools and activity drivers.

A number of distinct practical stages in the ABC implementation are as follows:

- **Staff Training:** The co-operation of the workforce is critical to the successful implementation of ABC. They are closest to the process and most aware of the problems. Staff training should be, as far as possible, jargon-free, and create an awareness of the purpose of ABC. It should be non-threatening in nature, stressing that increased efficiencies resulting from a successful implementation will mean rewards not redundancies. The need for the co-operation of staff in the concerted team effort, for mutual benefit, must be emphasised throughout the training activity.
- **Process Specification:** Informal, but structured, interviews with key members of personnel will identify the different stages of the production process, the commitment of resources to each, processing times and bottlenecks. The interviews will yield a list of transactions which may, or may not, be defined as 'activities' at a subsequent stage, but in any case provide a feel for the scope of the process in the entirety.
- **Activity Definition:** The problem must be kept manageable at this stage, despite the possibility of information overload from new data, much of which is in need of codification. The listed transactions must be rationalised in order to aggregate those in similar categories and eliminate those deemed immaterial. The resultant cost pools will likely have a number of different events, or drivers, associated with their incurrence.
- **Activity Driver Selection:** A single driver covering all of the transactions grouped together in an 'activity' probably does not exist. Multiple driver models could be developed if the data were available, but cost-benefit analysis has rarely shown these to be desirable. The inter-correlation of potential activity drivers will probably be so strong as to suggest that it really does not matter which one is selected. This argument might be employed to avoid the costly collection of data items otherwise not monitored, nor easily accessible.
- **Costing:** A single representative activity driver can be used to assign costs from the activity pools to the cost objects. If, for example, the number of engineering set-ups has been identified as a driver of process costs and the total set-up cost is ₹40,000 for a company producing four products (A, B, C, D) then the number of set-ups per product can be used to assign these costs. If product A requires 2 set-ups; B4 set-ups ; C24

and D10, then the average cost per set-up of ₹40,000/40 set ups = ₹1,000, a misleading figure taken at face value, which does not imply the different demands of the set up resource made by the different products. However, total set-up costs can be distributed to product groups in proportion to use, i.e., A:₹ 2,000, B:₹4,000, C:₹24,000 and D:₹10,000 and then assigned to individual units of product in proportion to the total level of output.

This procedure can then be repeated for all material activities.

1.3.7 ABC: A Decision Making Tool

It is a useful tool for many of the management decisions facing companies today. It can bring a picture of the operation to light that may not be obvious through other analysis tools. Specifically, ABC is useful in analyzing specific segments of an organization. This might include a market line, a group of products (even a single product), a customer, or an employee. The ABC is implemented infollowingdecisions:

- ABC is a complement to total quality management (TQM). It provides quantitative data that can track the financial impact of improvements implemented as part of the TQM initiative. Some have even suggested that ABC is the most important concept introduced since TQM. Amoco Performance Products, Transparent Container Co. and Fellowes Manufacturing Co. are a few companies that have utilized the ABC/TQM modeling concept to improve performance and profitability.
- Wholesale distributors can gain significant advantage in the decision-making process through implementation of ABC concepts. The expansion of line offerings has brought about difficult decisions for the distributor. Using traditional financial data, overhead burden is distributed equally across the product line. Introduction of new products or vendors might also introduce variance to the overhead. For instance, the need to support a special storage area for control or environmental reasons, or the need of new handling equipment will increase overall operational costs. These costs will be spread over the product line, reducing margin on existing products and reducing the cost impact of the new items. ABC models the costs back to the activity. The burden created by the new product is correctly reflected. This allows the existing merits while leaving the new line to justify itself.
- Other decisions that can be assisted by ABC include facility and resource expansion. Often the basis for relocation or opening of a new distribution center is based on cost associations. Reduction in freight or other logistics costs can offset the expense of the new facility, staff or equipment. When the numbers used are enterprise-based, the return might not develop as expected. The ABC model can identify the specific cost elements being targeted, providing a much clearer picture from which management can act.
- Decision support for human resources can be augmented by ABC. Where activity, and therefore cost, can be associated to an individual, new levels of financial performance

can be determined. This might be appropriate in cases of branch management or sales. Adding or deleting resource slots can be determined based on costs of activities as well. The added data provided through ABC can present a number of options, including outsourcing, productivity improvements through automation, and a determination of employee/revenue ratios.

- Companies who wish to determine price based on cost plus markup basis find ABC method of costing very relevant and are able to determine competitive prices for their products.
- Using Traditional absorption costing, overheads may get distributed equally across all product lines. ABC traces costs back to the activity and the consumption of resources by each product. Thus, product line profitability can be determined in more realistic terms.
- Other areas where ABC system can be relevant include market, make or buy decisions, transfer pricing, plant close – down decisions, evaluation of offshore production or outsourcing a process, capital investment decisions, etc.

In summary, activity-based costing is a management decision-making tool. It provides financial support data structured in a fashion fundamentally different from accounting data provided in the general ledger. By associating cost to the activity, a clear relationship can be established between sources of activity demand and the related costs. This association can benefit the distributor in determining where costs are being incurred, what is initiating the costs and where to apply efforts to curb inflationary costs. This can be of particular value in tracking new products or customers. It can also provide tracking of logistics costs, one of the fastest growing areas of expense to the distribution operation.

1.3.8 Activity Based Cost Management (ABM)

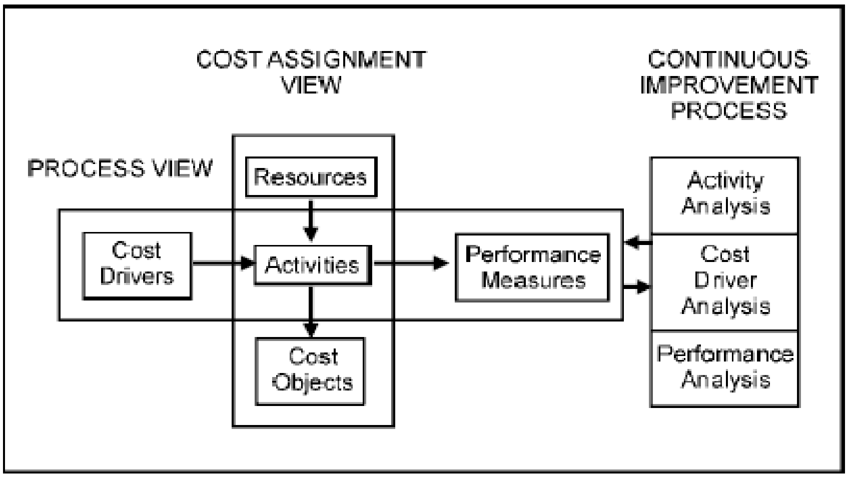
Empirical studies of ABC implementation have frequently shown that the greater benefit derived from its adoption are in Cost Management rather in providing accurate product cost. The term Activity based management (ABM) is used to describe the cost management application of ABC. The use of ABC as a costing tool to manage costs at activity level is known as Activity Based Cost Management (ABM). ABM is a discipline that focuses on the efficient and effective management of activities as the route to continuously improving the value received by customers. ABM utilizes cost information gathered through ABC. Through various different types of analysis, ABM manages activities rather than resources. It determines what drives the activities of the organisationand how these activities can be improved to increase the profitability.

Consortium for Advanced Management International (CAM) defines ABM as *"adds a dynamic, continuous improvement dimension to the more static ABC model".*

Interestingly, it has been observed that Japanese accountants began exploring activity-based techniques in the early 1990s following movement in the United States toward the ABM model.

CAM-1 defines ABM as:*"A discipline that focuses on the management of activities as the route to improving the value received by the customer and the profit achieved by providing this value. This discipline includes cost driver analysis, activity analysis, and performance measurement. Activity-Based Management draws on Activity-Based Costing as its major source of information."*

Activity-Based Management Model



Source: From the CAM-I Glossary of Activity-Based Management, Edited by Norm Raffish and Peter B. B. Turney, (Arligton: CAM-I, 1991).

Figure given above represents ABM model in a chart developed for CAM-1. In commenting on this model, one of its co-developers stated, "*ABC supplies the information, and ABM uses this information in various analysis designed to yield continuous improvement."*

- **Cost Driver Analysis:** The factors that cause activities to be performed need to be identified in order to manage activity costs. Cost driver analysis identifies these causal factors.

  For example, a cost driver analysis study might determine that slow processing of customer invoices results largely from lack of training of the customer invoice associates. This lack of training is thus a cost driver of the customer invoice processing activity. It is one of the factors causing this activity to take place (in this case, inefficiently). Managers have to address this cost driver to correct the root cause of the slow processing problem. To accomplish this task, managers might decide that an internal training program for customer invoice associates should be designed and implemented to increase the speed of customer invoice processing.

  The time saving from the improved training may lead to a reduction in the number of customer invoice associated and, thus, to lower costs for the "customer invoice processing" activity. Of course, real salary cost savings occur only if the number of jobs in the organisation actually decreases (e.g., through attrition) or if displaced customer invoice associates are redeployed to VA activities, thus canceling the need to hire new employees from outside. The tangible cost savings and intangible benefits from the customer invoice processing improvements should be compared with both the tangible and intangible costs of the new training program in a cost-benefit analysis.

The hypothetical customer invoice processing example shows that the identification and analysis of cost drivers (causal factors) is a necessary first step toward improving the cost-effectiveness of activities and cost management through ABM.

- **Activity Analysis:** Activity analysis, defined in section (a), identifies the activities of an organisation and the activity centres (or activity cost pools) that should be used in an ABC system. Activity analysis also identifies Value Added (VA) and Non Value Added (NVA) activities. The degree to which activities are grouped together into activity centres depends on the costs and benefits of the alternatives. The number of activity centres is likely to change over time as organisational needs for activity information evolve.

  For example, only a few activity centres may be used in an initial ABC pilot study. As managers become more accustomed to the initial ABC system and find the output useful, they may request a more detailed and refined ABC model.

- **Performance Analysis:** Performance analysis involves the identification of appropriate measures to report the performance of activity centres or other organisational units, consistent with each unit's goals and objectives. Performance analysis aims to identify the best ways to measure the performance of factors that are important to organisations in order to stimulate continuous improvement.

1.3.9 Business application of ABM

ABM views the business as a set of linked activities that ultimately add value to the customer. ABM is based on the premise that activities consume costs. Therefore, by managing activities costs will be managed in long term. Activities may be grouped in such a way as to describe the total process. For example, serving a particular customer involves a number of discrete activities, but the sum total of these activities represents the process by which the client is serviced.ABM classifies each activity within a process as value-added activities or non-value added activities.

- **Value-Added Activities (VA):** The value-added activities are those activities which are indispensible in order to complete the process.The customers are usually willing to pay (in some way) for these services. For example polishing furniture by a manufacturer dealing in furniture is a value added activity.

- **Non-Value-Added Activities (NVA):**The NVA activity represents work that is not valued by the external or internal customer.NVA activities do not improve the quality or function of a product or service, but they can adversely affect costs and prices. Non-value added activities create waste, result in delay of some sort, add costs to the products or services and for which the customer is not willing to pay. Moving materials and machine set up for a production run are examples of NVA activities. The preparation of tax returns and other compliance work by organisations do not directly benefit the customers of their products and services, but because they are required by law. They are not considered NVA activities.

By measuring activities rather than traditional departmental costs, business can focus on cross functional processes in order to identify non-value adding activities and pinpoint the time drives of cost at each stage. The goal of the ABCM is to make customer needs to be satisfied while making fewer demands for resources. Current research suggests that customers have perceived needs in four areas, all of which must be satisfied simultaneously.

The customers require- Lower costs, Higher quality, Faster response time&Greater innovation.

To satisfy these needs ABM currently being used for a variety of business applications.Such as:

- **Cost Reduction:** ABCM helps the organisation to identify costs against activities and to find opportunities to streamline or reduce the costs or eliminate the entire activity, especially if there is no value added. It is particularly useful in identifying and quantifying process waste and providing vehicle for continuous process improvement through continuous cost reduction.
- **Activity Based Budgeting:** Activity based budgeting analyse the resource input or cost for each activity.It provides a framework for estimating the amount of resources required in accordance with the budgeted level of activity. Actual results can be compared with budgeted results to highlight both in financial and non-financial terms those activities with major discrepancies from budget for potential reduction in supply of resources. It is a planning and control system which seeks to support the objectives of continuous improvement. It means planning and controlling the expected activities of the organization to derive a cost-effective budget that meet forecast workload and agreed strategic goals. The three key elements of activity based budgeting are as follows:-

  — Type of work to be done

  — Quantity of work to be done

  — Cost of work to be done
- **Business Process Re-engineering:** Business process re-engineering involves examining business processes and making substantial changes to how organisationcurrently operates. ABCM is a powerful tool for measuring business performance, determining the cost of business output and is used as a means of identifying opportunities to improve process efficiency and effectiveness. A business process consists of linked set of activities. For example purchasing of materials might be considered as business process consist of activities such as receiving a purchase request, identifying supplies, preparing purchase orders, mailing purchase orders and performing follow up. One way the process might be reengineered by sending the production schedule direct to the suppliers and to enter into a contractual agreement to deliver materials according to the production schedule. The end result might be permanent reduction or elimination of some activities like raising a requisition every

time there is a need for materials, identifying potential suppliers each time, waiting for their bid which may result in a delay of the production process and thereby hamper the organisation's goals

- **Benchmarking:**Benchmarking is a process of comparing of ABC-derived activity costs of one segment of company with those of other segments. It requires uniformity in the definition of activities and measurement of their costs.
- **Performance Measurement:**Manyorganisations are now focusing on activity performance as a means of facing competitors and managing costs by monitoring the efficiency and effectiveness of activities. Activity performance measures consist of measures relating to costs, time, quality and innovation. For instance, in the current era of globalisation, the overall goal for any company is to produce a quality product at a competitive price. But the quality is not something which one can apply somewhere in the production process or assume will happen automatically. Product quality starts with the correct design. The next stages are high quality raw material inputs, quality processing and work, and proper handling and packaging etc. The various performance measures of quality are:

  Area Measures

Quality of purchased component	$\longrightarrow$	zero defects
Quality of output	$\longrightarrow$	% yield
Customer awareness	$\longrightarrow$	orders; number of complaints

1.3.10 Benefits of Activity Based Cost Management

- Provision of excellent basis and focus for cost reduction.
- Provides operational management with a clear view of HOW to implement an ActivityBased budget.
- Provision of clear understanding of the underlying causes of business processingcosts.
- Provision of excellent basis for effectiveness of management decision making.
- Identification of key process waste elements, permit management prioritisation and leverage of key resources.

1.3.11 Difference between ABC and ABM

The ABC refers to the technique for determining the cost of activities and the output that those activities produce. It is the logical distribution of overhead i.e. overhead should be distributed on the consumption of resources consumed by goods and services. The aim of ABC is to generate improved cost data for use in managing a company's activities.

The ABM is a much broader concept.It refers to the management philosophy that focuses on the planning, execution and measurement of activities as the key to competitive advantage.

1.3.12 Activity Based Budgeting (ABB)

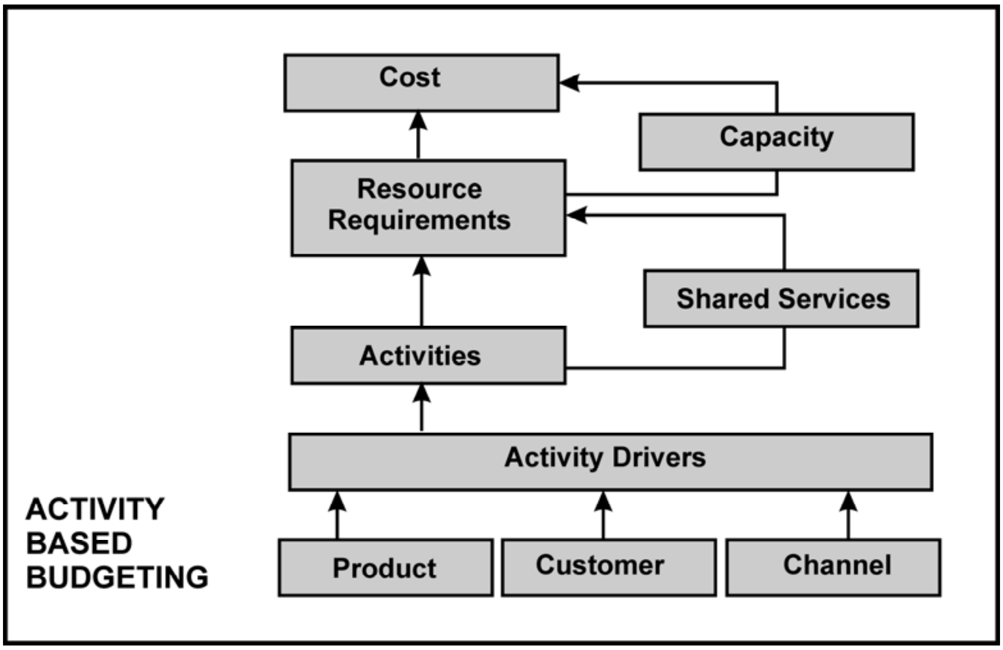
Activity-based budgeting is a process of planning and controlling the expected activities for the organisation to derive a cost-effective budget that meets forecast workload and agreed strategic goals. An activity-based budget is a quantitative expression of the expected activities of the firm, reflecting management's forecast of workload and financial and non-financial requirements to meet agreed strategic goals and planned changes to improve performance.

Thus, the key elements of ABB are:

- Type of work/activity to be performed;
- Quantity of work/activity to be performed; and
- Cost of work/activity to be performed.

ABB focuses on the activity/business processes. Resources required are determined on the expected activities and workload. The objective is to bring in efficiency into the system. So, in the process of budget preparation, many key questions need to be addressed and properly answered

Activity Based Budgeting (ABB) is a technique for enhancing the accuracy of financial forecasts and increasing management understanding. When automated, ABB can rapidly and accurately produce financial plans and models based on varying levels of volume assumptions. Also, ABB eliminates much of the needless rework created by traditional budgeting techniques. ABB analyzes the products or services to be produced, what activities are required to produce those products or services, and finally what resources need to be budgeted to perform those activities. Simply said, ABB is the reversing of the ABC process to produce financial plans and budgets.



1.3.13 Concept of 'ABC' followed by 'Coca Cola Enterprises Belgium'

Coca Cola Enterprises Belgium (CCEB) produces, distributes and sells the different brands of 'The Coca-Cola Company'. In the field of distribution, it is also CCEB who services the entire Belgian and Luxembourg market. CCEB is present in more than 85,000 points of sale (supermarkets, grocery stores, companies, hospitals, cinemas, amusement parks, sport center). By the end of 2009, CCEB employed more than 2,500 employees in Belgium and Luxembourg with revenue of €1.1 billion.

Like many other companies, Coca-Cola Enterprises Belgium was confronted with an increasing Cost to Serve (CTS) due to a changing customer landscape. This created a challenge to which CCEB needed to formulate decisive answers to stay on track towards achieving their growth path and their company objectives.

When companies are confronted with increasing CTS, it is essential to analyse the organisation, its revenues & costs and its processes down to the most detailed level of information. It is this data that give the true reasons behind certain evolutions so that management can take fact-based decisions. When having such a challenge at hand, Activity Based Costing is the most advanced and complete method to gain this information. Thus, CCEB wanted to use the information from ABC analysis to formulate:

— Cost/Profit modeling

— Performance Modeling and

— Set-up an internal recharge mechanism to sales

By implementing Activity-Based Costing, CCEB obtained the right information that enabled them to harmonise and streamline the processes of their different distribution centers.

This made it possible for CCEB to calculate the costs in a fair and transparent way, so that the sales force is charged correctly according to the complexity that Supply Chain had to deal with. From the capacity insights that CCEB got from Activity-Based Costing, multiple initiatives were derived that ultimately led to the redesign of the regional distribution strategy, including:

— Optimizing efficiency and capacity within the logistic department

— Designing the most efficient processes, based on the time equations from ABC Analysis

— Implementing "best practice" processes in the CCEB distribution centers

These actions successfully reduce their Cost to Serve and still be in line with the new corporate S&M-strategy.

1.3.14 Illustrations

Illustration1

A company produces three products A, B and C for which the standard costs and quantities per unit are as follows:

Products	***A***	***B***	***C***
Quantity produced	*10,000*	*20,000*	*30,000*
Direct material/p.u. (₹)	*50*	*40*	*30*
Direct labour/p.u. (₹)	*30*	*40*	*50*
Labour hours/p.u.	*3*	*4*	*5*
Machine hours/p.u.	*4*	*4*	*7*
No. of purchase requisitions	*1,200*	*1,800*	*2,000*
No. of set ups	*240*	*260*	*300*

Production overhead split by departments— Department 1 = ₹11,00,000

— Department 2 = ₹15,00,000

Department 1 is labour intensive and Department 2 is machine intensive

Total labour hours in Department 1 = 1,83,333

Total machine hours in Department 2 = 5,00,000

Production overhead split by activity	*— Receiving/inspecting*	*₹ 14,00,000*
	— Production scheduling/machine set up	*₹ 12,00,000*
		₹26,00,000

Number of batches received/inspected = 5,000

Number of batches for scheduling and set-up = 800

You are required to:

(i) Prepare Product Cost Statement under traditional absorption costing and Activity Based Costing method.

(ii) Compare the results under two methods.

Solution

Traditional absorption costing

Absorption rates:

$$\text{Department 1} = \frac{₹\ 11,00,000}{1,83,333 \text{ labour hours}} = ₹6 \text{ per labour hour}$$

$$\text{Department 2} = \frac{₹\ 15,00,000}{5,00,000 \text{ machine hours}} = ₹3 \text{ per machine hour}$$

Product cost statement

Products	**A** **(₹)**	**B** **(₹)**	**C** **(₹)**
Direct materials	50	40	30
Direct labour	30	40	50
Overhead :			
Department 1	18 *(3 hrs × ₹ 6)*	24 *(4 hrs × ₹ 6)*	30 *(5 hrs × ₹ 6)*
Department 2	12 *(4 hrs × ₹ 3)*	12 *(4 hrs × ₹ 3)*	21 *(7 hrs x ₹ 3)*
Total cost p.u.	110	116	131

Activity based costing

Cost driver rates:

$$\text{Receiving/inspecting} = \frac{₹\ 14,00,000}{5,000 \text{ (No. of batches received/inspected)}}$$

$$= ₹\ 280 \text{ per requisition}$$

$$\text{Production scheduling/Machine set ups} = \frac{₹\ 12,00,000}{800 \text{ (No. of setups)}} = ₹\ 1,500 \text{ per set up}$$

Product cost statement (per unit)

Products	A	B	C
	(₹)	(₹)	(₹)
Direct materials (₹)	50	40	30
Direct labour (₹)	30	40	50
Overhead:			
Receiving	34*	25	19
Production scheduling	36**	20	15
Total cost per unit	150	125	114

{₹280 × 1,200} ÷ 10,000 units = ₹34*; similarly ₹25 and ₹ 19
(₹1,500 × 240) ÷ 10,000 units = ₹36**; similarly ₹20 and ₹15

	A	B	C
Traditional System	110.00	116.00	131.00
ABC	150.00	125.00	114.00
Difference	(40.00)	(9.00)	17.00

The two absorption methods produce different results. Product C appears to be much more expensive using the traditional method than it does with ABC, while product A is the opposite. If it is assumed that ABC is more accurate, which it may or may not be, then product C would be overpriced on the traditional method and sales would presumably be poor as a consequence–assuming competitors supply more cheaply. Product A would be the opposite: sales would be high and it is possible that the company would unknowingly make a loss per unit on product A.

[Note: Calculation rounded to nearest rupee]

Illustration2

The following information provides details of costs, volume & cost drivers for a particular period in respect of ABC Ltd. for product X, Y and Z

		Product X	***Product Y***	***Product Z***	***Total***
1.	*Production and sales (units)*	*30,000*	*20,000*	*8,000*	
2.	*Raw material usage (units)*	*5*	*5*	*11*	
3.	*Direct material cost (₹)*	*25*	*20*	*11*	*12,38,000*
4.	*Direct labour hours*	*1.33…*	*2*	*1*	*88,000*
5.	*Machine hours*	*1.33…*	*1*	*2*	*76,000*
6.	*Direct labour cost (₹)*	*8*	*12*	*6*	
7.	*Number of production runs*	*3*	*7*	*20*	*30*
8.	*Number of deliveries*	*9*	*3*	*20*	*32*
9.	*Number of receipts (2 × 7)**	*15*	*35*	*220*	*270*
10.	*Number of production orders*	*15*	*10*	*25*	*50*

**The company operates a just-in-time inventory policy, and receives each component once per production run.*

Overhead costs:	₹	
Set-up	*30,000*	
Machines	*7,60,000*	
Receiving	*4,35,000*	
Packing	*2,50,000*	
Engineering	*3,73,000*	
		₹18,48,000

In the past the company has allocated overheads to products on the basis of direct labour hours. However, the majority of overheads are related to machine hours rather than direct labour hours.

The company has recently redesigned its cost system by recovering overheads using two volume related bases: machine hours and a materials handling overhead rate for recovering overheads of the receiving department. Both the current and the previous cost system reported low profit margins for product X, which is the company's highest-selling product. The management accountant has recently attended a conference on activity-based costing, and the overhead costs for the last period have been analysed by the major activities in order to compute activity-based costs.

From the above information you are required to:

(a) Compute the product costs using a traditional volume-related costing system based on the assumption that:

(i) All overheads are recovered on the basis of direct labour hours (i.e. the company's past product costing system);

(ii) The overheads of the receiving department are recovered by a materials handling overhead rate and the remaining overheads are recovered using a machine hour rate (i.e. the company current costing system).

(b) Compute product costs using an activity-based costing system.

Solution

(a) Computation of the product cost using a traditional volume related costing system based on assumption that:

(i) All overheads are recovered on the basis of direct labour hours (i.e. the company product costing system)

Statement showing the product cost

Products	**X**	**Y**	**Z**
	(₹)	**(₹)**	**(₹)**
Direct labour	8	12	6

Direct materials	25	20	11
Overhead	28	42	21
(*Refer to working note*)	*(1.33 hrs.× ₹ 21)*	*(2 hrs. × ₹ 21)*	*(1 hrs. × ₹ 21)*
Total	61	74	38

Working note :

Overheads to be charged to products

$$\text{Direct labour overhead rate} = \frac{\text{Total OH}}{\text{Total DLH}}$$

=₹21 per direct labour hour.

(ii) The overheads of the receiving department are recovered by a material handling overhead rate & the remaining overheads are recovered by using a machine hour rate (i.e. the company current costing system)

Products	**X (₹)**	**Y (₹)**	**Z (₹)**
Direct labour	8	12	6
Direct materials	25	20	11
Material handling overhead	8.78	7.03	3.87
(Refer to working note)	(₹ 25 × 35.14%)	(₹ 20 × 35.14%)	(₹ 11×35.14%)
Other overheads	24.79	18.59	37.18
(machine hour basis) (Refer to working note)	(1.33 hrs x ₹ 18.59)	(1.00 hrs x ₹ 18.59)	(2.00 hrs x ₹ 18.59)
Total	66.57	57.62	58.05

Overheads to be charged to products:

$$\text{Material handling overhead rate} = \frac{\text{Reveiving Dept. OH}}{\text{Direct material cost}} = \frac{₹\ 4{,}35{,}000}{₹\ 12{,}38{,}000} \times 100$$

$$= 35.14\% \text{ of direct material cost}$$

$$\text{Machine hour overhead rate} = \frac{\text{Other OH}}{\text{Machine Hrs.}} = \frac{₹\ 14{,}13{,}000}{76{,}000 \text{ hrs.}}$$

$$= 18.59 \text{ per machine hours}$$

(b) Statement showing the product costs using an activity based costing system.

Products	**X**	**Y**	**Z**
	(₹)	**(₹)**	**(₹)**
Direct labour	8	12	6

Direct materials	25	20	11
Machine overheads	13.33	10	20
(Refer to working note1)	(1.33 hrs× ₹ 10)	(1 hrs× ₹ 10)	(2 hrs×₹ 10)
Set-up costs	0.10	0.35	2.50
(Refer to working note 2(i))	$\left(\frac{₹\,1{,}000 \times 3 \text{ runs}}{30{,}000 \text{ units}}\right)$	$\left(\frac{₹\,1{,}000 \times 7 \text{ runs}}{20{,}000 \text{ units}}\right)$	$\left(\frac{₹\,1{,}000 \times 20 \text{ runs}}{8{,}000 \text{ units}}\right)$
Receiving	0.81	2.82	44.30
(Refer to working note 2(ii))	$\left(\frac{₹\,1{,}611 \times 15 \text{ receipts}}{30{,}000 \text{ units}}\right)$	$\left(\frac{₹\,1{,}611 \times 35 \text{ receipts}}{20{,}000 \text{ units}}\right)$	$\left(\frac{₹\,1{,}611 \times 220 \text{ receipts}}{8{,}000 \text{ units}}\right)$
Packing	2.34	1.17	19.53
(Refer to working note 2(iii))	$\left(\frac{₹\,7{,}812 \times 9 \text{ deliveries}}{30{,}000 \text{ units}}\right)$	$\left(\frac{₹\,7{,}812 \times 3 \text{ deliveries}}{20{,}000 \text{ units}}\right)$	$\left(\frac{₹\,7{,}812 \times 20 \text{ deliveries}}{8{,}000 \text{ units}}\right)$
Engineering	3.73	3.73	23.31
(Refer to working note 2(iv))	$\left(\frac{₹\,7{,}460 \times 15 \text{ orders}}{30{,}000 \text{ units}}\right)$	$\left(\frac{₹\,7{,}460 \times 10 \text{ orders}}{20{,}000 \text{ units}}\right)$	$\left(\frac{₹\,7{,}460 \times 25 \text{ orders}}{8{,}000 \text{ units}}\right)$
Total manufacturing cost	53.31	50.07	126.64

Working note:

1. Machine overhead rate per hour $= \frac{₹\,7{,}60{,}000}{76{,}000 \text{ hrs.}} = ₹10$

2. The cost per transaction or activity for each of the cost centres is as follows:

   (i) *Set-up cost*

   Cost per setup $= \frac{₹\,30{,}000}{30} = ₹1{,}000$

   (ii) *Receiving Cost*

   Cost per receiving order $= \frac{₹\,4{,}35{,}000}{270} = ₹1{,}611$

   (iii) *Packing Cost*

   Cost per packing order $= \frac{₹\,2{,}50{,}000}{32} = ₹7{,}812$

(iv) *Engineering*

Cost per production order $= \dfrac{₹\ 3{,}73{,}000}{50} = ₹7{,}460$

Illustration3

You have been appointed as a management consultant by XYZ ltd – a key manufacturer of machining tools. You need to analyse how application of activity-based costing (ABC) to costing of the company's product lines would improve product costing and help it price its product offerings in a more efficient manner.

Details of the four products and relevant information are given below for one period:

Product	***P***	***Q***	***R***	***S***
Output in units	*150*	*120*	*60*	*90*
Costs per unit	*₹*	*₹*	*₹*	*₹*
Direct material	*50*	*60*	*40*	*80*
Direct labour	*32*	*24*	*18*	*20*
Machine hours (per unit)	*5*	*4*	*3*	*2*

The four products are similar and are usually produced in production runs of 15 units and sold in batches of 10 units.

The production overhead is currently absorbed by using a machine hour rate, and the total of the production over head has been analysed as follows:

	₹
Machine department costs (rent, Business, rates, depreciation and Supervision)	*18,960*
Set-up costs	*5,600*
Stores receiving	*4,000*
Inspection/quality control	*1,620*
Material handling and dispatch	*7,980*

You have identified 'cost drivers' to be used are as listed below for the overhead costs shown:

Cost	***Cost Driver***
Set-up costs	*Number of production runs*
Stores receiving	*Requisitions raised*
Inspection/quality control	*Number of production runs*
Materials handling and dispatch	*Orders executed*

The number of requisitions raised on the stores was 20 for each product and the number of orders executed was 42, each order being for a batch of 10 of a product.

Requirements

(a) Calculate the total costs for each product if all overhead costs are absorbed on a machinehour basis.

(b) Calculate the total cost of each product, using activity-based costing.

(c) Compare the two costs under the two scenarios and identify the implications this could have on pricing and profit.

Solution

(a) We first calculate the Overhead Recovery Rate on machine hour basis

$$\text{Machine hour absorption rate} = \frac{\text{Total Overheads}}{\text{Total machine hours}}$$

Total Overheads = 18,960+5,600+4,000+1,620+7,980 = ₹ 38,160

Total Machine Hours

Units		Machine hours (per unit)	Hrs
P	150	5	750
Q	120	4	480
R	60	3	180
S	90	2	180
			1,590

Thus Overhead Recovery Rate = ₹ 38,160/1,590 hrs. = ₹ 24 per machine hour

Total costs based on machine hour basis

	P (₹)	Q (₹)	R (₹)	S (₹)
Direct material	50.00	60.00	40.00	80.00
Direct labour	32.00	24.00	18.00	20.00
Production overhead	120.00	96.00	72.00	48.00
Production cost/unit	202.00	180.00	130.00	148.00
Out put in units	150	120	60	90
Total production cost	₹ 30,300	₹ 21,600	₹ 7,800	₹ 13,320

(b) Overheads absorbed based on ABC

Overhead costs	Level of activity	Activity Driver	Cost/activity (₹)
Machine department costs	18,960	1,590	11.92/hour
Set-up costs	5,600	28*	200/run
Stores receiving costs	4,000	80**	50/requisition
Inspection/quality costs	1,620	28*	57.86/run
Material handling and dispatch	7,980	42	190 /order

Workings

*No. of production runs = output in units/15

(150+120+60+90)/15 =420/15

=28

**No. of requisitions raised = No. of products × 20 = 4 × 20 = 80

	P (₹)	Q (₹)	R (₹)	S (₹)
Direct materials	50.00	60.00	40.00	80.00
Direct labour	32.00	24.00	18.00	20.00
Machine dept costs [Note (a)]	59.60	47.68	35.76	23.84
Set-up costs [Note (b)]	13.33	13.33	13.33	13.33
Stores receiving [Note (c)]	6.67	8.33	16.67	11.11
Inspection [Note (d)]	3.86	3.86	3.86	3.86
Material handling [Note (e)]	19.00	19.00	19.00	19.00
Production cost/unit	184.46	176.20	146.62	171.14
Output in units	150	120	60	90
Total production costs	27,669	21,144	8,797	15,403

	P	Q	R	S
Note (a)	(5 hrs × ₹11.92)	(4 hrs × ₹11.92)	(3 hrs × ₹11.92)	(2 hrs × ₹11.92)
Note (b)	(₹200 / 15 units)	(₹200 / 15 units)	(₹200 / 15 units)	(₹200 / 150 units)
Note (c)	$\left(\frac{₹\ 50 \times 20 \text{ units}}{150 \text{ units}}\right)$	$\left(\frac{₹\ 50 \times 20 \text{ units}}{120 \text{ units}}\right)$	$\left(\frac{₹\ 50 \times 20 \text{ units}}{60 \text{ units}}\right)$	$\left(\frac{₹\ 50 \times 20 \text{ units}}{90 \text{ units}}\right)$
Note (d)	(₹57.86/15units)	(₹57.86/15 units)	(₹57.86/15 units)	(₹57.86/15 units)
Note (e)	(₹190/10 units)	(₹190/10 units)	(₹190/10 units)	(₹190/10 units)

(c) Comparison of the two unit costs calculated in (a) and (b) above.

Product	P (₹)	Q (₹)	R (₹)	S (₹)
Based on machine				
Hour rate	202.00	180.00	130.00	148.00
ABC method	184.46	176.20	146.62	171.14
Difference	17.54	3.80	(16.62)	(23.14)

Thus we find that there is a substantial difference in the product cost under the traditional and ABC methods. If the company were to apply a constant margin to cost price in order to determine the selling price, we find that P and Q would be priced higher than the ABC determined rate and R and S would be underpriced.

Illustration 4

A manufacturing company produces Bail Pens that are printed with the logos of various companies. Each Pen is priced at ₹ 5. Costs are as follows:

Cost Driver	***Unit Variable Cost (₹)***	***Level of Cost Driver***
Units Sold	*2.5*	*—*
Setups	*225*	*40*
Engineering hours	*10*	*250*

Other Data:

Total Fixed Costs (conventional) *₹ 48,000*

Total Fixed Costs (ABC) *₹ 36,500*

Required:

1. *Compute the break-even point in units using activity-based analysis.*
2. *Suppose that company could reduce the setup cost by ₹ 75 per setup and could reduce the number of engineering hours needed to 215. How many units must be sold to break even in this cost?*

Solution

1. Break Even Units:

[Fixed Costs + (Setup Cost × Setups) + (Engineering Cost × Engineering Hours)]/(Sale Price – Variable Cost)]

= [36,500 + (₹ 225 × 40) + (₹ 10 × 250)] /(₹5 – ₹2.5)

= 19,200 units

2. = [36,500 + (₹150 × 40) + (₹10 × 215)]/(₹5 – ₹2.5)

= 17,860 units

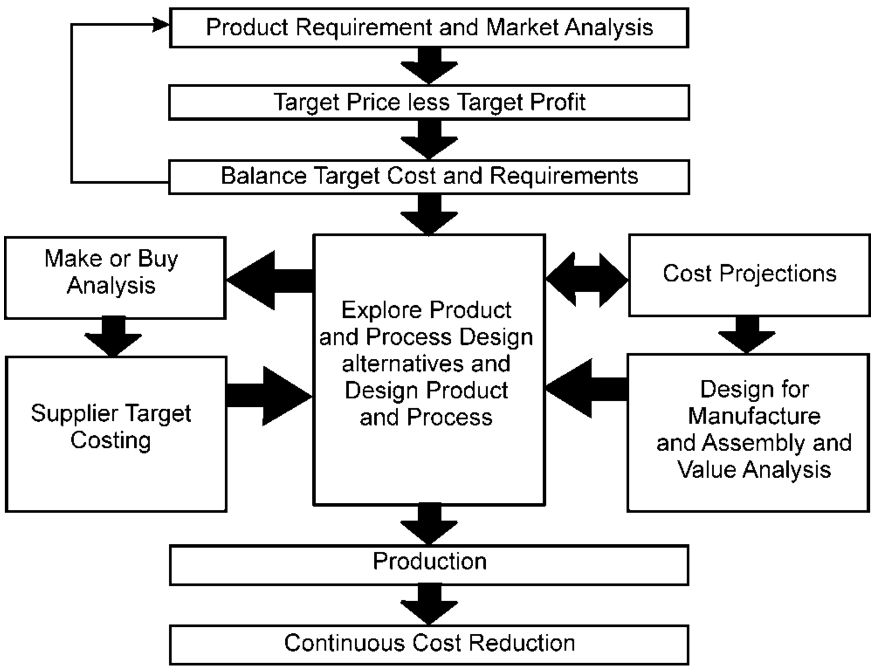
1.4 Target Costing

1.4.1 Target costing has been described as a process that occurs in a competitive environment, in which cost minimization is an important component of profitability. This newer approach of product costing may take into account initial design and engineering costs, as well as manufacturing costs, plus the costs of distribution, sales and services.

It can be defined as “a structured approach to determining the cost at which a proposed product with specified functionality and quality must be produced, to generate a desired level of profitability at its anticipated selling price”.

A critical aspect of this definition is that it emphasizes that target costing is much more than a management accounting technique. Rather, it is an important part of a comprehensive management process aimed at helping an organization to survive in an increasingly competitive environment. In this sense the term “target costing” is a misnomer: it is not a product costing system, but rather a management technique aimed at reducing a product's life-cycle costs.

Target Cost Concept



Target costing is almost the exact opposite of cost plus margin modeling where a company produces a product with no cost structure in mind.Once the product is built they add a profit margin on top to arrive at the final price.In Target costing, we first determine what price we think the consumer will pay for our product.We then determine how much of a profit margin we expect and subtract that from the final price.The remaining amount left is what is available as a budget to be used to create the product.

1.4.2 Advantages of Target Costing

- Proactive approach to cost management
- It reinforces top-to-bottom commitment to process and product innovation, and is aimed at identifying issues to be resolved, in order to achieve some competitive advantage.
- Target costing starts with customer's study or market study. It helps to create a company's competitive future with market-driven management for designing and manufacturing products that meet the price required for market success.
- It uses management control systems to support and reinforce manufacturing strategies; and to identify market opportunities that can be converted into real savings to achieve the best value rather than simply the lowest cost.

- Target costing ensures proper planning well ahead of actual production and marketing.
- Implementation of Target Costing enhances employee awareness and empowerment.
- Foster partnership with suppliers
- Minimize non value-added activities.
- Encourages selection of lowest cost value added activities
- Reduced time to market
- Target Costing takes a market – driven approach towards cost, in which value is defined not only by what customers demand but also by what they are willing to pay for. This strategy introduces a discipline in which planning focus shifts to those costs that create value and meet the needs of the customer.By involving and educating customers, target costing provides a process that allows teams to make intelligent trade-offs between features, functionality and cost, resulting in designs that are better suited to customers' quality and price expectations.

1.4.3 Main features of Target Costing System

The main features or practices followed in different Japanese companies can be understood by going through the following points:

- Target costing is viewed as an integral part of the design and introduction of new products. As such, it is part of an overall profit management process, rather than simply a tool for cost reduction and cost management. The first part of the process is driven by customer, market and profitability considerations. Given that profitability is critical for survival, a target profit margin is established for all new product offerings. The target profit margin is derived from the company's long-term business plan, which incorporates its long-term strategic intent and profit margins. Each product or product line is required to earn at least the target profit margin.
- For any given product, a target selling price is determined using various sales forecasting techniques. Critical to setting the target selling price are the design specifications (reflecting certain levels of functionality and quality) of the new product. These specifications are based on customer requirements and expectations and are often influenced by the offerings of competitors. Importantly, while setting the target selling price, competitive conditions and customers' demands for increased functionality and higher quality, without significant increases in price, are clearly recognised, as charging a price premium may not be sustainable. Hence, the target selling price is market-driven and should encompass a realistic reflection of the competitive environment.
- Integral to setting the target selling price is the establishment of target production volumes, given the relationship between price and volume. The expected targets volumes are also critical to computing unit costs, especially with respect to capacity-

related costs (such as tooling costs), as product costs are dependent upon the production levels over the life cycle of the product. Once the target selling price and required profit margin have been determined, the difference between these two figures indicates the allowable cost for the product. Ideally, the allowable cost becomes the target cost for the product. However, in many cases the target cost agreed upon will exceed the allowable cost, given the realities associated with existing capacities and capabilities.

- Establishing Cost Reduction Targets: The next stage of the target costing process is to determine cost reduction targets. Some firms will do this by estimating the "current cost" of the new product. The current cost is based on existing technologies and components, but encompasses the functionalities and quality requirements of the new product. The difference between the current cost and the target cost indicates the required cost reduction that is needed. This amount may be divided into a target cost-reduction objective and a strategic cost-reduction challenge. The former is viewed as being achievable (yet still a very challenging target), while the latter acknowledges current inherent limitations. After analysing the cost reduction objective, a product-level target cost is set which is the difference between the current cost and the target cost-reduction objective.
- It should be noted that a fair degree of judgement is needed where the allowable cost and the target cost differ. As the ideal is to produce at the allowable cost, it is important that the difference is not too great. Once the product-level target cost is set, however, it generally cannot be changed, and the challenge for those involved is to meet this target.
- Having achieved consensus about the product-level target cost, a series of intense activities commence to translate the cost challenge into reality. These activities continue throughout the design stage up until the point when the new product goes into production.

1.4.4 Components of Target Costing System

Typically, the total target is broken down into its various components, each component is studied and opportunities for cost reductions are identified. These activities are often referred to as **value engineering (VE)** and **value analysis (VA)**. Value engineering involves searching for opportunities to modify the design of each component or part of a product to reduce cost, but without reducing functionality or quality of the product. Value analysis entails studying the activities that are involved in producing the product to detect non-value-adding activities that may be eliminated or minimized to save costs, but without reducing the functionality or quality of the product. Where components are sourced from suppliers (which is often the case in the automotive industry), target prices are established for each part and the company's employees work with the suppliers to ensure that the targets are achieved. Overall, the aim of the process is to ensure that when production commences, the total cost will meet the target, and profit goals will be achieved. There is also an ongoing continuous improvement program, known as **kaizen costing**, that focuses on the reduction of waste in the production process, thereby further lowering costs below the initial targets specified during the design phase.

While the above description captures the essential features of the target costing process, it should be emphasized that successful, target costing requires careful planning, attention to detail and a strong degree of commitment from those involved. The description, however, does not provide any insights into what is entailed in implementing a target costing approach in an organization.

Here are some of the issues that are dealt with during a value engineering review:

- **Can we eliminate functions from the production process?**

  This involves a detailed review of the entire manufacturing process and determine the non- value added activities. By eliminating them, one can take their associated direct or overhead costs out of the product cost. However, these functions were originally put in for a reason, so the engineering team must be careful to develop work-around steps that eliminate one or more activities from the original set of functions and be sure enough that eliminating these activities will not hamper the value added activities in any manner.

- **Can we eliminate some durability or reliability?**

  It is possible to design an excessive degree of sturdiness into a product. For example, a vacuum cleaner can be designed to withstand a 1-ton impact, although there is only the most vanishing chance that such an impact will ever occur; designing it to withstand an impact of 100 pounds may account for 99.999% of all probable impacts, while also eliminating a great deal of structural material from the design. However, this concept can be taken too far, resulting in a visible reduction in durability or reliability, so any designs that have had their structural integrity reduced must be thoroughly tested to ensure that they meet all design standards.

- **Can we minimize the design?**

  This involves the creation of a design that uses fewer parts or has fewer features. This approach is based on the assumption that a minimal design is easier to manufacture and assemble. Also, with fewer parts to purchase, less procurement overhead is associated with the product. However, reducing a product to extremes, perhaps from dozens of components to just a few molded or prefabricated parts, can result in excessively high costs for these few remaining parts, since they may be so complex or custom made in nature that it would be less expensive to settle for a few extra standard parts that are more easily and cheaply obtained. Also a proper trade-off between price and quality is necessary in this context.

- **Can we design the product better for the manufacturing process?**

  Also known as design for manufacture and assembly (DFMA), this involves the creation of a product design that can be created in only a specific manner. For example, a toner cartridge for a laser printer is designed so that it can be successfully inserted into the printer only when the sides of the cartridge are correctly aligned with the printer opening; all other attempts to insert the cartridge will fail. When used for the assembly of an entire product, this approach ensures that a product is not incorrectly manufactured or assembled, which would call for a costly disassembly or (even worse) product recalls from customers who have already received defective goods.

- **Can we substitute parts?**

  This approach encourages the search for less expensive components or materials that can replace more expensive parts currently used in a product design. It is becoming an increasingly valid approach since new materials are being developed every year. However, sometimes the use of a different material impacts the types of materials that can be used elsewhere in the product, which may result in cost increases in these other areas, for a net increase in costs. Thus, any parts substitution must be accompanied by a review of related changes elsewhere in the design. This step is also known as component parts analysis and involves one extra activity—tracking the intentions of suppliers to continue producing parts in the future; if parts will not be available, they must be eliminated from the product design.

- **Can we combine steps?**

  A detailed review of all the processes associated with a product sometimes reveals that some steps can be consolidated, which may mean that one can be eliminated (as noted earlier) or that several can be accomplished by one person, rather than having people in widely disparate parts of the production process perform them. This is also known as process centering. By combining steps in this manner, we can eliminate some of the transfer and queue time from the production process, which in turn reduces the chance that parts will be damaged during these transfers.

- **Can we take supplier's assistance?**

  Another approach to value engineering is to call on the services of a company's suppliers to assist in the cost reduction effort. These organizations are particularly suited to contribute information concerning enhanced types of technology of materials, since they may specialize in areas that a company has no information about. They may have also conducted extensive value engineering for the components they manufacture, resulting in advanced designs that a company may be able to incorporate into its new products. Suppliers may have also redesigned their production processes, or can be assisted by a company's engineers in doing so, producing cost reductions or decreased production waste that can be translated into lower component costs for the company.

- **Is there a better way?**

  Though this step sounds rather vague, it really strikes at the core of the cost reduction issue—the other value engineering steps previously mentioned focus on incremental improvements to the existing design or production process, whereas this one is a more general attempt to start from scratch and build a new product or process that is not based in any way on preexisting ideas. Improvements resulting from this step lend to have the largest favourable impact on cost reductions but can also be the most difficult for the organization to adopt, especially if it has used other designs or systems for the production of earlier models.

A mix of all the value engineering steps noted above must be applied to each product design to ensure that the maximum permissible cost is safely reached. Also, even if a minimal amount of value engineering is needed to reach a cost goal, one should conduct the full range of value engineering analysis anyway, since this can result in further cost reductions that improve the margin of the product or allow management the option of reducing the product's price, thereby creating a problem for competitors who sell higher-priced products.

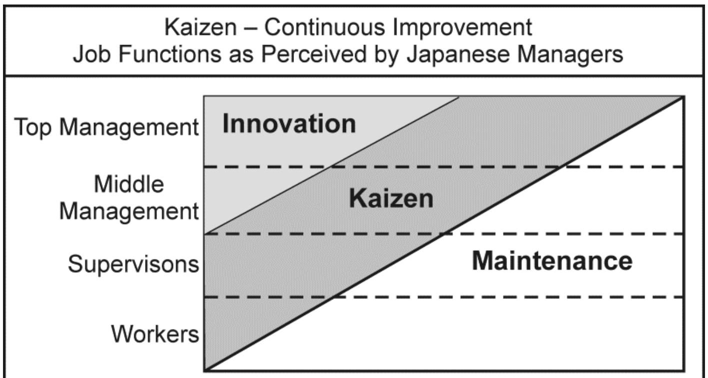
Kaizen Costing

CIMA defines *"Kaizen as Japanese term for continuous improvement in all aspects of an entity's performance at every level. See continuous improvement"*.

Kaizen Costing is a Japanese term for a number of cost reduction steps that can be used subsequent to issuing a new product design to the factory floor. Some of the activities in the kaizen costing methodology include the elimination of waste in the production, assembly, and distribution processes, as well as the elimination of work steps in any of these areas. Though these points are also covered in the value engineering phase of target costing, the initial value engineering may not uncover all possible cost savings. Thus, kaizen costing is really designed to repeat many of the value engineering steps for as long as a product is produced, constantly refining the process and thereby stripping out extra costs. The cost reductions resulting from kaizen costing are much smaller than those achieved with value engineering but are still worth the effort since competitive pressures are likely to force down the price of a product over time, and any possible cost savings allow a company to still attain its targeted profit margins while continuing to reduce cost.

The type of cost reduction program used for target costing has an impact on the extent of cost reduction, as well as on the nature of the components used in a product. When a design team elects to set cost reduction goals by allocating specific cost reduction amounts to major components of an existing product, it tends to focus on finding ways to make incremental cost reductions rather than focusing on entirely new product configurations that might both radically alter the product's design and lower its cost. This approach is most commonly used during the redesign of products already in the market. Another cost reduction approach is to allocate cost reductions based on the presence of certain product features in a product design. This method focuses the attention of the design team away from using the same components that were used in the past, which tends to produce more radical design changes that yield greater cost savings. However, the latter approach is also a riskier one, since the resulting product concepts may not work, and also requires so much extra design work that the new design may not be completed for a long time. Therefore, the second method is generally reserved for situations where a company is trying to create products at a radically lower cost than previously.

All the changes noted in this section that are necessary for the implementation and use of the target costing methodology represent a massive change in mind-set for the product design personnel of any company because they require the constant cooperation of many departments and rapid, voluminous communications between them, not to mention heightened levels of trust in dealing with suppliers. All these concepts run counter to the traditional approach.



Innovation	Drastic Improvements in Current Processes
Kaizen	Small Continuous Improvements in Current Process
Maintenance	Activities Directed to Maintaining Current Technological, Managerial, & Operating Standards

It is no coincidence that the traditional design process defines each of the departments that take part in the process. These departments tend to guard their turf jealously, which is a major impediment to realizing a smoothly functioning set of product development teams. Only the most active support from senior management can enforce the new approach of drawing product design team members from all these castles and having them work together amicably.

1.4.5 Problems with Target Costing

Though the target costing system results in clear, substantial benefits in most cases, it has a few problems that one should be aware of and guard against. These problems are as follows:

- The development process can be lengthened to a considerable extent since the design team may require a number of design iterations before it can devise a sufficiently low-cost product that meets the target cost and margin criteria. This occurrence is most common when the project manager is unwilling to "pull the plug" on a design project that cannot meet its costing goals within a reasonable time frame. Usually, if there is no evidence of rapid progress toward a specific target cost within a relatively short period of time, it is better to either ditch a project or at least shelve it for a short time and then try again, on the assumption that new cost reduction methods or less expensive materials will be available in the near future that will make the target cost an achievable one.
- A large amount of mandatory cost cutting can result in finger-pointing in various parts of the company; especially if employees in one area feel they are being called on to provide a disproportionately large part of the savings. For example, the industrial engineering staff will not be happy if it is required to completely alter the production

layout in order to generate cost savings, while the purchase staff is not required to make any cost reductions through supplier negotiations. Avoiding this problem requires strong interpersonal and negotiation skills on the part of the project manager.

- Representatives from number of departments on the design team can sometimes make it more difficult to reach a consensus on the proper design because there are too many opinions regarding design issues. This is a major problem when there are particularly stubborn people on the design team who are holding out for specific product features. Resolving out is difficult and requires a strong team manager, as well as a long-term commitment on the part of a company to weed out those who are not willing to act in the best interests of the team.
- Effective implementation and use requires the development of detailed cost data. This can be really costly and may not be profitable for the company when a detailed cost-benefit analysis is done.
- Use of target costing may reduce the quality of products due to the use of cheap components which may be of inferior quality.
- For every problem area outlined have the dominant solution is retaining strong control over the design teams, which calls for a good team leader. This person must have an exceptional knowledge of the design process, good interpersonal skills, and a commitment to staying within both time and cost budgets for a design project.

1.4.6 Cost Accountant's Role in a Target Costing Environment

- The cost accountant should be able to provide for the other members of the design team a running series of cost estimates based on initial designs sketch, activities based costing reviews of production processes, and "best guess" costing information from suppliers based on estimated production volumes. Essentially in the earliest stages of a design, the cost accountant works with vague costing information and so must be able to provide estimates within a high-low range costs, gradually tightening this estimated cost range as more information becomes available.
- The cost accountant should also be responsible for any capital budgeting requests generated by the design team since he or she has the knowledge of the capital budgeting process, how to fill out the required forms, and precisely what types of equipment are needed for the anticipated product design. The cost accountant also becomes the key contact on the design team for answers to any questions from the finance staff regarding issues or uncertainties in the capital budgeting proposal.
- The cost accountant should work with the design team to help it understand the nature of various costs (such as cost allocations based on an activity-based costing system), as well as the cost-benefit trade-offs of using different design or cost operations in the new product.
- In addition, the cost accountant is responsible for tracking the gap between the current

cost of a product design and the target cost that is the design team's goal, providing an itemization of where cost savings have already been achieved and where there has not been a sufficient degree of progress.

- Finally, the cost accountant must continue to compare a product's actual cost to the target cost after the design is completed, and for as long as the company sells the product. This is a necessary step because management must know immediately if costs are increasing beyond budgeted levels and why these increases are occurring.

There are particular qualifications that a cost accountant must have to be assigned to a target costing team. Certainly, one is having a good knowledge of company products as well as their features and components. Also, the cost accountant must know how to create an activity based costing system to evaluate related production costs, or at least interpret such costing data developed by someone else. Further, he or she must work well in a team environment, proactively assisting other members of the team in constantly evaluating the costs of new design concepts. In addition, he or she should have good analytical and presentation skills, since the ongoing costing results must be continually presented not only to other members of the team but also to the members of the milestone review committee. Thus, the best cost accountant for this position is an outgoing person with several years of experience within a company or industry.

1.4.7 Impact of Target Costing on Profitability

Target costing can have a startlingly large positive impact on profitability, depending on the commitment of management to its use, the constant involvement of cost accountants in all phases of a product's life cycle, and the type of strategy a company follows.Target costing improves profitability in two ways.

- It places such a detailed continuing emphasis on product costs throughout the life cycle of every product that it is unlikely that a company will experience runaway costs; also, the management team is completely aware of costing issues since it receives regular reports from the cost accounting members of all design teams.
- It improves profitability through precise targeting of the correct prices at which the company feels it can field a profitable product in the marketplace that will sell in a robust manner. This is opposed to the more common cost-plus approach under which a company builds a product, determines its cost, tacks on a profit and then does not understand why its resoundingly high price does not attract buyers. Thus, target costing results not only in better cost control but also in better price control.

Target costing is really part of a larger concept called concurrent engineering, which requires participants from many departments to work together on project teams rather than having separate departments handle new product design only after they have been handed off from the preceding department in the design chain. Target costing removes the barriers between departments and provides way for a united effort by all members of the organisation towards achievement of the enterprise's goals.

Target Costing	Traditional Costing
Production Specification	Production Specification
↓	↓
Target Price and Volume	Product Design
↓	↓
Target Profit	Estimated Cost
↓	↓
Target Cost	Target Cost
↓	↓
Product Design	Target Price

The review of product costs under the target costing methodology is not reserved just for the period up to the completion of design work on a new product. On the contrary, there are always opportunities to control costs after the design phase is completed, though these opportunities are fewer than during the design phase. Therefore, cost accountants should not be pulled from a design team once the final drawings have left the engineering department. Instead, they should regularly monitor actual component costs and compare them to planned costs, warning management whenever significant adverse variances arise. Also, cost accountants should take a lead role in the continuing review of supplier costs to see if they can be reduced, perhaps by visiting supplier facilities, as well as constantly reviewing existing product designs to see if they can be improved, and by targeting for elimination waste or spoilage on the production floor. Therefore, the cost accounting staff must be involved in all phases of a product's life cycle if a company is to realize profitability improvements from target costing to the fullest extent.

A company's strategy can also has its impact on profitability. If it constantly issues a stream of new products, or if its existing product lines is subject to severe pricing pressure, it must make target costing a central part of its strategy so that the correct price points are used for products and actual costs match those originally planned. However, there are other strategies, such as growth by geographical expansion of the current product line (as is practiced by retail stores) or growth by acquisition, where there is no particular need for target costing—these companies make their money in other ways than by a focused concentration on product features and costs. For them, there may still be a role for target costing, but it is strictly limited by the reduced need for new products.

If the issues presented here are properly dealt with by a management team, it should find that target costing is one of the best accounting methods available for improving profitability. It is indeed one of the most pro-active systems found in the entire range of accounting knowledge.

1.4.8 Target Costing Data Flow

- Data can be obtained from central accounting data base carefully stocked formsuch a variety of sources as accounts payable, billing, bills of materials and inventory records.

- In initial stages of product design, the cost accountant must make the best possible guesses regarding the cost of proposed designs.
- The cost accountant may include the best estimate an additional estimate of the highest possible cost that will be encountered. This additional information lets management know whether there is a significant degree of risk that the project may not achieve its desired cost target.
- Data can also be obtained from competitor's information collected by the marketing staff or an outside research agency. This database contains information about the prices at which competitors are selling their products, as well as the prices of ancillary products and perhaps also the discounts given at various price points. It can also include market share data for individual products or by firm, the opinion of customers regarding the offerings of various companies, and the financial condition of competitors. This information is mostly used to determine the range of price points at which a company should sell its existing or anticipated products.
- Sometimes information is compiled by a combined effort of the marketing and engineering staffs through a process called reverse engineering. This source can also serve as a data base for the project team.
- Engineering staff also compiles their own cost data relating to different designs/components. This data is collected over the years and can be useful for target costing.
- The final database available to the cost accounting member of a design team contains information regarding the previous quality, cost and on-time delivery performance of all key suppliers, as well as the production capacity of each one.

1.4.9 Most Useful Situations for Target Costing

Target costing is most useful in situations where the majority of product costs are locked in during the product design phase. This is the case for most manufactured products, but few services. In the services area, such as consulting, the bulk of all activities can be reconfigured for cost reduction during the "production" phase, which is when services are being provided directly to the customer. In the services environment the "design team" is still present but is more commonly concerned with streamlining the activities conducted by the employees providing the service, which can continue to be enhanced at any time, not just when the initial services process is being laid out.

Whenever a new and innovative approach to doing business is discovered, the question arises as to which clients and potential clients might this methodology provide an appropriate fit. In addition, and consistent with many new financial or operational approaches, target costing may not be for everyone. Some companies, which seem to benefit most from target costing, are those, which maintain the following criteria:

- Assembly-oriented industries, as opposed to repetitive-process industries that produce homogeneous products;
- Involved heavily with the diversification of the product lines;
- Use technologies of factory automation, including computer-aided design, flexible manufacturing systems, office automation, and computer-aided manufacturing;
- Have experienced shorter product life cycles where the pay-back for factory automation typically must be achieved in less than eight years;
- Must develop systems for reducing costs during the planning, design and development stages of a product's life cycle;
- Are implementing management methods such as just-in-time, value engineering, and total quality control.

The above listing is not completely exhaustive as a variety of factors are at work to promote the usefulness of target costing in other companies. First, products are experiencing shortening life cycles, so the design phase of a product is critical to managing costs.Manufacturing costs are driven primarily by the characteristics of the products and the process used to manufacture them. Manufacturing processes are determined by the nature of the product and the expected volume to be produced. Therefore, to a great extent, costs are determined in the design stage.

Another factor which encourages the use of target costing is product diversity. The types of products manufactured by companies have increased rapidly in recent years. Target costing, in both the design and production stages, helps manage costs effectively.

However, applying target costing in the design stage has the greatest cost reduction potential and bottom-line impact.

Target Costing &Fast-Food Restaurant: Design team can lay out the floor plan of a fast-food restaurant, with the objectiveof creating an arrangement that allows employees to cover the shortest possible distances while preparing food and serving customers; this is similar to the design of a new product. However, unlike a product design, this layout can be readily altered at any time if the design team can arrive at a better layout, so that the restaurant staff can continue to experience high levels of productivity improvement even after the initial design and layout of the facility. In this situation costs are not locked in during the design phase, so there is less need for target costing.

Target Costing & Chemical Production Industry: Another situation where target costing results in less value is the production of raw materials, such as chemicals. In this case there are no design features for a design team to labour over; instead, the industrial engineering staff tries to create the most efficient possible production process, which has little to do with cost reduction through the improvement of customer value by creating a product with a high ratio of features to costs.

1.4.10 Target Costing Control Points

Control Points which should be taken care of in all target costing projects:

- **Identification of Principal control point:** Experience shows that there always comes a point, where the cost of maintaining the design team exceeds the savings gardened from additional iterations. It is also necessary that most products should be launched within a reasonably short time or they will miss the appropriate market, where they will beat the delivery of competing products to the market. This emphasis that the principal control points over the course of target costing programme should be properly taken care of.
- **Point of Go/No Go decision:** If target costing is not reached, management retains power to abandon the design project. There comes a point, when actual performance is very close to expected performance in matter of cost incurrence.
- **Milestone can be in terms of timer or points:** A milestone can be in terms of time, say one month. It can also be on the points in design process, at which specific activities are completed.

1.4.11 Implementing a Target Costing System

A target costing initiative requires the participation of several departments. Because there are so many participants in the process from so many departments, some of whom have different agendas in regard to what they want the program to produce. Design projects can be delayed by squabbling or by an inability to drive down design or production costs in a reasonably efficient manner. This delay may lead to serious cost overruns in the cost of the design team itself, which can lead to abrupt termination of the entire targets costing system by the management team. However, these problems can be mitigated or completely eliminated by ensuring that the steps listed here are completed when the target costing system is first installed:

- **Create a project charter:** The target costing effort should begin with a document, approved by senior management that describes its goals and what it is authorized to do. This document, known as the project charter, is essentially a subset of the corporate mission statement and related goals as they pertain to the target costing initiative. Written approval of this document by the senior management group provides the target costing effort with a strong basis of support and direction in all subsequent efforts.
- **Obtain a management sponsor:** The next step is to obtain the strongest possible support from a management sponsor. This should be an individual who is well positioned near the top of the corporate hierarchy, believes strongly in the goals of target costing, and will support the initiative in all respects—obtaining funding, lobbying other members of top management, working to eliminate road blocks, and ensuring that other problems are overcome in timely manner. This person is central to the success of target costing.
- **Obtain a budget:** The target costing program requires funds to ensure that one or more well-staffed design teams can complete target costing tasks. The funding should be based on a formal allocation of money through the corporate budget, rather than a

parsimonious sub allocation grudgingly granted by one or more departments. In the first case the funds are unreservedly given to the target costing effort, whereas in the latter case, they can be suddenly withdrawn by a department manager who is not fully persuaded of the need for target costing or who suddenly finds a need for the money elsewhere.

- **Assign a strong team manager:** Because the typical target costing program involves so many people with different backgrounds and represents so many parts of a company, it can be difficult to weld the group together into a smoothly functioning team focused on key objectives. The best way to ensure that the team functions properly is to assign to the effort a strong team manager skilled in dealing with management, the use of project tools, and working with a diverse group of people. This manager should be a full-time employee, so that his or her complete attention can be directed toward the welfare of the project.
- **Enroll full-time participants:** A target costing team member puts the greatest effort into the program when he or she is focused only on target costing. Thus, it is essential that as many members of the team as possible be devoted to it full-time rather than also trying to fulfill other commitment elsewhere in the company at the same time. This may call for the replacement of these individuals in the departments they are leaving so that there are no emergencies requiring their sudden withdrawal back to their "home" departments to deal with other work problems. It may even be necessary to permanently assign them to a target costing program, providing them with a single focus on ensuring the success of the target costing program because their livelihood are now tied to it. As discussed above, a full time Cost Accountant should be employed for target costing who carries out the cost-benefit analysis on a continuous basis.
- **Use project management tools:** Target costing can be a highly complex effort especially for high-cost products with many features and components. To ensure that the project stays on track, the team should use all available project management tools, such as Microsoft Project (for tracking the completion of specific tasks), a company database containing various types of costing information, and a variety of product design tools. All these items require assured access to many corporate databases, as well as a budget for whatever computing equipment is needed to access this data.

The main focus of the step described in this section is to ensure the fullest possible support for target costing by all available means—management, money and staff. Only when all these elements are in place and concentrated on the goals at hand does a target costing program have the greatest chance for success.

1.4.12 Concept of 'Target Costing'followed by 'A Textile Manufacturer in the USA'

A major textile manufacturer in the USA introduced a target costing system. Beginning the project, they realised the need for better-cost management and have worked to overcome that issue.

The company's journey has led them to a realization that cost management is different from other accounting efforts, and has undertaken a target costing program to help them build profits and decrease the cost of their products at the design stage. They recognised the importance of breaking down the traditional barriers of the firm toward cost management. This approach was addressed using a series of three strategies.

Strategy 1

Separate the functions of managerial and financial accounting so that each could serve its customer to the best advantage. This separation is important to any management accounting system evolution.

Strategy 2

Achieve a level of accurate product costing. The accuracy school of cost management obviously is concerned with accurate product costs. Although this goal sometimes sounds simplistic, it is not always that easy to get product costs as close to actual as possible.

Strategy 3

Going from the accuracy school to target costing. Costing products accurately is a worthy goal. Accurate product costing, however, in and of itself does little to improve the firm's position and does nothing to reduce costs. This strategy includes the discovery that an overwhelming majority of costs were created and built into the products before the manufacturing process ever begins.

1.4.13 Illustrations

Illustration1

You are a manager of ABC Paper Mills and have recently come across a particular type of paper, which is being sold at a substantially lower rate by another company PQR Ltd than the price charged by your own mill. The Value chain for use of a tonne of such paper for PQR Ltd is: PQR Ltd.→Merchant → Printer→ Customer.

PQR Ltd. sells this particular paper to Merchant at the rate of ₹ 1,466 per tonne. PQR pays for freight which amounts to ₹ 30 per tonne. Average returns and allowances amount to 4% of sales and approximately equal₹60 per tonne.

The Value chain of your company through which the paper reaches the ultimate customer is similar to that of PQR. However, your mill does not sell directly to the Merchant, the latter receiving the paper from huge Distribution Center maintained by your Company at Punjab.

Shipment cost from the Mill to the Distribution Center is ₹ 11 per tonne while the Operating Costs in the Distribution Center are estimated at ₹ 25 per tonne. The Return on Investment required by the Distribution Center for the investments made, amount to an estimate ₹ 58 per tonne.

Calculate the "Mill Manufacturing Target Cost" for this particular paper ofABC Paper Mills. Assume that the ROI expected by ABC is ₹ 120 per tonne of paper.

Solution

Particulars		Amount in ₹
Sale Price of PQR Ltd. to Merchant		1,466
Less: Reduction towards - Freight paid by PQR Ltd.	₹ 30	
Returns and Allowances (given)	₹ 60	90
Target Sale Price for ABC Paper Mills		1,376
Less: Target Profit margin for ABC Paper Mills = Overall ROI expected (given)		120
Target Cost for ABC Paper Mills (Overall, i.e. Mill + Distribution)		1,256
Less:Value Addition at Distribution Center Level		
(a) Shipping cost + Operating cost	₹ 11+₹ 25 = ₹ 36	
(b) ROI for Distribution Center	= ₹ 58	94
Target Cost at Mill Level		1,162

Illustration2

A company has the capacity of production of 80,000 units and presently sells 20,000 units at ₹ 100 each. The demand is sensitive to selling price and it has been observed that every reduction of ₹ 10 in selling price the demand is doubled. What should be the target cost at full capacity if profit margin on sale is taken as 25%?

What should be the cost reduction scheme if at present 40% of cost is variable with same % of profit? If Rate of Return is 15%, what will be maximum investment at full capacity?

Solution

(a) Maximum capacity 80,000 units

Presented sales 20,000units @ ₹100 p.u.

Selling price/unit	Demand
100	20,000
90	40,000
80	80,000= Full capacity

Therefore, target Price = 80

Target cost/unit = 80 –25% of sales

= 80 –20 = 60 p.u.

Total Target Cost = 80,000 units x ₹ 60 p.u. = ₹ 48 lakhs

(b) At present

Totalcost/unit = 100 –25% of 100 = ₹ 75

Variable cost/unit = 40% of cost i.e. 75 = ₹30

		₹
Cost/Unit		75
Less: Variable cost/unit		30
Fixed cost		45 p.u.
Total fixed cost (₹45× 80,000 units)	=	₹ 36 lakhs
Variable cost (80,000 units x₹30)	=	₹24 lakhs

RequiredCost reduction following value engineering is ₹ 12 lakhs.

(c) Rate of return 15%

Profit p.u.= 25% of 80 = 20/unit

Profit before tax = 20/unit x 80,000 units = 16 lakhs

ROCE = (PBT/Investments)

Investments = (PBT/ROCE) = 16 lakhs/15% = ₹106 lakhs.

Illustration3

Sterling Enterprises has prepared a draft budget for the next year as follows:

Quantity	***10,000 units***
Sales price per unit	*30*
Variable costs per unit:	
Direct Materials	*8*
Direct Labour	*6*
Variable overhead (2 hrs × Re. 0.50)	*1*
Contribution per unit	*15*
Budgeted Contribution	*1,50,000*
Budgeted Fixed costs	*1,40,000*
Budgeted Profit	*10,000*

The Board of Directors is dissatisfied with this budget, and asks a working party to come up with an alternate budget with higher target profit figures.

The working party reports back with the following suggestions that will lead to a budgeted profit of ₹25,000.The company should spend ₹28,500 on advertising, &put the target sales price up to ₹32 per unit.It is expected that the sales volume will also rise, in spite of the price rise, to 12,000 units.

In order to achieve the extra production capacity, however, the work force must be able to reduce the time taken to make each unit of the product.It is proposed to offer a pay and productivity deal in which the wage rate per hour in increased to ₹4.The hourly rate for variable overhead will be unaffected.

Ascertain the target labour time required to achieve the target profit.Prepare a revised budget giving effect to the above suggestions.

Solution

Revised Budget

Quantity (Units)(a)	12,000
Sales price per unit	32.000
Less: Variable costs per unit: Direct Materials	8.000
Less: Direct Labour (1.75 hrs x ₹4)	7.000
Less: Variable overhead (1.75 hrs × Re. 0.50)	0.875
Contribution per unit(b)	16.125
Budgeted Contribution (a) x (b)	193,500
Less: Budgeted Fixed costs (₹ 140,000+₹ 28,500)	168,500
Budgeted Profit	25,000

Working Note:

Calculation of target labour time required to achieve the target profit

	(₹)
Target profit	25,000
Add: Fixed cost	1,40,000
Add: Additional Advertisement	28,500
Total contribution (a)	1,93,500
Required. Sales (volume) (b)	12,000
Contribution/unit (a/b)	16.125
Target Selling price/unit	32.000
Less: Contribution/unit	16.125
Target variable cost p.u.	15.875
Less: material cost p.u.	8.000
Labour + Variable overhead	7.875

Let the target number of labour hours per unit be x

Labour: x hr. @ 4	4x
Variable overhead: x hr. @ 0.5	0.5x
Therefore, 4.5x =	7.875
x (hr.)	1.75
Labour Time/unit	1.75
Present Labour time	2.00
Time reduced	0.25 hr.

Illustration4

E-Tech Ltd. manufactures and sells computers peripherals to several retail outlets throughout the country. Josaph is the manager of the printer division. Its two largest-selling printers are Z2001 & Z2002. The manufacturing cost of each printer is calculated using E-Tech's activity based costing system. E-Tech has one direct manufacturing cost category (direct materials) and the following five indirect manufacturing cost pools.

Indirect manufacturing cost pool	*Allocation Rate (₹)*
1. Materials handling	₹ 0.60 per part
2. Assembly management	₹ 20.00 per hour of assembly time
3. Machine insertion of parts	₹ 0.35 per machine inserted part
4. Manual insertion of parts	₹ 1.05 per manually inserted part
5. Quality testing	₹ 12.50 per testing hour

Product characteristics of Z2001 and Z2002 are as follows:

Product	Z2001	Z2002
Direct materials costs	₹ 203.75	₹ 146.05
Number of parts	43.00	23.00
Hours of assembly time	2.00	1.00
Number of machine – inserted parts	24.00	15.00
Number of manually inserted parts	18.00	8.00
Hours of quality testing time	1.00	1.00

A foreign competitor has introduced products very similar to Z2001 and Z2002. Given their announced selling prices, to maintain the company's market share and profits, Josaph estimated the Z2001 to have manufacturing cost of approximately ₹ 295 and Z2002 to have a manufacturing cost of approximately ₹ 180. He calls a meeting of product designers and manufacturing personnel at the printer division. They all agreed to have the ₹ 295 and ₹ 180 figures become target costs for designed version of Z2001 and Z2002 respectively. Product designers examine alternative ways of designing printer with comparable performance but lower costs. They come up with the following revised designs for Z2001 and Z2002 (termed Z2001 – REV and Z2002 – REV, respectively)

Particulars	Z2001 – REV	Z2002 – REV
Direct materials cost	₹ 190.60	₹ 131.55
Number of parts	36.00	19.00
Hours of assembly time	1.00	2.00
Number of machine – inserted parts	29.00	14.00
Number of manually – inserted parts	6.00	5.00
Hours of quality testing time	2.00	1.00

Required:

a) Compute the present costs of products Z2001 and Z2002 using ABC system.

b) *Compute the manufacturing costs of Z2001 – REV and Z2002 – REV. How do they compare with the ₹295 and ₹180 target costs?*

Solution

Statement showing manufacturing cost

		Z2001 ₹/unit		**Z2002 ₹/unit**
Material:		203.75		146.05
Overhead:				
Material handling	43×0.60	25.80	23×0.60	13.80
Assembly Management	2×20.00	40.00	1×20.00	20.00
Machine insertion	24×00.35	8.40	15×00.35	5.25
Manual insertion	18×01.05	18.90	8×01.05	8.40
Quality testing	1x12.50	12.50	1x12.50	12.50
Present cost		309.35		206.00
Target cost		295.00		180.00

	Revised Z2001		**Revised Z2002**	
		₹/unit		**₹/unit**
Material		190.60		131.55
Overhead:				
Material handling	36×0.60	21.60	19×0.60	11.40
Assembly Management	1×20.00	20.00	2×20.00	40.00
Machine insertion	29×00.35	10.15	14×00.35	4.90
Manual insertion	6×01.05	6.30	5×01.05	5.25
Quality testing	2x12.50	25.00	1x12.50	12.50
Present cost		273.65		205.60
Target cost		295.00		180.00
		Achieved		Not achieved

Illustration5

ABC Electronics Ltd makes audio player model "AB-100".This model has 80 components. ABC sells 10,000 units each month at ₹3,000 per unit.The cost of manufacturing is ₹2,000 per unit or ₹200 lakhs per month for the production of 10,000 units.Monthly manufacturing costs incurred (in ₹Lakhs) are as follows:

Direct materials costs	*100.00*
Direct manufacturing labour costs	*20.00*
Machining costs	*20.00*
Testing costs	*25.00*

Rework costs	*15.00*
Ordering costs	*0.20*
Engineering costs	*19.80*
	200.00

Labour is paid on piece rate basis, therefore, ABC considers, direct manufacturing labour costs as variable cost.

The following additional information is available for "AB-100":

(a) *Testing and inspection time per unit is 2 hours*

(b) *10 per cent of "AB-100" manufactured are reworked*

(c) *It currently takes 1 hour to manufacture each unit of "AB-100".*

(d) *ABC places two orders per month for each component, each component being supplied by a different supplier.*

ABC has identified activity cost pools and cost drivers for each activity.The cost per unit of the driver for each activity cost pool is as follows:

Manufacturing Activity	***Description of Activity***	***Cost Driver***	***Cost per unit of Cost Driver***
Machining costs	*Machining components*	*Machine hours of Capacity*	*₹200*
Testing costs	*Testing components and Finished Products. (Each unit of AB 100 is Tested individually)*	*Testing hours*	*₹125*
Rework costs	*Correcting and fixing errors*	*Units reworked*	*₹1,500*
Ordering costs	*Ordering of components*	*Number of orders*	*₹125*
Engineering costs	*Designing and managing of Products and processes*	*Engineering hrs.*	*₹198*

Over a long – run horizon, each of the overhead costs described above varies with chosen cost drivers.

In response to competitive pressure ABC must reduce the price of its product to ₹2,600 and to reduce the cost by at least ₹400 per unit.ABC does not anticipate increase in sales due to price reduction.However, if it does not reduce price it will not be able to maintain the current sales level.

Cost reduction on the existing model is almost impossible.Therefore, ABC has decided to replace "AB-100" by a new model "AB-200", which is a modified version of "AB-100".The expected effect of design changes are:

(a) *The number of components will be reduced to 50.*

(b) *Direct materials costs to be lower by ₹200 per unit.*

(c) Machining time required to be lower by 20%.

(d) Direct Manufacturing required costs to be lower by ₹20 per unit.

(e) Testing time required to be low by 20%.

(f) Rework to decline to 5%

(g) Machining capacity and engineering hours capacity to remain the same.

(h) ABC currently outsources the rework on defective units.

Required:

Compare the manufacturing cost per unit of "AB-100" and "AB-200".

Assume that the cost per unit of each cost driver for "AB-100" continues to apply to "AB-200".

Solution

Comparisons of Manufacturing Cost Per Unit

Units	**"AB-100" 10,000 ₹ /unit**	**"AB-200" 10,000 ₹ /unit**
Material 100 lakhs/10,000	1,000	800
Direct wages 20 L/10,000	200	180
Machining Cost	200	160
Test cost	250	200
Rework cost	150	75
Ordering cost	2	1.25
Engineering cost	198	198
Total Manufacturing Cost	2,000	1,614.25

Illustration6

A Company has sales of 1,00,000 units at a price of ₹ 200 per unit and a profit of ₹40 lakhs in the current year. Due to stiff competition, the company has to reduce its price of product next year 5% to achieve same volume target of sales. The cost structure and profit for the current year is given as below:

Particulars	***₹in Lakhs***
Direct Materials	*60*
Direct Wages	*45*
Variable Factory Overhead	*20*
Fixed Overheads including Sales and Admin Expenses	*35*
Total Cost	*160*

To achieve the target cost to maintain the same profit, the Company is evaluating the proposal to reduce Labour cost and Fixed Factory overhead. A vendor supplying the Machine suitable for the Company's *operations has offered an advanced technology Semi- Automatic Machine of ₹20 lakhs as replacement of Old machine worth ₹5 lakhs. The Vendor is agreeable to take back the old machine at ₹ 2.70 lakhs only. The Company's policy is to charge depreciation at 10% on WDV. The Maintenance Charge of the existing machine is ₹ 1.20 lakhs p.a. whereas there will be warranty of services free of cost for the New Machine first two years. There are ten supervisors whose salary is ₹ 1.50 lakhs p.a. The New machine having Conveyor Belt is expected to help in cost cutting measures in the following ways:*

Improve productivity of workers by 20%

Cut-down material wastage by 1%

Elimination of services of Supervisors because of automatic facilities of the machine

Saving in packaging cost by ₹ 1.5 lakhs

Assuming Cost of Capital to be 15%, calculate how many supervisors should be removed from the production activities to achieve the Target Cost.

Solution:

For the same quality, sales value will reduce by 5% of (1,00,000 units X₹ 200) = ₹10 lakhs. For maintaining the same amount of profit, cost also has to be reduced by ₹ 10 lakhs, which can be achieved as under –

Particulars	**₹in lakhs**
Savings: Reduction in wages (Note:Due to higher labour productivity, Wages will be 45/1.20 = ₹37.50 lakhs)	7.50
Elimination of wastage of Materials= 1% of ₹ 60 lakhs	0.60
Packaging Cost (given)	1.50
Maintenance Cost (given)	1.20
Sub-Total Savings (A)	10.80
Costs: Loss in Disposal of Old Machine (₹5 lakhs – ₹ 2.7 lakhs)	-2.30
Difference in Depreciation (₹20 lakhs – ₹5 lakhs) x 10%	-1.50
Cost of Capital Investment (₹20 lakhs x 15%)	-3.00
Sub-Total Costs (B)	6.80
Effective Cost Reduction before considering removal of supervisors (A) – (B)	4.00
Additional reduction required for meeting Target Cost, by removing supervisors = (₹ 10 lakhs – ₹ 4 lakhs)	6.00

Hence, number of supervisors to be removed =₹6 lakhs / ₹ 1.50 lakhs per supervisor = 4 supervisors.

Illustration7

PT Ltd. produces and markets Air Conditioner. It ensures after sales service through PK Ltd. The big appliances are serviced at customer's residence while small appliances are serviced at workshop of PK Ltd.

The material supplied to PK Ltd. is charged at cost at 15%. PK Ltd. charges customers at 20% over the above price. For labour, the company receives 8% of the rate fixed for work done under the after-sales service agreement and 10% of the rate fixed in case of jobs not covered under the agreement from PK Ltd. 65% by value of the total work undertaken by PK Ltd. was for big appliances and rest accounted for small appliances during the previous year.

The company decides to carry out all or some of the work itself and has chosen one area in the first instance. During the previous year the company earned a profit of ₹1,08,000 as detailed below from PK Ltd. for the area chosen:

	Materials ₹	***Labour*** ₹
Under after –sales service agreement	*30,000*	*50,000*
For jobs not covered under the agreement	*10,000*	*18,000*

The company forecasts same value of work in that area for the ensuing period. The following three options are under consideration of the management:

(1) To set up a local service Centre to provide service for small appliances only. The existing system is to continue for big appliances.

(2) To set up a local services Centre to provide service for big appliances only. The existing system is to continue for small appliances.

(3) To set up a local service Centre to provide service to all appliances. The existing system then stands withdrawn.

The relevant costs for carrying out jobs under the above options are as under:

(₹'000)

	Option -1	***Option -2***	***Option -3***
Heat, rent, light etc.	*55*	*25*	*70*
Management costs	*50*	*42*	*80*
Service staff costs	*110*	*210*	*355*
Transport costs	*18*	*120*	*110*

You are required to find out the most profitable option.

Solution

Statement showing value of total work undertaken by PK Ltd. at customer's price

	(₹'000)
Material cost (for appliances covered under agreement)[Refer to workings]	276

Material cost (for appliances not covered under agreement) [Refer to workings]		92
Labour cost (for appliances covered under agreement) [Refer to workings]		625
Labour cost (for appliances not covered under agreement) [Refer to workings]		180
Total receipts		1,173
Break up of receipts:		
Big appliances	65%	762
Small appliances	35%	411

Profitability Statement

			(₹ '000)
	Option 1	**Option 2**	**Option 3**
Income:			
Big appliances	70.20 (65%×₹108)	762.00	762
Small appliances	411.00	37.80 (35%× ₹108)	411
Total receipts: (A)	481.20	799.80	1,173
Costs:			
Material [Refer to workings]	93.33	173.33	266.66
Heat, rent, light etc.,	55.00	25.00	70.00
Management costs	50.00	42.00	80.00
Service staff costs	110.00	210.00	355.00
Transport costs	18.00	120.00	110.00
Total costs: (B)	326.33	570.33	881.66
Profit: [(A-B)]	154.87	229.47	291.34

Recommendation: Option 3 is most profitable one.

Working Notes:

1. Material and labour cost (for appliances under after sales agreement):	(₹)
(i) Cost of Material per unit charged to customer's by PK Ltd (120% of ₹115)	138.00

Cost of material charged to customer's by PK Ltd. (₹30,000/₹15×₹138)	2,76,000
(ii) Cost of labour charged to customers by PK Ltd. (₹50,000 / ₹8 x ₹100)	6,25,000
2. Material and labour cost (for appliances not covered under sales agreement):	
(i) Cost of material charged to customer's by PK Ltd. (₹10,000 / ₹15 x ₹138)	92,000
(ii) Cost of labour charged to customers by PK Ltd. (₹18,000 / ₹10 x ₹100)	1,80,000
3. Material Cost to PT ltd:	
(₹276,000+₹92,000) / ₹138 x ₹100	2,66,667
Option 1	
To set up a local service Centre to provide service for small appliances (35% Share)	
Material Cost ₹2,66,667x 35%	93,333
Option 2	
To set up a local service Centre to provide service for big appliances (65% Share)	
Material Cost ₹2,66,667 x65%	1,73,333
Option 3	
To set up a local service Centre to provide service to all appliances (100% Share)	
Material Cost ₹2,66,667 x 100%	2,66,667

1.5 Life Cycle Costing

1.5.1 Life cycle costing is different to traditional cost accounting system which report cost object profitability on a calendar basis i.e. monthly, quarterly and annually.

CIMA defines life cycle costing as "the practice of obtaining over their life time, the best use of physical asset at the lowest cost of entity".

In contrast life cycle costing involves tracing cost and revenues on a product by product basis over several calendar periods. Costs and revenue can be analysed by time period, but the emphasis is on cost revenue accumulation over the entire life cycle of each product.

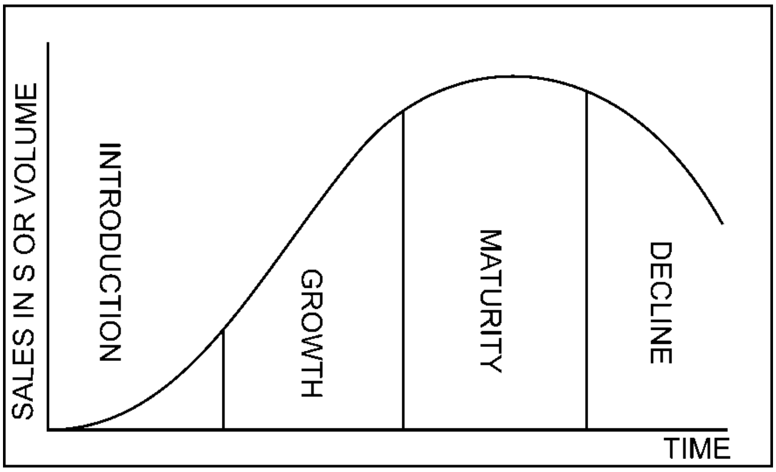
Now, we will discuss life cycle costing for products.

1.5.2 Product life cycle

Each product has a life cycle. The life cycle of a product vary from a few months to several years'. Product life cycle is thus a pattern of expenditure, sales level, revenue and profit over the period from new idea generation to the deletion of product from product range.

The life cycle of a product consists of four phases viz., Introduction; Growth; Maturity; Saturation and Decline.

- **Introduction:**During introductory phase, a product is launched into the market. Its customers are innovators. Competition is almost negligible and profits are non-existent.
- **Growth:**Under growth phase, sales and profits rise, at a rapid pace. Competitors enter the market often in large numbers. As a result of competition, profits starts declining near the end of the growth phase.
- **Maturity:**During the phase of maturity sales continue to increase, but at a decreasing rate. When sales level off, profits of both producers and middlemen decline. The main reason is intense price competition; some firms extend their product lines with new models.
- **Saturation and decline:**At last a point comes, when it starts appearing that market has bought enough of the product. Decline in sales volume characterizes this last phase of the product life cycle. The need or demand for product disappears. Availability of better and less costly substitutes in the market accounts for the arrival of this phase.



1.5.3 Characteristics of product life cycle

The major characteristics of product life-cycle concept are as follows:

- The products have finite lives and pass through the cycle of development, introduction,growth, maturity, decline and deletion at varying speeds.
- Product cost, revenue and profit patterns tend to follow predictable courses through the product life cycle. Profits first appear during the growth phase and after stabilising during the maturity phase, decline thereafter to the point of deletion.

- Profit per unit varies as products move through their life cycles.
- Each phase of the product life-cycle poses different threats and opportunities that give rise to different strategic actions.
- Products require different functional emphasis in each phase-such as an R&D emphasis in the development phase and a cost control emphasis in the decline phase.
- Finding new uses or new users or getting the present users to increase their consumption may extend the life of the product.

Product Life Cycle Characteristics
Industry Phase

	Introduction	Growth	Maturity	Decline
Number of firms	Small, growing fast	Large	Shakeout, number stabilizes	Small, exit
Market Size	Small	Large	Large	Declining
Market	Fast	Fast	Slow	Negative
Entry	Large	Medium	Low	Negative
Market Leader Characteristics	Existing Reputation Innovator, Educator, Flexible, Total solution	Existing Reputation Marketing, Quality production, Modular	Reputation for quality, low cost production and distribution, specialization	Low cost, serves niche market
Profits	Negative	Low, but sharply increasing	High, then declining	Low, then negative
Product Varieties	Few and Growing	Increases, then declines to few dominant designs	Increasing specialization & segmentation	Declining
Distribution	One-stop shopping	Various outlets	Superstores, direct sales	Minimum cost
Investment	High	High	Low	Negative
Average Costs	High, falling	Medium, falling	Low	Low
Foreign Trade	Low exports	Increasing exports	Initially exports, then imports	Imports
Customer Name	Innovator/ Early Adopter	Early Majority	Late Majority	Laggard
Customer	Sophistication,	Performance	Price/performance	Low price

needs	Features, Fit			
Customer Knowledge	Low	Medium	High	High, Declining
Product Complexity	High	Product simplified	Low: standardization & commoditization	Low
Information collection	Demand, Customer awareness and satisfaction,	Marketing, relative performance, product	Cost reduction, new markets, competitive threats	Capacity reduction, new uses, new features
Marketing Strategy	Promotion of brand and market	Build brand identity	Segment market; steal business	New promotion; bitter attacks

1.5.4 Various stages of product life cycle

Typically the life cycle of a manufactured product will consist of the following stages:

- **Market research:** Before any investment in made the investor must believe that what the company proposes to make can be sold at a price which will permit a profit to be made. This usually means that market research will establish what product the customer wants, how much he is prepared to pay for it and how many he will buy.
- **Specification:** When market research has established what is to be made, it will be necessary to turn the general statement of requirements into a detailed specification which will tell the designer and manufacturing engineer precisely what is required. The design specification will give such details as required life, maximum permissible maintenance costs, maximum permissible manufacturing cost, the number required, the delivery date, the required performance of the product.
- **Design:** With a precise specification, the designers can produce the drawings and process schedules which define the geometry of the product and some of the manufacturing processes.
- **Prototype manufacture:** From the drawings it will be possible to manufacture a small number of the product. These prototypes will be used to develop the product and eventually to demonstrate that it meets the requirements of the specification.
- **Development:** When a product has been made for the first time, it is necessary to prove that it meets the requirements of the specification. In fact, when a product is first made it rarely meets the requirements of the specification and changes have to be made until it does. This period of testing and changing is 'development'. Development can be very expensive and often generates a large negative cash flow before any products have been sold and hence, before any positive cash flows have been generated.

- **Tooling:** When a product is shown to meet the requirements of the specification and if calculations suggest that it will be profitable, the decision will be made to make it to sell. This is not a decision that will be taken lightly because, in many cases, the decision to make a product for sale is commitment to tool up for production. Tooling up for production can mean building a production line costing several lakhs of rupees, building expensive jigs, buying special purpose machine tools or, in some other say, making a very large initial investment.
- **Manufacture:** The manufacture of a product involves the purchase of the raw materials, the purchase of bought out components, the use of labour to make and assemble the product, and the use of supervisory labour.
- **Selling:** When the product is fit to sell and available, it may be necessary to spend money on a campaign to sell the product.
- **Distribution:** In the process of selling the product, it must be distributed to the sales outlets and to the customers.
- **Product support:** When the product has been bought, the customer will expect it to be supported. The manufacturer or supplier will have to make sure that spares and expert servicing are available for the life of the product. The manufacturer or the supplier may even have to offer free servicing and parts replacement during the early life of the product.
- **Decommissioning or Replacement:** When a manufacturing product comes to an end, the plant used to build the product must be re-used, sold, scrapped, or decommissioned in a way that is acceptable to society.

1.5.5 Product life cycle costing

It is an approach used to provide a long term picture of product line profitability, feedback on the effectiveness of life cycle planning and cost data to clarify the economic impact of alternatives chosen in the design, engineering phase etc. It is also considered as a way to enhance the control of manufacturing costs. The thrust of product life cycle costing is on the distribution of costs among categories changes over the life of the product, as does the potential profitability of a product. Hence it is important to track and measure costs during each stage of a product's life cycle.Product life cycle costing is important due to the following features:

- Product life cycle costing involves tracing of costs and revenues of each product over several calendar periods throughout their entire life cycle. Costs and revenues can be analysed by time periods, but the emphasis is on cost and revenue accumulation over the entire life cycle for each product.
- Product life cycle costing traces research and design and development costs etc., incurred to individual products over their entire life cycles, so that the total magnitude of

these costs for each individual product can be reported and compared with product revenues generated in later periods.

Life cycle costing therefore ensures that costs for each individual product can be reported and compared with product revenues generated in later periods. Hence the costs are made more visible.

1.5.6 Benefits of product life cycle costing

The benefits of product life cycle costing are summarized as follows:

- The product life cycle costing results in earlier actions to generate revenue or to lower costs than otherwise might be considered. There are a number of factors that need to the managed in order to maximise return on a product.
- Better decisions should follow from a more accurate and realistic assessment of revenues and costs, at least within a particular life cycle stage.
- Product life cycle thinking can promote long-term rewarding in contrast to short-term profitability rewarding.
- It provides an overall framework for considering total incremental costs over the entire life span of a product, which in turn facilitates analysis of parts of the whole where cost effectiveness might be improved.

1.5.7 Pricing Strategies for new products entering the market

The pricing strategy for a new product should be developed so that the desired impact on the market is achieved while the emergence of competition is discouraged. Two basic strategies that may be used in pricing a new product are skimming pricing and penetration pricing.

- **Skimming Pricing Strategy:**Skimming pricing is the strategy of establishing a high initial price for a product with a view to "skimming the cream off the market" at the upper end of the demand curve. It is accompanied by heavy expenditure on promotion. A skimming strategy may be recommended when the nature of demand is uncertain, when a company has expended large sums of money on research and development for a new product, when the competition is expected to develop and market a similar product in the near future, or when the product is so innovative that the market is expected to mature very slowly. Under these circumstances, a skimming strategy has several advantages. At the top of the demand curve, price elasticity is low. Besides, in the absence of any close substitute, cross-elasticity is also low. These factors, along with heavy emphasis on promotion, tend to help the product make significant inroads into the market. The high price also helps segment the market. Only non price-conscious customers will buy a new product during its initial stage. Later on, the mass market can be tapped by lowering the price. If there are doubts about the shape of the demand curve for a given product and the initial price is found to be too high, price may be slashed. However, it is very difficult to start low and then raise the price.

Raising a low price may annoy potential customers, and anticipated drops in price may retard demand at a particular price. For a financially weak company, a skimming strategy may provide immediate relief. This model depends on selling enough units at the higher price to cover promotion and development costs. If price elasticity is higher than anticipated, a lower price will be more profitable and “relief giving.” Modern patented drugs provide a good example of skimming pricing.

- **Penetration Pricing Strategy:**Penetration pricing is the strategy of entering the market with a low initial price so that a greater share of the market can be captured. The penetration strategy is used when an elite market does not exist and demand seems to be elastic over the entire demand curve, even during early stages of product introduction. High price elasticity of demand is probably the most important reason for adopting a penetration strategy. The penetration strategy is also used to discourage competitors from entering the market. When competitors seem to be encroaching on a market, an attempt is made to lure them away by means of penetration pricing, which yields lower margins. A competitor’s costs play a decisive role in this pricing strategy because a cost advantage over the existing manufacturer might persuade another firm to enter the market, regardless of how low the margin of the former may be. One may also turn to a penetration strategy with a view to achieving economies of scale. Savings in production costs alone may not be an important factor in setting low prices because, in the absence of price elasticity, it is difficult to generate sufficient sales. Finally, before adopting penetration pricing, one must make sure that the product fits the lifestyles of the mass market. For example, although it might not be difficult for people to accept imitation milk, cereals made from petroleum products would probably have difficulty in becoming popular. How low the penetration price should be differs from case to case.

1.5.8 Costs included in different stages of Product Life cycle

Development phase	R&D cost/Design cost
Introduction phase	Promotional Cost/Capacity costs
Growth/ Maturity phase	Manufacturing, distribution and product support costs
Decline /replacement phase	Plant reuse, sale and related costs.

1.5.9 Uses of Product Life cycle

- As a Planning tool, it characterizes the marketing challenges in each stage and poses major alternative strategies, i.e application of kaizen
- As a Control tool, the PLC concept allows the company to measure product performance against similar products launched in the past
- As a forecasting tool, it is less useful because sales histories exhibit diverse patterns and the stages vary in duration.

1.6 Value Chain Analysis

1.6.1 **C**ompetitive advantage for a company means not just matching or surpassing their competitors, but discovering what the customers want and then profitably satisfying, and even exceeding, their expectations. As barriers to inter-regional and international trade are diminishing and as access to goods and services are growing, customers can locate after identification and acquire the best of what they want, at an acceptable price, wherever it is in the world. Under growing competition and, hence, rising customer expectations, a company's penalty for complacency becomes even greater**.**

A strategic tool to measure the importance of the customer's perceived value is value chain analysis. By enabling companies to determine the strategic advantages and disadvantages of their activities and value-creating processes in the marketplace, value chain analysis becomes essential for assessing competitive advantage.

The aspect of value chain analysis is addressed to managers, and more specifically to management accountants, who may lead efforts to implement value chain analysis in their organisations.

The concepts, tools and techniques of value chain analysis apply to all those organisations which produce and sell a product or provide a service.

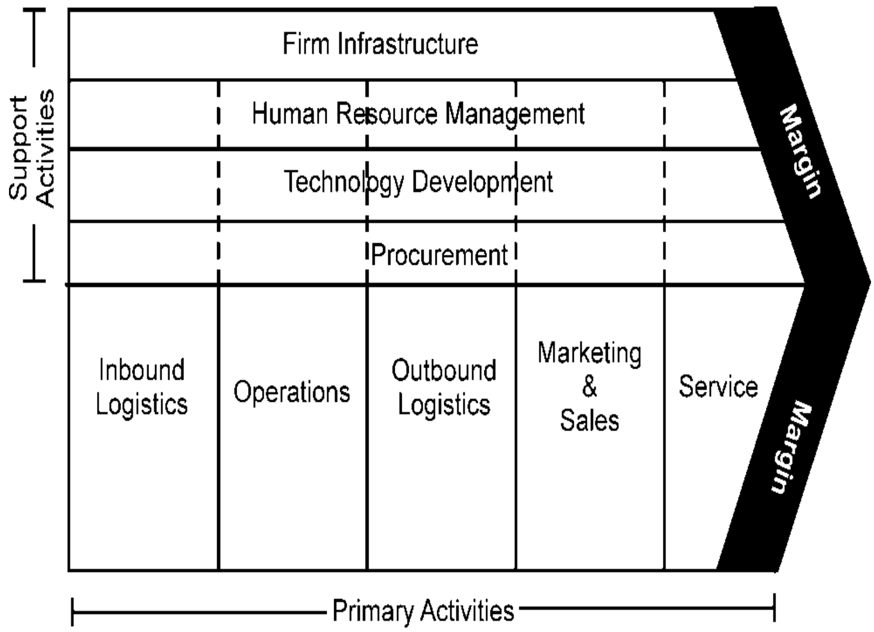
Porter describes the value chain as *"internal processes or activities a company performsto design, produce, market, deliver and support its product."* He further stated that *"a firm's value chain and the way it performs individual activities are a reflection of its history, its strategy, its approach of implementing its strategy, and the underlying economics of the activities themselves."*

Classification of business activities for Value Chain Analysis:

Porter classified business activities under two heads.

- **Primary activities:**Primary activities are directly involved in transforming inputs into outputs and delivery and after-sales support to output. In other words they include:
  - ✓ Inbound logistics: Material handling and warehousing;
  - ✓ Operations: Transforming inputs into final product;
  - ✓ Outbound Logistics: Order processing and distribution;
  - ✓ Marketing and Sales: Communication, pricing and channel management, and
  - ✓ Service: Installation, repair and parts replacement.
- **Support activities:** are the activities which support primary activities. They are handled by the organisation'sstaff functions and include the following :
  - ✓ Procurement:Purchasing of raw materials, supplies and other consumable items as well as assets.
  - ✓ Technology Development:Know-how, procedures and technological inputs needed in every value chain activity.

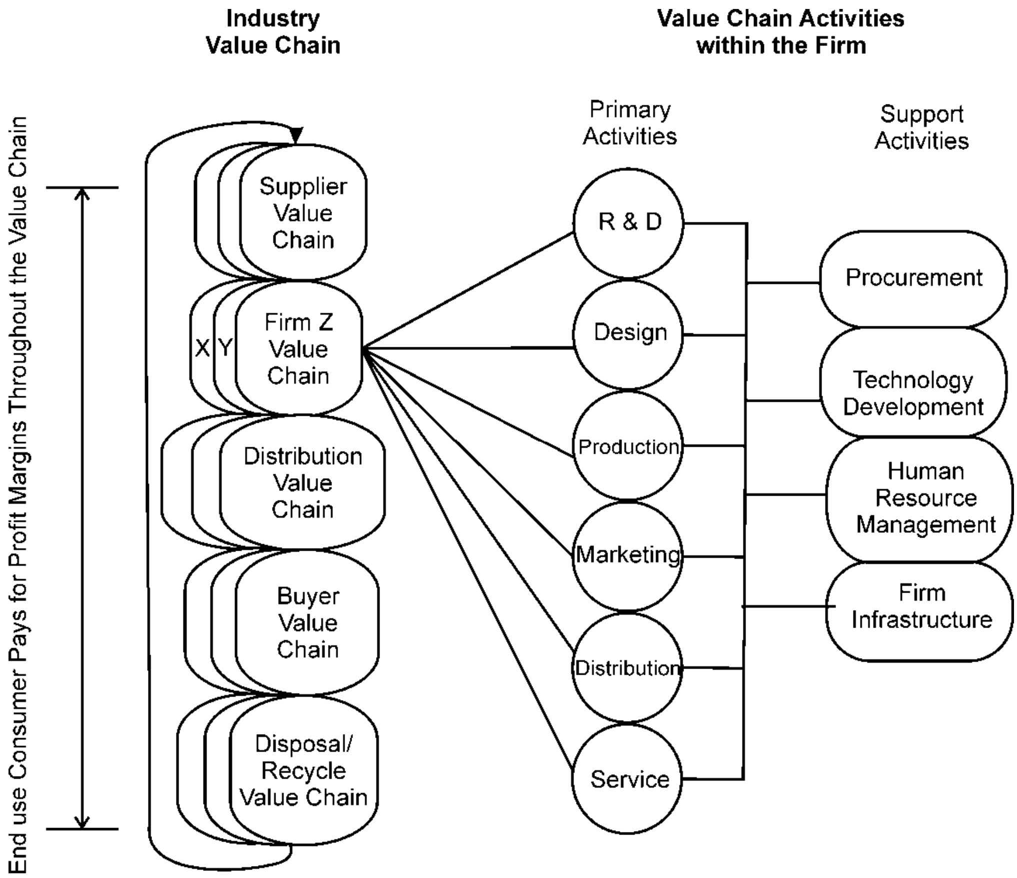
- ✓ Human Resource Management:Selection, promotion and placement, appraisal, rewards; management development; and labour/employee relations.
- ✓ Firm Infrastructure:General management, planning, finance, accounting, legal, government affairs and quality management.



John Shank and V. Govindarajan described the value chain in broader terms. According to them *"the value chain for any firm is the value-creating activities all the way from basic raw material sources from component suppliers through to the ultimate end-use product delivered into the final consumers' hands."* This description views the firm as part of an overall chain of value-creating processes.

<u>Industry Value Chain</u>refers to the series of activities, which add value to the product supplied to the industry.The industry value chain starts with the value-creating processes of suppliers, who provide the basic raw materials and components. It continues with the value creating processes of different classes of buyers or end-use consumers, and culminates in the disposal and recycling of materials.

The industry value chain and the value chain activities within the firm are compared in below diagram.



<u>Value Analysis</u> is a systematic interdisciplinary examination of factors affecting the cost of a product or service in order to devise means of achieving the specified purpose at the required standard of quality and reliability at the target cost.

The aim of value engineering is to achieve the target cost by:

- Identifying improved product designs that reduce the product's cost without sacrificing its utility and/or
- Eliminating unnecessary functions that increase the product's costs and for which customers are not prepared to pay extra.

Value analysis or value engineering is one of the most widely used cost reduction techniques. It can be defined as a technique that yields value improvements.

It investigates into the economic attributes of value. It attempts to reduce cost through Design change, Modification of material specification, Change in the source of supply and so on.

It emphasizes on finding new ways of getting equal or better performance from a product at a lesser cost without affecting its quality, function, utility and reliability. For example, the function of a fastener is to join two or more parts. Value analysis examines the value of this function in terms of alternative methods such as welding, taping, stapling, etc. in view of the stress and vibrations involved in a specific application.

In value analysis each and every product or component of a product is subjected to a critical examination so as to ascertain its utility in the product, its cost, cost benefit ratio and better substitute, etc. When the benefits are lower than the cost, advantage may be granted by giving up the activity concerned or replacing it for betterment. The best product is one that will perform satisfactorily at the lowest cost.

The various steps involved in value analysis are:

- identification of problem,
- collecting information about function, design, material, labour, overhead costs, etc. of the product and finding out the availability of the competitive products in the market and
- exploring and evaluating alternatives and developing them.

In other words, value analysis brings out clearly the areas where the cost of the product can be reduced by pointing out:

- Unnecessary items, components in a product to be removed.
- Possibility of substitution with reduced cost without affecting its quality.
- Possibility of overall simplification in design manufacture,etc. of a product.

1.6.2 Competitive advantage and customer value

In order to survive and prosper in an industry, firms must meet two criteria:

- They must supply what customers want to buy and
- They must survive competition.

A firm's overall competitive advantage derives from the difference between the value it offers to customers and its cost of creating that customer value.

Competitive advantage with regard to products and services takes two possible forms.

- An offering or *differentiation* advantage. If customers perceive a product or service as superior, they become more willing to pay a premium price relative to the price they will have to pay for competing offerings.
- Relative low-cost advantage, under which customers gain when a company's total costs undercut those of its average competitor.

Differentiation Advantage: It occurs when customers perceive that a business unit's product offering (defined to include all attributes relevant to the buying decision) is of higher quality, involves fewer risks and/or outperforms competing product offerings. For example, differentiation may include a firm's ability to deliver goods and services in a timely manner, to produce better quality, to offer the customer a wider range of goods and services, and other factors that provide unique customer value.

Once a company has successfully differentiated its offering, management may exploit the advantage in one of two ways viz., either; increase price until it just offsets the improvement in customer benefits, thus maintaining current market share; or price below the “full premium” level in order to build market share.

Low-Cost Advantage: A firm enjoys a relative cost advantage if its total costs are lower than the market average. This relative cost advantage enables a business to do one of the two things; price its product or services lower than its competitors’in order to gain market share and still maintain current profitability; or match with the price of competing products or services and increase its profitability.

Many sources of cost advantage exist; access to low-cost raw materials; innovative process technology; low-cost access to distribution channels or customers; and superior operating management. A company might also gain a relative cost advantage by exploiting economies of scale in some markets.

The relationship between low-cost advantage and differentiation advantage has been illustrated below.

Competitive Advantage Through Low Cost and/or Differentiation

Relative Differentiation Position	Inferior (**Relative Cost Position**)	Superior (**Relative Cost Position**)
Superior	DifferentiationAdvantage	Differentiationwith Cost Advantage
Inferior	Stuck-in-the-Middle	Low-Cost Advantage

Superior relative cost position offers equivalent customer value for a lower price. Superior relative differentiation position offers better customer value for an equivalent price.

Organisations which fail to gain competitive advantage through low cost or superior differentiation, or both, are “stuck-in-the-middle.” For instance, several American bicycle makers, found themselves in this position during 1980s. These companies lacked a cost advantage and failed to foresee the emerging mountain bike market. By contrast, Cannondale captured market share after introducing its large-diameter frame bicycle.

1.6.3 The Value Chain Approach for Assessing Competitive Advantage

Most corporations define their mission as one of creating products or services. For these organisations, the products or services generated are more important than any single step within their value chain. In contrast, other companies are acutely aware of the strategic importance of individual activities within their value chain. They thrive by concentrating on those activities that allow them to capture maximum value for their customers and themselves.

These firms use the value chain approach to better understand which segments, distribution channels, price points, product differentiation, selling propositions and value chain configurations will yield them the greatest competitive advantage.

The way the value chain approach helps these organisations to assess competitive advantage includes the use of following steps of analysis:

1) **Internal Cost Analysis:**Organisations use the value chain approach to identify sources of profitability and to understand the cost of their internal processes or activities. The principal steps of internal cost analysis are:

   - **Identify the firm's value-creating processes:** To identify a firms value-creating processes, the firm must de-emphasise its functional structure. Most large businesses still organise themselves as cost, revenue, profit and investment centres. These and other organisational sub-units, such as departments, functions, divisions or separate companies that are frequently used for control purposes are not very useful for identifying value-creating processes. Adopting a process perspective requires a horizontal view of the organisation, beginning with product inputs and ending with outputs and customers.

     Processes are structured and measured sets of activities designed to produce a specified output for a particular customer or market. Emphasising process means focusing not on what work is done but on how work is done within the organisation.

     While an organisation's hierarchical structure typically lays out responsibilities and reporting relationships, its process structure shows how the organization delivers customer value. While it is not possible to measure or improve hierarchical structure in any absolute sense, processes lend themselves to such measures as cost, time, output quality and customer satisfaction.

     Because processes normally cut across functional areas, defining process boundaries is not always a straightforward task. People associated with a particular business process may view it in different ways. For example, the new product development process could start with marketing surveys or with delivery of product requirements from marketing to development engineering. The process could end with the release of product specifications or with shipment of the first order. Process boundaries should be defined independently of the way in which activities are organised.

     Selecting the appropriate activity category may be anything but straightforward. The key is to classify value activities according to their true contribution to the firm's competitive advantage. For example, if order processing is important to a firm's customer interactions, then this activity should be classified under marketing.

Management at American Airlines, for example, handed its marketing unit the task of developing and implementing the carrier's SABRE computerised reservation system. The result is a significant competitive advantage that left the other airlines scrambling to copy the system. Even mighty United Airlines has failed to match American's installed base of terminals in travel agencies.

- **Determine the portion of the total cost of the product or service attributable to each value creating process:** The next step is to trace or assign costs and assets to each value-creating process identified. Although firms maintain internal reports and cost accounting information, this information may not align with their processes. Companies might have to reclassify their data or conduct cost studies to assign costs and assets to each process. However, instead of a detailed cost study, rough estimates to assign costs to their value-creating processes may be useful.

  A full-cost approach provides the best estimate of life-cycle costs for evaluating the strategic cost advantage of a firm's value-creating process. Without adopting this approach, a firm risks sacrificing product development costs to short-term profits or for example, the savings in factory labour that an organisation gains through using flexible manufacturing systems, robotics and computer-integrated manufacturing might be offset by the high cost or computer software programmers. The information systems support costs should be allocated to the value-creating processes that benefit from the new systems as part of the full cost.

  For estimating the full cost of each value-creating activity, the full utilisation of the capacity of the activity or its practical capacity, is normally used. Facility managers and equipment vendors are useful sources of capacity estimates. If estimates of full capacity vary widely, a firm could perform the analysis with the resulting costs to assess the sensitivity of the analysis to the different capacity measures. When costs vary dramatically, companies should seek more information for a more realistic long-term estimate of capacity.

  Although many of the processes identified may be instrumental for achieving competitive advantage, various value-creating processes may have differing effects on a firm's costs or products. Companies selling pencils, pens or paper clips, for example, are unlikely to concern themselves with after-sales service. But customer support is a vital part of the competitive strategy for makers of computers or high-speed copiers.

- **Identify the cost drivers for each process:** The next step of internal cost analysis is to identify the factor or cost determinants for each value-creating process. By understanding what factors drive costs, a firm can assign priorities among its cost improvement initiatives. In order to determine its relative cost advantage, a firm should also know the cost factors of its competitors.

  While management accounting systems may contain the total cost of each value-creating process, they may not reveal the causes or factors for the significant individual costs. Using single output or volume measures (e.g., units, labour hours, sales in ₹) to assign costs is often misleading. Multiple cost drivers usually provide more useful information.

✓ **Structural cost drivers** consist of organisational factors that determine the economic structure driving the cost of a firm's products. These cost drivers reflect a firm's long-term decisions, which position the firm in its industry and marketplace. Structural cost drivers may change. For example, large pharmaceutical companies enjoy economies of scale that lower their unit costs for expensive R&D.

Scale	How big an investment to make in manufacturing, R&D, marketing and other resources?
Scope	What is the degree of vertical integration — Horizontal integration is more related to scale?
Experience or learning	How often has the firm already done this?
Technology	What process technologies are used within eachstep of the firm's value chain?
Complexity	How wide a line of products or services to offer to customers?

✓ **Executional cost drivers**capture a firm's operational decisions on how best to employ its resources to achieve its goals and objectives. These cost drivers are determined by management policy, style and culture. How well a firm executes its use of human and physical resources will determine its level of success or failure. For example, worker empowerment and flattened organisations are helping many firms in their continuous improvement efforts.

Work force involvement	Is the workforce involved in decisions and improvements inperformance?
Total quality management	Are the workforce and managers committed to total Quality in processes and products?
CapacityUtilization	What are the scale choices on maximum plant construction?
Plant layout efficiency	How efficient, against current norms, is the plant's layout?

Product configuration	Is the design or formulation of the product effective?
Linkages with suppliers and customers	Is the linkage with suppliers and customers exploited, according to the firm's value chain?

Few structural and executional cost drivers can be operationalised under existing management accounting systems in the cost analysis of the value chain. However, these cost drivers do offer an important reminder of the strategic decisions that firms need to make, or at least acknowledge, in designing their value-generating systems. Increasingly, companies are using activity-based costing to understand the resources/costs consumed by the activities and processes used in delivering their products and services.

- **Identify the links between processes:** While individual value activities are considered separate and discrete, they are not necessarily independent. Most activities within a value chain are interdependent. Firms must not overlook value chain linkages among interdependent activities that may impact their total cost.

  For example, cost improvement programs in one value chain process may lower or increase costs and/or revenues in other processes. Transfers of goods and services from one value chain process to another increases cost. Eliminating these transfers reduces the costs of purchasing, invoicing and other record keeping functions.

  Tandem Computers eliminated its costs of purchase orders, invoicing and other functions by jointly developing a detailed bar code process with its suppliers. By improving its upstream design and engineering processes for the Taurus, Ford saved on downstream production and customer service costs. Using fewer floppy drives and motherboards in its PCs has enabled IBM to halve its delivered cost in two years.

  As sources of competitive advantage, these relationships or linkages among activities can be as important as the activities themselves. Such linkages may also offer sustainable competitive advantage, because their subtle, complex nature makes them difficult for competitors to imitate.

- **Evaluate the opportunities for achieving relative cost advantage:** In many organisations, cost reductions are made across the board (e.g., "eliminate 10 per cent from every department"). Because these firms do not reduce their costs strategically, this effort usually fails. More often than not, across-the-board cost reduction misconstrues the underlying problem. The point is not to become more efficient at insignificant activities, but to better meet customer demands.

Using the value chain approach, a company goes beyond simple across-the-board cuts and attempts to lower cost and improve efficiency within each value-creating process. For instance, a company might negotiate lower costs of process inputs such as wages or purchases, or evaluate make-or-buy options.

Reducing process input costs often means negotiating lower wages or moving production to countries with cheaper labour costs. Suppliers might be willing to drop their prices if the company negotiates long-term contracts. Companies also use buyer-seller partnerships to gain advantages in cost, quality, time, flexibility, delivery and technology.

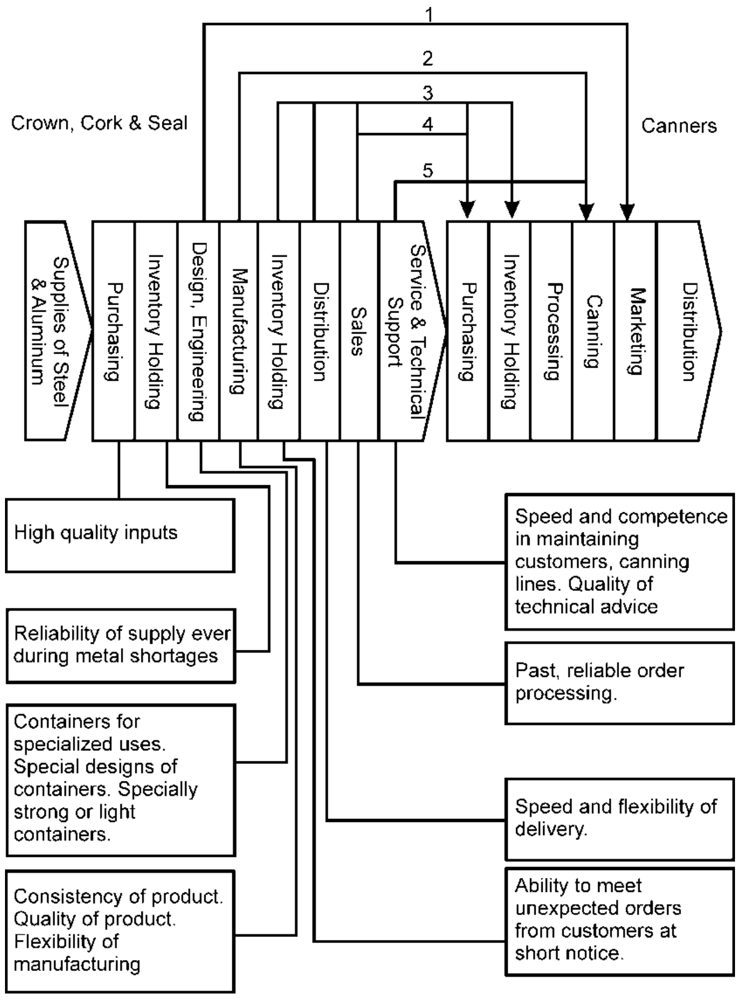
Some processes may offer more opportunities for improvement than others. In order to get the most out of its cost reduction programs, a company should prioritise its value-creating process. Under the 80:20 rule, 20 per cent of the value-creating processes often account for 80 per cent of total costs.

2) **Internal Differentiation Analysis:** The value chain approach is also used by organisations to identify opportunities for creating and sustaining superior differentiation. In this situation, the primary focus is on the customer's perceived value of the products and services.As with internal cost analysis, internal differentiation analysis requires firms to first identify their value-creating processes and primary cost drivers.They are then ready to perform a differentiation analysis using the following guidelines:

- **Identify the customers'value-creating processes:**To pursue a superior differentiation strategy, a firm's processes must enhance those of its customers. Thus, a firm should carefully study the value-creating processes of its customers.

> Following diagram presents such an analysis for Crown, Cork and Seal Company (CCS), a metal can maker, and its customers in the late 1970s. The metal container industry was characterised by low growth, low profits and intense competition. The CCS succeeded with a differentiation strategy, which is usually very difficult to accomplish in a commodity-type business. Two different groups of customers—food and beverage canners—accounted for 80 per cent of the metal containers produced.

VALUE CHAIN FOR CROWN, CORK AND SEAL COMPANY



1. Designing distinctive can for customers may assist their own marketing activities.
2. Consistent can quality lowers customers' canning costs by avoiding breakdowns and holdups on their canning lines.
3. By maintaining high stocks and offering speedy delivery, customers can economize on their own stockholding (they may even be able to move to a just in time system of can supply).
4. Efficient order processing can reduce customers' ordering costs.
5. Capable and fast technical support can reduce the costs of breakdowns on canning lines.

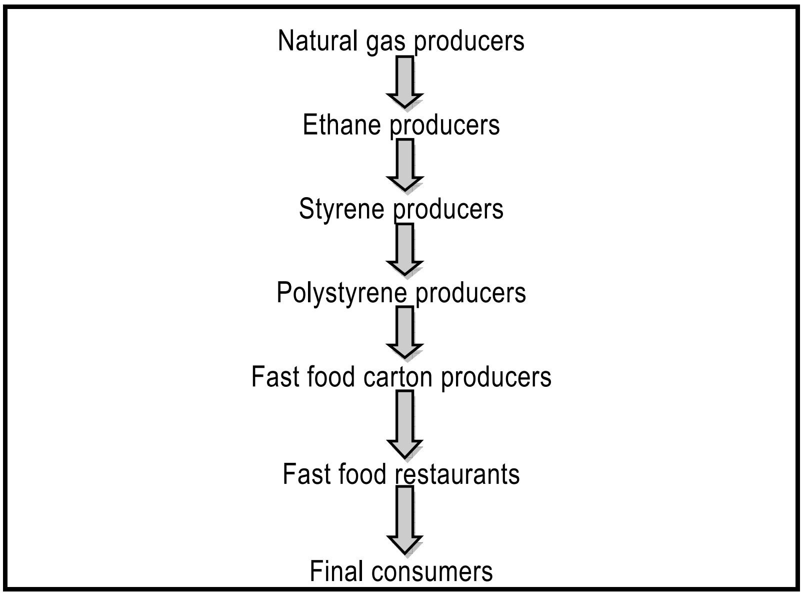
- **Evaluate differentiation strategies for enhancing customer value:** The key to successful differentiation under the value-chain approach is to identify the value creating processes that distinguish a firm's products or services from those of its competitors. In making this distinction, customer value is emphasised.The ways customer value can be enhanced through differentiation include:
  - ✓ **Product features-**that are esthetically appealing or functionally superior. Forexample, the Mercedes-Benz automobile accomplished this feat so well for years that its name became synonymous with the highest level of quality-people would describe a product as the "Mercedes-Benz" of its category;
  - ✓ **Marketing channels -** that provide desired levels of responsiveness, convenience, variety and information;
  - ✓ **Service and support -**tailored to end-user and channel member sophistication and urgency of need;
  - ✓ **Brand or image positioning -**that lends greater appeal to the company's offerings on critical selection criteria. For many years, this quality image has allowed the American Express Co. to command a significant price premium in the highly competitive financial services market; and
  - ✓ **Price -** including both net purchase price and cost savings available to the customer through the use of the product and service.
- **Determine the best sustainable differentiation strategies:** For a firm to achieve superior differentiation, it must utilise the best mix of resources in creating value for its customers. In order to prioritise its processes as sources of differentiation, a company must determine what attributes of each process enhance customer value.

  The more unique a firm's resources and skills, the more sustainable is its differentiation advantage over competitors.

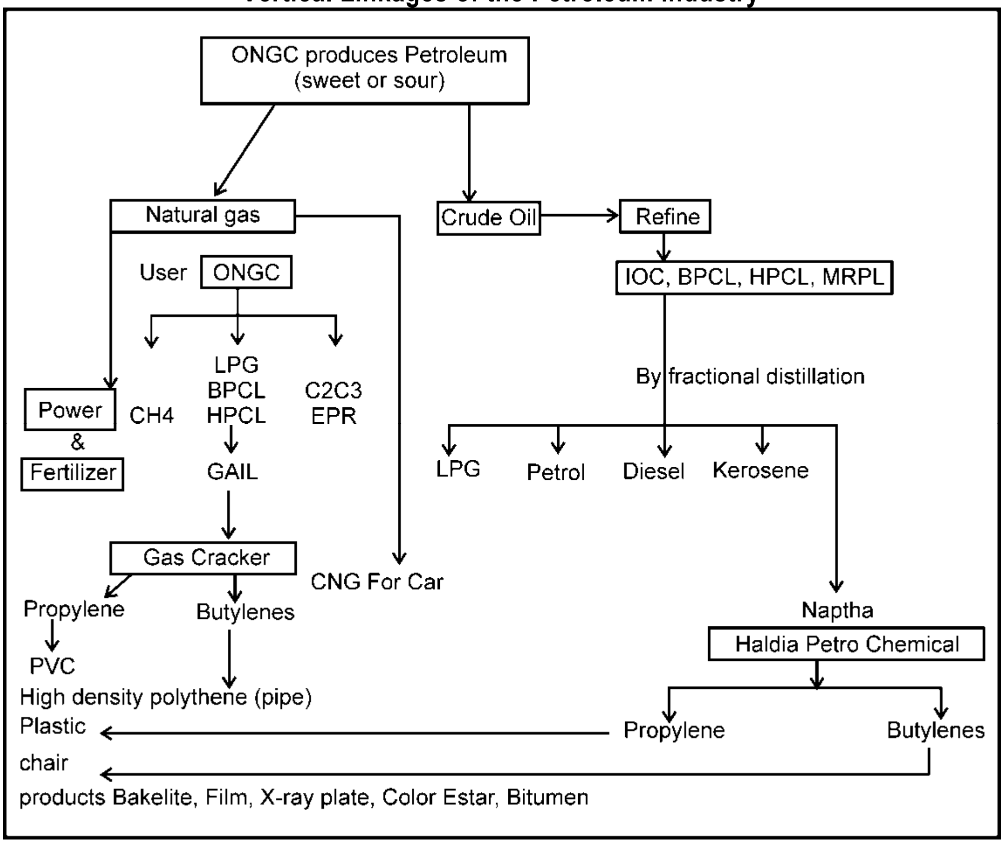
3) **Vertical Linkage Analysis:** Linkages among value-creating processes do not end with the activities within a firm. The greatest competitive advantage may come out of linkages between a firm's value-creating activities and those of its suppliers, channels or users.

   Vertical linkage analysis is a much broader application of internal cost and differentiation analysis that includes all upstream and downstream value-creating processes throughout the industry. Vertical linkage analysis considers all links from the source of raw materials to the disposal and/or recycling of the product. Following diagrams outlines the vertical links involved in the production of "fast food" containers and "Petroleum Industry".

Vertical linkages in the 'Production of Plastic Food Containers'



Vertical Linkages of the Petroleum Industry



Vertical linkage can reveal which activities are the most (and least) critical to competitive advantage (or disadvantage). For example, Swiss watchmakers succeeded for years as relatively small, labour-intensiveassemblers. Then came the 1970s and the advent of low-cost, mass-produced watches. The Swiss responded by restructuring their industry to gain economies of scale similar to those enjoyed by their new global competitors.

Vertical linkage analysis includes the following steps:

- **Identify the industry's value chain and assign costs, revenues and assets to value-creating processes:**Because vertical linkages can be complex and intangible, they are often overlooked by organisations. For example, the petroleum industry consists of numerous value-creating processes or activities, including exploration, production, refining, marketing and distribution. These processes define the value chain for this industry. One company may participate in all parts of this value chain; another firm may participate in only a few. This diversity of operations and organisations makes it difficult to adopt a standard approach for identifying industry value chain processes.

  Few firms have information systems that can identify and analyse these subtle relationships. For example, profitability and return on assets are key measures of competitive advantage throughout an industry's value chain. It can be extremely difficult to obtain pertinent information for these measures, including operating costs, revenues and assets for each process throughout the industry's value chain. However, this information is necessary to calculate a rate of return on assets for each value chain process.

  Obtaining the replacement or current cost of physical assets used by a value-creating activity is a necessary but often-complex undertaking. Historical or book values usually provide inadequate measures of current investment. Plant engineers, equipment vendors and independent appraisal professionals may be consulted to help establish current asset values. Likewise, establishing prices for transferring goods and services along value chain processes requires an understanding of market or competitive-based rates. If at least one firm competes in each stage of value creation, then competitive market prices are available. If not, then a company must use judgement in determining a transfer price that incorporates a normal profit margin on full costs. For long-term strategic decision-making, companies should use full cost under conditions of full capacity for the value activity. While several measures of capacity exist, the best measure should represent the long-term utilisation of the value activity's assets (sometimes called "practical capacity").

  Publicly available financial reports produced by firms throughout the industry value chain can provide key financial information. Typically, this information is neither in the proper format nor disaggregated enough to accommodate vertical linkage analysis. Significant analysis, data manipulation and judgement may be necessary to obtain the appropriate information for each value chain process.

For intermediate transfers between processes, competitive market prices, if available, should be substituted for the internal transfer prices. For example, competitive market prices for a single link in the value chain may be obtained from individual firms that operate only in that link of the chain. For long-term cost estimation, full costs should be used rather than marginal, variable or incremental costs.

- **Diagnose the cost drivers for each value-creating process:**Traditional management or cost accounting systems often assign costs by using a single output measure of operating activity, such as output volume. For vertical linkage analysis, a single measure is inadequate to capture the underlying cost categories. Direct labour-based measures may be appropriate for labour-intensive activities; operating hours may be appropriate for machine-based activities.
- **Evaluate the opportunities for sustainable competitive advantage:**By nature, competitive advantage is relative. In an ideal world, a firm can gauge its competitive position by knowing its competitor's value chains and the rates of return on each. In reality, however, this may be rather difficult; the competitor's internal cost, revenue and asset data for its processes are generally unavailable. Sufficient qualitative information usually exists on a firm's major value-creating processes and the strategies for each. By understanding how other companies compete in each process of the industry value chain, a firm can use the qualitative analysis to seek out competitive niches even if financial data areunavailable.

  Value chains for three competitors in the rapidly changing telecommunications industry—AT&T, NYNEX and IBM—are listed below in table, along with the strategic differences for each firm (Hax and Majluf, 1991). The strategic differences reflect varying structural and executional cost drivers. In marketing, for instance, AT&T started with no organisation but with significant name recognition. The regional marketing scale of NYNEX and the worldwide marketing scale of IBM are important cost advantages.

Value Chain Differences: The Telecommunications Industry

Value Chain Processes	AT & T	Strategic Differences NYNEX	IBM
Procurement	Owns manufacturing Branch (Western Electric)	Free to use any Supplier it wants	Owns Rolm, CPE manufacturer
Technology Development	Technological leadership through Bell Labs	Focus on software products	Strong R & D in computer hardware and software technologies

Operations	National presence High quality of equipment through heavy capital expenditure Similar communications standards nationwide Strongest national telecommunications network	Regional monopoly Innovative equipment from outside suppliers High-quality regional network through heavy capital investment	Global presence Leading computer technology Partnership with MCI
Marketing and Sales	New emphasis on marketing (still weak) High name recoginism Long-term relationship with clients Recruits computer executives	Use of Bell logo Focus on top 1,000 corporate customers Sales and distribution centres close to customers	Strong reputation for marketing excellence Already sells to most major corporations Experianced Sales force
Source:*Hax and Majluf. 1991*			

Finding innovative ways to perform value-creating activities helps firms to improve their overall performance and achieve competitive advantage. In order to thrive in the mature, highly competitive meat packing industry, for example, Iowa Beef Processors built its plants near cattle ranches, thus eliminating the high cost of shipping cattle to northern processing plants. In order to lower its costs, Tropicana froze slabs of orange juice concentrate near the orange groves in Florida and shipped the slabs to its large markets in the Northeastern U. S. Only then did the company mix the concentrate with water, thus avoiding the lengthy and costly shipment of water.

Increased global competition forces firms to focus on worldwide sustainable competitive advantage. Global competition cites following four major factors that influences national competitive advantage:

- ✓ **Factor conditions:** The nation's position in factors of production, such as skilled labour or infrastructure, necessary to compete in a given industry;
- ✓ **Demand conditions:** The nature of domestic demand for the industry's product or service;
- ✓ **Related and supporting industries:** The presence or absence in the nation of

supplier industries and related industries that are internationally competitive; and

- ✓ **Firm strategy, structure and rivalry**: The conditions in the nation governing how companies are created, organised and managed and the nature of domestic rivalry.

Geographic scope can allow firms to gain substantial competitive advantages by sharing or co-coordinating similar value activities in different places. The importance of this advantage is illustrated by the recent success of firms with a global scope, such as Canon (Japan), Caterpillar (U.S.), N. V. Philips (Netherlands) and Siemens (West Germany). These firms sell and service their products in practically every corner of the globe.

1.6.4 Strategic Frameworks for Value Chain Analysis

Value chain analysis requires a strategic framework or focus for organising internal and external information, for analysing information, and for summarising findings and recommendations. Because value chain analysis is still evolving, no uniform practices have yet been established. However, borrowing recent concepts from strategists and organisation experts, three useful strategic frameworks for value chain analysis are:

- **Industry Structure Analysis:** Michael Porter developed a five factors model as a way to organise information about an industry structure to evaluate its potential attractiveness.

  Under this model, the profitability of an industry or market — measured by the long-term return on investment of the average firm — depends largely on five factors that influence profitability.

  Factors which influence profitability are:

  - ✓ **Bargaining power of buyers**: The degree of buyer power generally depends on:
    - ➢ customer concentration (the higher the concentration of customers, the greater is their negotiation leverage);
    - ➢ the propensity for customers to integrate backward (the higher the propensity for backward integration, the greater the bargaining leverage);
    - ➢ costs of switching suppliers (the lower the switching costs, the greater the buyer’s leverage); and
    - ➢ the number of alternative suppliers (the greater the number, the greater the customer’s leverage).
  - ✓ **Bargaining power of suppliers**: Just as powerful buyers can squeeze profits by putting downward pressure on prices, suppliers squeeze profits by increasing input costs. The same factors that determine the power of buyers also determine the power of suppliers. The bargaining power of suppliers and buyers relative to the firm depends on the relationships between their value chains. Bargaining power will be a function of relative strengths, in particular, value activities that depend on one another.

Identifying the specific activities involved and the nature of their strengths and relationships can give important insights into the power balance between buyer and seller, and how it may be altered for the firm's benefit.

- ✓ **Threat of substitute products or services**:The potential for profit in an industry is determined by the maximum price that customers are willing to pay. This depends primarily on the availability of substitutes. When few substitutes exist for a product — e.g., gasoline — consumers are willing to pay a potentially high price. If close substitutes for a product exist, then there is a limit to what price customers are willing to pay. Any price increase will then cause some customers to switch to substitutes. A thorough understanding of the value chains of buyers as they relate to the firm's product can help in assessing (and combating) the threat of substitution.
- ✓ **Threat of new entrants:** If an industry is earning a return on invested capital above the cost of capital, that industry will act as a magnet to firms outside the industry. Unless the entry of new firms is barred, the rate of profit must fall to the competitive level. Even the mere threat of entry may be sufficient to ensure that established firms constrain their prices to the competitive level.
- ✓ **Intensity of competition**:Markets experiencing rapid growth typically see less intense competition. Rival companies can usually satisfy profitability and growth without having to take market shares from their competitors.

  The variety and nature of the value chains of competitors shape many of the characteristics of an industry. The relative importance of economies of scale versus economies of scope, for example, depends on the kind(s) of technology employed in competitors'value chains. The stability of the industry and of its competitive situation also relates to what happens to the value chains of firms in the industry. The effectiveness of low cost versus differentiation strategies depends on the nature of users'value chains, and on how competitors value chains interact with those of both sellers and users.

Since these five forces are ever-changing, Porter's framework needs to be employed as a *dynamic* analytical tool. This is because competition is a dynamic process; equilibrium is never reached and industry structures are constantly being reformed.

A major difficulty in industry structure analysis lies in defining the specific industry. No industry has clear boundaries either in terms of products or geographical areas. For example, does one analyse the industry environment of Ford as the "transportation equipment" industry, the "motor vehicles and equipment" industry or the "automobile" industry?

To overcome the difficulty of defining an industry, the concept of *substitutability* can be applied to a firm's supply and demand chains. On the demand side, if buyers are willing to substitute one product for another — e.g., Toyotas for Fords — then the manufacturers belong in a single industry. However, this guideline does not always hold. For example, customers may be unwilling to substitute Apple Macintosh computers for H.P. computers, even though both manufacturers belong to the same industry. On the supply side, if two manufacturers can make each other's products, then they belong to a single industry.

- **Core Competencies Analysis:** Industry structure analysis is well suited to describing thewhat of competitiveness, i.e., what makes one firm or one industry more profitable than another. But understanding the particulars of such advantages as low cost, quality, customer service and time to market may still leave the question of why largely unanswered. For example, why do some companies seem able to continually create new forms of competitive advantage while others seem able only to observe and follow? Why some firms are net advantage creators and others net advantage imitators? For assessing competitive advantage it is necessary not only to keep score of existing advantages — what they are and who has them — but also to discover what it is that drives the process of advantage creation. Industry structure analysis is much better suited to the first task than to the second.

  Thus, industry structure analysis must be supplemented by an equally explicit core competence focus. Organisations need to be viewed not only as a portfolio of products or services, but also as a portfolio of core competencies.

  Core competencies are created by superior integration of technological, physical and human resources. They represent distinctive skills as well as intangible, invisible, intellectual assets and cultural capabilities. Cultural capabilities refer to the ability to manage change, the ability to learn and team working. Organisations should be viewed as a bundle of a few core competencies, each supported by several individual skills.

  Core competencies are the connective tissue that holds together a portfolio of seemingly diverse businesses. They are the lingua franca that allows managers to translate insights and experience from one business setting into another. Core competence-based diversification reduces risk and investment and increases the opportunities for transferring learning and best practice across business units.

  For instance, Microsoft's only factory asset is its human imagination. This company has excelled in inventing new ways of using information technology for a wide variety of end users.In contrast, using its core competence in information processing, Xerox developed icons, pull-down menus and the computer mouse, but failed to exploit the marketplace.

  A core competence is identified by the following tests:

  ✓ **Can it be leveraged?**—does it provide potential access to a wide variety of markets?

  ✓ **Does it enhance customer value?** — does it make a significant contribution to the perceived customer benefits of the end product?

  ✓ **Can it be imitated?**—does it reduce the threat of imitation by competitors?

  Applying the value chain approach to core competencies for competitive advantage includes the following steps:

- ✓ **Validate core competencies in current businesses:** Core competencies should tie together the portfolio of end products and help a firm excel in dominating its industry. For example, Corning Glass's core competence is its ability to melt specialty glass. Pyrex, television bulbs, headlamps and optical wave guides are just a few of the products of this successful producer. Procter & Gamble's R&D expertise and marketing/distribution skills provide a significant competitive advantage in a wide range of mass consumer products (e.g., Ivory, Tide, Folgers, Crisco, Pampers).

  Core competencies need to be continually validated. In the early 1970s, Timex held half of the global market for watches with its core competence in low-cost management of precision manufacturing. By the mid-1970, the watch industry moved to digital technology, making Timex's core competence irrelevant.

- ✓ **Export or leverage competencies to the value chains of other existing businesses:** The same set of core competencies can be exploited in multiple businesses by exporting core competencies to the value chains of other existing businesses.

  Examples:

  - ➢ **Honda.**One of Honda's core competencies is designing and producing small engines. By exporting this core competence to a wide variety of business lines, the company seeks to have six Hondas in every garage: autos, motorcycles, snowmobiles, lawnmowers, snow blowers, chain saws and power tools. Other Honda core competencies are dealership management and shorter product development cycles.
  - ➢ **Marriott Corp.**Marriott Corp. has core competencies in food service and hospitality skills, standardised hotel operating procedures, and a shared procurement and distribution system. Besides employing these core competencies in hotels, the company uses them in its other businesses, including institutional food service, consumer food and restaurants, cruise ships and theme parks.
  - ➢ **AT&T.**AT&T extended its core competence as an efficient processor of customer accounts by entering the credit card business. Kimberly Clark's entry into disposable diapers extended its core competence in the design of paper products.

- ✓ **Use core competencies to reconfigure the value chains of existing businesses:** While firms may manage their existing value chains better than their competitors, sophisticated firms work harder on using their core competencies to reconfigure the value chain to improve payoffs. Otherwise, competitors may exploit opportunities.

Examples:

- **Japanese watch-makers**-Japanese watch-makers side-stepped traditional distribution channels in favour of mass merchandisers such as department store chains. By efficiently consolidating freight, Emery Freight dominated the air freight industry and was consistently a leader in profitability in U. S. industry. Federal Express reconfigured the air freight business by focusing on the overnight delivery of small packages.
- **Tetra-Pak**-Tetra-Pak is an excellent example of a firm that reconfigured the value chain in the packaging industry for dairy products and orange juice. Tetra-Pak designed filling machine for its aseptic packages and changed the packaging industry. Exhibit 5 illustrates Tetra-Pak's changes to the value chain.

  How Tetra-Pak reconfigured the value chain?

Filling	**Transport**	**Retail**	**Customers Display**
Make container on site	No refrigerated trucks	Low store handling	Longer shelf life
Tetra Pak specialized equipment	No wasted space in filling & packing	No need to refrigerate & less space is required	No need to refrigerate & less space is required

- **IKEA-**Another example of a value chain reconfiguration is IKEA, which grew from a small, Swedish mail-order furniture operation to one of the world's largest retailers of home furnishings (Normann& Ramirez, 1993). As illustrated below, IKEA selected numerous factors to offer prices that are 25-50 per cent lower than those of competitors.

  How IKEA reconfigured the furniture industry?

Value Chain	**Major Choice**
Design	Simple, high quality, designed to lower cost
Parts	Standard & common, global supplier network
Assembly	By the customer
Transport/stocking	Computerised system for suppliers & warehouses
Marketing	Scandinavian image
Display	Focus on designs, not pieces, to create value
Home delivery	By the customer

Source :Normann and Ramirez, 1993

✓ **Use core competencies to create new value chains:** With strong core competencies in its existing businesses, an organisation can seek new customers by developing new value chains.

Examples

Federal Express (FedEx): FedEx transferred its expertise in the delivery of small packages to contract new business with L.L. Bean for overnight distribution. Disney has exported its people-moving skills to urban mass transit for Oakland, California.

- **Segmentation Analysis:** Industries are sometimes collections of different market segments. Vertically integrated industries are good examples of a string of natural businesses from the source of raw material to the end use by the final consumer. Several firms in the paper and steel industries are vertically integrated. Not all firms in an industry participate in all segments.

  If the nature and intensity of Porter's five forces or the core competencies vary for various segments of an industry, then the structural characteristics of different industry segments need to be examined. This analysis will reveal the competitive advantages or disadvantages of different segments. A firm may use this information to decide to exit the segment, to enter a segment, reconfigure one or more segments, or embark on cost reduction/differentiation programs.

  Differences in structure and competition among segments may also mean differences in key success factors among segments.

  Using the value chain approach for segmentation analysis, Grant (1991) recommended five steps:

  ✓ **Identify segmentation variables and categories:** There may be literally millions of ways to divide up the market into segments. Typically, an analysis considers between five to ten segmentation variables. These variables are evaluated on the basis of their ability to identify segments for which different competitive strategies are (or should be) pursued.

    The selection of the most useful segment-defining variables is rarely obvious. Industries may be subdivided by product lines, type of customer, channels of distribution and region/geography. The most common segmentation variables considered are type of customer and product related, as illustrated below.

    Approaches to defining Segmentation Variables-

    Customer Characteristics

Geographic	Small communities as markets for discount stores
Type of organisation	Computer needs of restaurants versus manufacturing firms versus banks versus retailers
Size of firm	Large hospitalversus mediumversus small
Life-style	Jaguar buyers tend to be more adventurous, less conservative than buyers of Mercedes-Benz and BMW
Sex	The Virginia Slims cigarettes for women

Age	Cereals for children versus adults
Occupation	The paper copier needs of lawyers versus bankers versus dentists

Product-Related Approaches

Use type	Appliance buyer-home builder, remodeler,homeowner
Usage	The heavy potato user - the fast-food outlets
Benefits sought	Dessert caters - those who are calorie-consciousversus those who are more concerned with convenience
Price sensitivity	Price-sensitive Honda Civic buyer versus the luxury Mercedes-Benz buyer
Competitor	Those computer users now committed to IBM
Application	Professional users of chain saws versus the homeowner
Brand loyalty	Those committed to IBM versus others

The first set of variables describes segments in terms of general characteristics unrelated to the product involved. Thus, a bakery might be concerned with geographic segments, focusing on one or more regions or even neighbourhoods. It might also divide its market into organisational types such as at-home customers, restaurants, dining operations in schools, hospitals and so on. Demographics can define segments representing strategic opportunities such as single parents, professional women and elderly people.

The second category of segment variables includes those that are related to the product. One of the most frequently employed is usage. A bakery may employ a very different strategy in serving restaurants that are heavy users of bakery products than restaurants that use fewer bakery products. Zenith made a niche for itself in the very competitive personal computer industry by focusing on government, which is the largest computer user.

Segmenting by competitor is useful because it frequently leads to a well-defined strategy and a strong positioning statement. Thus, a target customer group for the Toyota Cressida consists of buyers of high-performance European cars such as the BMW. The Cressida is positioned against the BMW as offering comparable performance for a substantially lower cost.

- ✓ **Construct a segmentation matrix:** After customer and product related variables have been selected for identifying different segments, a segmentation matrix can be developed. Two or more dimensions may be used to partition an industry.Restaurants could be divided into four dimensions; types of cuisine, price range, type of service (e.g., sit-down, buffet, cafeteria, take-out, fast food) and location.

Examples:

British Frozen Food Industry- A segmentation matrix for the British frozen foods industry is presented below. Five types of product and five channels of distribution are used to construct the two-dimensional segmentation matrix consisting of 25 potential segments. However, not every cell in the matrix may be relevant. Empty cells may represent future opportunities for products or services.

		Distribution Channels				
		Supermarkets		Independent Grocery Retailers	Specialist Freezer Stores	Caterers
		Producers' Brands	Retailers' Brands			
Product Types	Vegetables					
	Fruits					
	Meat Products					
	Desserts					
	Convenience ReadyMeals					

The above matrix identifies five categories of frozen food, and five distribution channels. While the basic distinction of customers is between retail and catering, within retailing there are three distinct categories of outlet; supermarkets, independent grocery stores, and specialist retailers of frozen foods ("home freezer centres"). In addition, different market conditions exist for processors supplying frozen foods for sale under their own brand names as opposed to those supplying frozen foods for sale under the brand name of the retailer.

✓ **Analyse segment attractiveness:**Competitive assessments using industry structure analysis or core competencies analysis can also be used to evaluate the profitability of different segments. However, the competitive focus shifts to an analysis of the different segments.

Examples:

Frozen Foods Industry-In the frozen foods industry segmentation, independent grocers and caterers may be willing to substitute fresh fruits and vegetables for frozen goods. Therefore, the threat of substitutes within the segments and from outside sources must be carefully examined.In addition, the interrelationship among segments must be carefully considered. For example, caterers may purchase frozen food items from supermarkets at bargain prices. Segments may be natural buyers, sellers or substitutes for one another.

Automobile Industry - In the automobile industry, the luxury car and sports car segments were high-priced, high-margin products with less intense competition than other automobile segments. The introduction of high-quality, lower-priced Acura, Lexus and Infinity autos changed the competitive structure of these high-priced segments.

- ✓ **Identify key success factors for each segment:**Quality, delivery, customer satisfaction, market share, profitability and return on investment are common measures of corporate success. In this regard, each segment must be assessed using the most appropriate key success factors. Cost and differentiation advantages should be highlighted by these measures.
- ✓ Examination of differences among segments in buyers'purchase criteria can reveal clear differences in key success factors.
- ✓ **Analyse attractiveness of broad versus narrow segment scope:** A wide choice of segments for an industry requires careful matching of a firm's resources with the market. The competitive advantage of each segment may be identified in terms of low cost and/or differentiation.

Examples:

Sharing costs across different market segments may provide a competitive advantage.Gillette broadened its shaving systems to include electric shavers through its 1970 acquisition of Braun.Lipton recently entered the bottled iced-tea market.On the other hand, when the Toro Company broadened its distribution channels for its snow blowers and lawnmowers to include discount chains, it almost went bankrupt. Felling betrayed, a number of Toro's dealers dropped its products.

Taking a narrow segment focus may leave a firm vulnerable to competitors.For instance, by relying solely on its lemon-lime soft drink, 7-Up left itself at a competitive disadvantage to Coca-Cola and Pepsi.

In many industries, aggressive firms are moving toward multiple-segment strategies. Campbell Soup, for example, makes its nacho cheese soup spicier for Texas and California customers and offers a Creole soup for Southern markets and a red-bean soup for Hispanic areas. In New York, Campbell uses promotions linking Swanson frozen dinners with the New York Giants football team; in the Sierra mountains, skiers are treated to hot soup samples. Developing multiple strategies is costly and often must be justified by an enhanced aggregate impact.

Some firms decide to avoid or abandon segments because of limited resources or because of uncertain attractiveness. For example, in the 1960s, IBM decided not to enter the mini-computer segment. This allowed upstart Digital Equipment Corp. to dominate this segment of the computer industry.

A segment justifying a unique strategy must be of worthwhile size to support a business strategy. Furthermore, that business strategy needs to be effective with respect to the target segment in order to be cost effective. In general, it is costly to develop a strategy for a segment. The question usually is whether or not the effectiveness of the strategy will compensate for this added cost.

1.6.5 Limitations of Value Chain Analysis

- **Non-availability of data:** Internal data on costs, revenues and assets used for value chain analysis are derived from financial information of a single period. For long term strategic decision making, changes in cost structures, market prices and capital investments etc. may not be readily available.
- **Identification of stages:** Identifying stages in an industry's value chain is limited by the ability to locate at least one firm that participates in a specific stage. Breaking a value stage into two or more stages when an outside firm does not complete in these stages is strictly judgment.
- **Ascertainment of cost, revenues and assets:** Finding the costs revenues and assets for each value chain activity poses/gives rise to serious difficulties. There is no scientific approach and much depends upon trial and error and experimentation methods.
- **Identification of cost drivers:** Isolating cost drivers for each value-creating activity, identifying value chain linkages across activities and computing supplier and customer profit margins present serious challenges.
- **Resistance from employees:**Value chain analysis is not easily understandable to all employees and hence may face resistance from employees as well as managers.
- **Science vs. Art:**Value chain analysis is not exact science. It is more "art" than preparing precise accounting reports.Certain judgments and factors of analysis are purely subjective and differ from person to person.

1.6.6 Organisational and Managerial Accounting Challenges

Value chain analysis offers an excellent opportunity to integrate strategic planning with management accounting to guide the firm to growth and survival. This change in focus for management accounting is necessary to maintain its critical role as the information profession.

The most significant challenge for senior management and management accountants is to recognise that the traditional, functional, internally oriented information system is inadequate for the firm engaged in global competition.

Another challenge for management accountants is to bring the importance of customer value to the forefront of managements strategic thinking. For many managers and firms, this requires a great deal of education and awareness. Management accountants should take the initiative to bring the value chain message to major players in the firm. Seminars, articles, value chain examples and company-specific applications are useful to illustrate the advantages of value chain analysis.

Although value chain analysis requires expertise in internal operations and information, it demands a great deal of external information. Management accountants must seek relevant financial and non-financial information from sources outside the organisation.

Management accountants must integrate databases and potential sources of timely information on competitive forces confronting the business. This calls for innovation and creativity in gathering and analysing information for management decisions.

Designing internal and external information systems to assist managers in planning, monitoring and improving value-creating processes is another challenge facing management accountants.

Information technology is improving daily but existing information systems are slow to change. Management accountants should solicit support from all senior managers for allocating resources to develop and improve value chain-oriented information systems.

Value chain analysis requires the cooperation of all managers involved in value chain processes, including engineers, designers, production managers, marketing managers and distribution managers. Leadership from the CEO is vital to successful cooperation of managers. The management accountant should ensure that the CEO is committed to value chain analysis and the organisational changes necessary for its successful implementation.

For many service companies, Porter's value chain model emphasising manufacturing firms may appear inappropriate. However, every organisation (banks, hospitals, airlines, professional firms) has a variety of primary and support value-creating activities to which value chain analysis applies. For example, a publishing company might have the following primary activities; information acquisition, editorial, production, distribution, sales and service. Support activities include new product and business development, technology assessment and development, human resource management and firm infrastructure. If strategy is seen as the pursuit of competitive advantage, the link between the formulation of service strategy and operational service delivery is vital.

1.6.7 Value Chain Analysis vs. Conventional Management Accounting

Information generated from the traditional management accounting systems, including cost accounting, is generally unsuitable for value chain analysis for a variety of reasons. Below table provides a comparison of value chain analysis and traditional management accounting.

	Traditional ManagementAccounting	**Value Chain Analysis in the Strategic Framework**
Focus	**Internal**	**External**
Perspective	Value Added	Entire set of linked activities from suppliers to end-use customers
Cost Driver Concept	Single cost driver (Cost is function ofVolume) Application at the overallfirm level (cost-volume-profit analysis)	Multiple cost drivers — Structuraldrivers (e.g. scale, scope, experience,technology and complexity) — Executional drivers (e.g. participative management and

		plant layout) A set of unique cost drivers for each value activity.
Cost Containment Philosophy	"Across the board" cost reductions	View cost containment as a function ofthe cost drivers regulating each value activity. Exploit linkages with suppliers Exploit linkages with customers Exploitprocess linkages within the firm "Spend to save".
Insights for Strategic Decisions	Somewhat limited	Identify cost drivers at the individual activity level, and develop cost/differentiation advantage either bycontrolling those drivers better thancompetitors by reconfiguring the value chain (e.g. Federal Express in maildelivery, and MCI in long distance telephone) For each value activity, askstrategic questions pertaining to: — Makeversus buy — Forward/backwardintegration Quantity and assess"supplier power" and "buyer power", and exploit linkages with suppliers and buyers.

Generally, traditional management accounting focuses on internal information. It often places excessive emphasis on manufacturing costs. It also assumes that cost reduction must be found in the "value-added" process, i.e., selling price less the cost of raw material.

Using a value added approach can be misleading, since there are many other purchased inputs such as engineering, maintenance, distribution and service. The value-added process starts too late because it ignores linkages with suppliers, and stops too early because it ignores linkages with customers.

The value chain approach encompasses external and internal data, uses appropriate cost drivers for all major value-creating processes, exploits linkages throughout the value chain, and provides continuous monitoring of a firm's strategic competitive advantage.

1.7 Cost Control and Cost Reduction

1.7.1 Cost control implies guidance a reputation of cost by executive action. For this purpose, the executives are provided with some yard stick such as standards or budgets with which the actual costs and performances are compared to ascertain the degree of achievement made. Therefore Cost Control involves continuous comparisons of actual with the standards or budgets to regulate the former. Standards or budgets once set up are not attended during the period or until some mistakes are discovered in standards.

Cost reduction is the achievement of real and permanent reduction in unit cost of products manufactured. It, therefore, continuously attempts to achieve genuine savings in cost of production distributing, selling and administration. It does not accept a standard or budget as or fined. It rather challenges the standards/budgets continuously to make improvement in them. It attempts to excavate, the potential savings buried in the standards by continuous and planned efforts. Cost control relax that dynamic approach, it usually dealt with variances leaving the standards intact.

Cost Reduction	Cost Control
1. Cost Reduction is the achievement of real and permanent reduction in unit cost of products manufactured.	1. Cost Control involves a comparison of actual with the standards or budgets, to regulate the actual costs.
2. Realistic savings in cost.	2. There could be temporary savings in cost.
3. Product's Utility, Quality and Characteristics are retained.	3. Quality Maintenance is not a guarantee.
4. It is not concerned with maintenance of performance according to standards	4. The process involves setting up a target, investing variances and taking remedial measures to correct them.
5. Continuous process of critical examination includes analysis and challenge of standards.	5. Control is achieved through compliance with standards. Standards by themselves are not examined.
6. Fully dynamic approach.	6. Less dynamic than Cost Reduction.
7. Universally applicable to all areas of business. Does not depend upon standards, though target amounts may be set.	7. Limited applicability to those items of cost for which standards can be set.
8. Emphasis here is partly on present costs and largely on future costs.	8. Emphasis on present and past behaviour of costs.
9. The function of Cost Reduction is to find out substitute ways and new means like waste reduction, expense reduction and increased production	9. Cont Control does competitive analysis of actual results with established standards.
10. Cost reduction is a corrective measure.	10. Cost Control is a preventive measure.

1.7.2 Application of cost control in material cost

Materials Cost is the price paid and the cost incurred by an organization in procuring materials for production. If material cost is effectively controlled we must have a proper system of material control and the following are the fundamental requirement of such a control:-

- Definite responsibility in respect of every function of material control should be specified and allocated.
- Proper co-ordination between the various sections/departments responsible for different function should be achieved.
- Purchasing function should be centralised as far as possible and entrusted to a competent person conversant with purchasing function.
- Controlled procedure should be standardised ad uniform forms and documents should be used all over the organisation.
- To facilitate the control procedures materials requirements budget and materials purchased budget should be prepared.
- Adequate provision for proper storage facilities and suitable arrangements for storing materials should be made.
- A proper system of stock control should be introduced and maintained.

1.7.3 Programme for Cost Reduction

The possibilities of reducing the cost of a product in the applications of cost reduction methods.The lines of approach in laying out a cost reduction plan are suggested below:

- **Product Design:** Costreduction starts with the design of the product. Product design being first step in manufacturing of a product, the impact of any economy or cost reduction effected their stage will be felt throughout the manufacturing life of the product. Design is therefore the most important field where cost reduction may be attempted.Efficient designing for a new product or improving the design for an existing product reduces cost in the following manner:-
  - ✓ Cheaper substitute, higher yield and less quantity and varieties of materials, cause reduction in cost.
  - ✓ Reduced time of operation and increased productivity reduce cost.
  - ✓ Cost of gigs, tools and fixtures are to be minimised.
  - ✓ Standardisation and simplification in variety increases productivity and reduces costs.
- **Organisation:** It is not possible to measure the extent of cost reduction resulting from an improvement in organisation nevertheless, economies are bound to be achieved if the following considerations are looked into:-
  - ✓ Definition of each function and responsibility.
  - ✓ Proper assignment of task and delegation of responsibility to avoid overlapping

- ✓ A suitable channel of communication between various management levels.
- ✓ Co-operation and closed relationship between the various executives.
- ✓ Removal of doubts and fiction.
- ✓ Encouragement to employees for cost reduction suggestion.

- **Factory Lay Out Equipment:-** A cost reduction programme should study the factory layout and the utilisation of the existing equipment to determine whether there is any scope of cost reduction by elimination of wastage of men, materials and maximum utilisation of the facilities available.

  The necessity for replacement of Plants, introduction of new techniques or expansion of facilities should be considered and various alternatives explored with a view to reducing costs.

- **Production Plan Programme and Method:-**Production control ensures proper planning of work by installing and efficient procedure and programme ordering correct machine and proper utilisation of materials, manpower and resources so that there is no waste of time and money due to wait for components, men, material etc. An efficient cost reduction programme should examine the following points relating to production control.
  - ✓ Whether wastage of manpower and material is kept to the minimum
  - ✓ Whether there is any scope for reducing idle capacity.
  - ✓ Whether the procedures for the control of stores and maintenance services are efficient.
  - ✓ Whether labour wastage may be reduced and productivity increased by eliminating faulty production method, plant layout and designs or introducing incentive schemes.
  - ✓ Whether there is scope for reduction of over head, whether a budgetary control system is in operation to ensure the control over overhead costs.

1.7.4 Cost reduction techniques

It may be extended to administrative, selling and distribution methods, personnel management, purchase and material control, financial management and other mischevious services. Tools and techniques for cost reduction:-

- Budgetary control and standard cost.
- Work study and organisation and method of procedure.
- Value analysis.
- Standardisation.

- Simplification and variety reduction.
- Economic batch quantity (E. B. Q.)
- Coding and classification.
- Improvement in design.
- Substitute material utilisation.
- Automation.
- Operational Research.
- Quality Control.
- Production Planning and Control.
- Inventory Control.
- Purchase Scheduling.
- Job evaluation and merit voting.
- Trainingand development.
- Business forecast.
- Market Research.

1.7.5 Illustrations

Illustration1

Even Forward Ltd. is manufacturing and selling two products: Splash and Flash at selling price of ₹ 3 and ₹4 respectively. The following sales strategy has been outlined for the year:—

(i) Sales planned for year will be ₹7.20 lakhs in the case of Splash and ₹3.50 lakhs in the case of Flash.

(ii) To meet competition, the selling price of Splash will be reduced by 20% and that of Flash by 12 ½ % .

(iii) Break- even is planned at 60%of the total sale of each product.

(iv) Profit for the year to be achieved is planned as ₹ 69,120 in the case of Splash and ₹ 17,500 in the case of Flash. This would be possible by launching a cost reduction programme and reducing the present annual fixed expenses of ₹1,35,000 allocated as ₹1,08,000 to Splash and ₹27,000 to Flash.

You are required to present the proposal in financial terms giving clearly the following information:

Number of units to be sold of Splash and Flash to break-even as well as the total number of units of Splash and Flash to be sold during the year.

Reduction in fixed expenses product-wise that is envisaged by the Cost Reduction Programme.

Solution

		Splash	Flash
(a)	Sales (₹)	7,20,000	3,50,000
(b)	Selling price per unit (Revised)	2.4	3.5
		(80% of 3)	(87.5% of 4)
(c)	Sales units [(a) / (b)]	3,00,000	1,00,000
(d)	BEP (%)	60%	60%
(e)	BEP –units [(c) × (d)]	1,80,000	60,000
(f)	Margin of Safety (%)	40%	40%
(g)	Margin of Safety (₹)	2,88,000	1,40,000
		(1,20,000 units x₹2.4)	(40,000 units x₹3.5)
(h)	Profit (given)	69,120	17,500
(i)	Previous Fixed cost (given)	1,08,000	27,000
(j)	New P/V ratio [(h) / (g) × 100)	24%	12.5%
(k)	Break Even Sales [(e) x (b)]	4,32,000	2,10,000
(l)	Revised fixed cost [(k) x (j)]	1,03,680	26,250
(m)	Reduction in fixed cost (l)-(i)	4,320	750

1.8 Computer-aided manufacturing

The manufacturing process is carried out by a range of machinery that, together with its concomitant software, comes under the collective heading of computer–aided manufacturing (CAM).

Maximum elements of CAM are computer numerical control (CNC) and robotics.

CNC machines are programmable machine tools. These are capable of performing a number of machining tasks, e.g. cutting, grinding, moulding, bending etc.

A program stores all the existing manufacturing activities and set-up instructions for a particular machine or bank of machines, providing facility of changing its configuration in a matter of seconds via the keyboard; changes to existing configurations and new configurations are easily accommodated. CNC therefore offers great flexibility, and reduces set-up times.

Human operators will tire and are error prone. CNC machines are able to repeat the same operation continuously in identical manner, with high accuracy level.

For Example the car producer found that the time taken to completely retool car body panel jigs in their intelligent body assembly system (IBAS) fell from 12 months to less than 3 months by reprogramming the process machinery by computer and using computerised jig robots.

1.9 Just in Time

1.9.1 A just in time approach is a collection of ideas that streamline a company's production process activities to such an extent that wastage of all kinds viz., of time, material, and labour is systematically driven out of the process. JIT has a decisive, positive impact on product costs. In this chapter we would review the various components of the JIT system and then discuss how its use affects different variety of costs, capital investments, and measurements.

CIMA defines:

"Just-in-time (JIT):*A System whose objective is to produce or to procure products or components as they are required by a customer or for use, rather than for stock. Just-in-time system Pull system, which responds to demand, in contrast to a push system, in which stocks act as buffers between the different elements of the system such as purchasing, production and sales".*

"Just-in-time production: *Production system which is driven by demand for finished products, whereby each component on a production line is produced only when needed for the next stage".*

"Just-in-time purchasing: *Purchasing system in which material purchases are contracted so that the receipt and usage of material, to the maximum extent possible, coincide".*

A JIT system comprises of a number of subcomponents, which are discussed in this chapter. A complete JIT system begins with production, includes deliveries to a company's production facilities, continues through the manufacturing plant, and even includes the types of transactions processed by the accounting system.

"Process that vastly reduces the amount of raw materials inventory and improves the quality of received parts"

- To begin with, a company must ensure that <u>it receives products/spare parts/materials from its suppliers on the exact date and at the exact time when they are needed</u>. For this reason the purchasing staff must investigate and evaluate every supplier, eliminate those which could not keep up with the delivery dates.
- In addition, <u>deliveries should be sent straight to the production floor for immediate use in manufactured products</u>, so that there is no time to inspect incoming parts for defects.
- Instead, <u>the engineering staff must visit supplier sites and examine their processes</u>, not only to see if they can reliably ship high-quality parts but also to provide them with engineering assistance to bring them up to a higher standard of product.
- As soon as suppliers certify for their delivery and quality, the concern must install a system, which may be as simplistic as a fax machine or as advanced as an electronic data interchange system or linked computer systems, that tells suppliers exactly how much of which parts are to be sent to the company.
- Drivers then bring small deliveries of product to the company, possibly going to the extreme of dropping them off at the specific machines that will use them first.

"Process in which a company reduces the amount of work-in-process, while also shrinking the number of products that can be produced before defects are identified and fixed, thereby reducing scrap costs"

- Next, we shorten the setup times for concern's machinery. In most of the factories equipment is changed over to new configurations as rarely as possible because the conversion is both lengthy and expensive. When setups take a long time, company management authorises long production runs, which spreads the cost of the setup over far more units, thereby reducing the setup cost on a per-unit basis. However with this approach too many products are frequently made at one time, resulting in product obsolescence, inventory carrying costs, and many defective products (because problems may not be discovered until a large number of items have already been completed). '*But under JIT system a different approach to the setup issue is followed which focuses on making a video tape of a typical set up, instead of reducing the length of equipments setups and thereby eliminating the need for long production runs to reduce per unit costs. A team of industrial engineers and machine users examines this tape, spotting and gradually eliminating steps that contribute to a lengthy setup'*. It is not unusual, after a number of iterations, to achieve setup times of minutes or seconds when the previous setup times were well into hours.
- It is not sufficient to reduce machine setup times because there are still problems with machines not being coordinated properly so that there is a smooth, streamlined flow of parts from machine to machine. In most of the companies there is such a large difference between the operating speeds of different machines that work-in-process inventory builds up in front of the slowest ones. Not only does this create an excessive quantity of work-in-process inventory, but defective parts produced by an upstream machine may not be discovered until the next downstream machine operator works his way through a pile of work-in-process and finds them. By the time this happens the upstream machine may have created more defective parts, all of which must now be destroyed or reworked. There are two ways to resolve both problems.
  - ✓ The first involves a **"kanban card,"** which is a notification card that a downstream machine sends to each machine that feeds it parts, authorizing the production of just enough components to fulfill the production requirements being authorized in turn by the next machine further downstream. This is also known as a "pull" system, since kanbans are initiated at the *end* of the production process, pulling work authorizations through the production system. With this approach, there is no way for work-in-process inventory to build up in the production system, since it can be created only with a kanban authorization.
  - ✓ The second way to reduce excessive work-in-process inventory and defective parts, is to, **group machines into working cells**. A working cell is a small cluster of machines which can be run by a single machine operator. This individual machine operator takes each output part from machine to machine within the cell; and thus there is no way for work-in-process to build up between machines. Also, this operator can immediately identify defective output which otherwise is difficult for

each machine of the cell. This configuration has the additional benefit of lower maintenance costs since the smaller machines used in a machine cell are generally much simpler than the large, automated machinery they replace. Also, because the new machines are so small, it is much easier to reconfigure the production facility when it is necessary to produce different products, avoiding the large expense of carefully repositioning and aligning equipment.

Both kanbans and machine cells should be used together—they are not mutually exclusive. By doing so a company can achieve extremely low product defect rates, as well as vanishingly small investments in work-in-process inventory.

- Before the preceding steps are completed, it becomes apparent that a major change must also be made in the work force. The traditional approach is to have one employee maintaining one machine, which is so monotonous that workers quickly lapse into apathy and develop a complete disregard for the quality of their work. Now, with full responsibility for a number of machines, as well as product quality, workers become much more interested in what they are doing. To enhance this situation the human resource development department of organisation must prepare and organise training classes to teach to employees how to operate a multitude of different machines, perform limited maintenance on the machines without having to call in the maintenance staff, spot product errors, understand how the entire system flows, and when to halt the production process to fix problems. In short, the workforce must be completely retrained and focused on a wide range of activities. This usually results in a reconfiguration of the compensation system as well, because the focus of attention shifts away from performance based to high production volumes and in the direction of performance based to high product quality.
- A major result of having an empowered workforce is that employees are allowed to stop their machines when they see a problem, and either fix it on the spot or immediately call in a repair team. In either case the result is immediate resolution of the bulk of performance problems. This one step has a profound impact on much of the manufacturing variance analysis. Historically, cost accountants compile all kinds of variance information at the end of each month, investigate problems in detail, and then present a formal problem analysis report to management a few weeks after the end of the month. However, because the production staff resolved the underlying issues within a few minutes of their occurence, the variance report becomes a complete waste of time. Management no longer cares what happened a month in the past because it is presently dealing with current problems that will not appear on cost accountant reports for weeks to come. In short, the quick response capabilities of a JIT system allows the cost accountant to omit a large amount of the variance reporting that was previously an important central job function.
- This approach also means that there is no need for suppliers to send invoices, since the company relies solely on its internal production records to complete payments.

"Processes in which company alters in supporting accounting system"

- Finally, the massive changes caused by a JIT system also requires several alterations in the supporting accounting systems. Because of the large number of daily supplier shipments, the accounting staff faces the prospect of going through a large pile of accounts payable paperwork. To make the problem worse there is no receiving paperwork, because the suppliers deliver parts directly to the production operation, so there is no way to determine if deliveries have been made. To avoid the first problem, accountants can switch to making a single consolidated monthly payment to each supplier. The second problem requires a more advanced solution. To prove that a supplier has delivered the part quantities which it claims it has, the accounting system that can determine the amount of finished products created during the period and then multiply these quantities by the parts listed on the bill of materials for each product, obtaining a total quantity for each part used. The accountants then pay suppliers based on this theoretical production quantity, which is also adjusted for scrap during the production process (otherwise suppliers—unfairly—will not be paid for their parts that are scrapped during the company's production process). This approach also means that there is no need for suppliers to send invoices, since the company relies solely on its internal production records to complete payments.

Clearly, the changes imposed by a JIT system are profound and can greatly improve company operations when installed and operated correctly. They can also have a profound effect on product costs.

So, JIT system aims at:

- Meeting customer demand in a timely manner
- Providing high quality products and
- Providing products at the lowest possible total cost.

The five main features of JIT production system:

- Organise production in manufacturing cells, a grouping of all the different types of equipment used to make a given product. Materials move from one machine to another where various operations are performed in sequence. **Material – handling cost are reduced.**
- Hire and retain workers who are multi-skilled so that they are capable of performing a variety of operations, including repairs and maintenance tasks. Thus, **labour idle time gets reduced.**
- Apply TQM to eliminate defects. As, there are tight link stages in the production line, and minimum inventories at each stage, defect arising in one stage can hamper the other stages. **JIT creates urgency for eliminating defects as quickly as possible.**
- Place emphasis on reducing set-up time which makes production in smaller batches economical and reducing inventory levels. Thus **company can respond to customer demand faster.**

- Carefully selected suppliers capable of delivering high quality materials in a timely manner directly at the shop – floor, reducing the material receipt time.

1.9.2 Impact of JIT System on:

- **Waste Costs-** A characteristic of the JIT system is its continuous focus on eliminating all waste from a system. This can be a waste of assets, excessive inventory. It can also be a waste of time, in the case of assets it may include unused assets for long periods of time (e.g., work-in-process inventory held in a production queue). It can also be a waste of materials, such as unnecessary levels of obsolete inventory, defective products, rework, and the like. When fully installed, a JIT system vastly reduces all these types of waste. When this happens, there is a sharp drop in several aspects of a product's costs.

  For example, by reducing the amount of work in process, machine operators can make out immediately, if an incoming part from another workstation is defective and can notify the preceding workstation of the problem before it makes any more parts, which reduces the quantity of rework that must be done. Since a standard quantity of rework labour is frequently included in a product's labour routing, a reduction here lowers the amount of labour cost charged to a product. Similarly, any material that would have been scrapped because of improper rework is no longer lost, so the standard amount of scarp noted on a product's bill of materials can now be reduced. This also decreases a product's cost.

  Overhead costs charged to a product also reduce as other types of waste decline. For example, by clustering machines into cells, the materials handling costs previously incurred in shifting materials between widely scattered machines can be eliminated. This reduces the amount of materials handling costs that used to be charged to overhead. Also, machine cells tend to reduce the amount of floor space needed since there is no longer a need for large aisles for the materials handling people to drive their forklifts through; by reducing floor space, one can also reduce facility costs, which no longer appear in the overhead cost pool. Another form of waste is the quality inspections once performed on many machines. Under the JIT system machine operators conduct their own quality checks, so there is less need for a separate group of supervisors/inspectors; accordingly, the cost of their pay can be eliminated from overhead costs. All these costs (and more) do not directly add value to a product, so they are wasteful costs that are subject to elimination. By doing so with a JIT system, there are fewer costs left to be charged to a product.

  A key focus of any JIT system is on reducing various kinds of wastage of time, so that the entire production process is concentrated on the time spent in actually producing products. For example, all inspection time is eliminated from the system as operators conduct their own quality checks. Similarly, all movement, which involves shifting inventory and work in process throughout various parts of the plant, can be eliminated by clustering machines together in logical groupings. Third, queue time is eliminated by not allowing inventory to build up in front of machines. Finally, one can eliminate storage time by clearing out excessive stocks of inventory and having suppliers deliver parts only as and when needed. By shrinking the amount of wastage time out of the manufacturing process, a company effectively eliminates activities that do not contribute to the value of a product, which in turn reduces the costs associated with them.

Another way in which waste is eliminated in a JIT system is to charge cost drivers to wasteful activities that accumulate costs. For example, overhead costs can be charged out based on the number of components in a product (since more parts require more purchasing activity and materials handling), the number of material moves (which is not a value-added activity), or the number of units scrapped. In this way the cost of these activities becomes apparent to management, and as a result, there will be considerable focus on reducing these cost drivers since the accounting system places so much emphasis on their total burdened costs. Then, when these cost drivers have been reduced to significant levels, the cost accountants can find other wasteful cost drivers and shift the allocation system to place the most emphasis on them. This directs management's attention toward their elimination, too. And so on. In this way the cost accounting system can be continually altered so that it has a direct, active role in reducing wasteful activities.

- **Overhead Costs:**The costs of material handling, facilities, and quality inspection decline when a JIT system is installed. In addition, the reduction of all types of inventory results in a massive reduction in the amount of space required for the warehouse facility. Since all costs associated with the warehouse are assigned to the overhead cost pool, the amount of overhead is reduced when the costs of staff, equipment, fixed assets, facilities, and rent associated with the warehouse are sharply cut back.

  There is also a shift of costs from the overhead cost pool to direct costs when machine cells are introduced. The reason for this change is that a machine cell generally produces only a small range of products, making it easy to assign the entire cost of each machine cell to these items. This means that the depreciation, maintenance, labour and utility costs of each cell can be charged straight to a product, which is preferable to the traditional approach of sending these costs to an overhead cost pool from which they are assigned to products in much less identified manner. Though this change does not represent a cost increase or reduction, it does increase the reliability of allocation for many more costs than that was previously the case.

  Despite the shift of many overhead costs to direct costs, there is still an overhead cost pool left over that must be allocated to products. However, given the large number of changes implemented as part of the JIT system, cost accountants may find that there are now better allocation bases available than the traditional direct labour allocation. For example, the amount of time a product takes in each work cell may be a better measure for allocating costs, instead of amount of space occupied in the work cells that create each product. No matter what allocation system is used, it is somewhat different from the old system, so there is a shift in the allocation of costs between different products.

  In short, overhead costs decline as some costs are eliminated, while other costs shift between products as more costs are charged directly to products and the remaining overhead costs are charged out using different allocation methods.

- **Other Costs:** When a JIT system in created, the amount of inventory retained in a company drops continuously. Raw materials inventory is reduced because suppliers deliver only small quantities of parts as and when they are needed. Work-in-process inventory drops because the conversion to machine cells and the use of kanban cards greatly reduces the need to pile up inventory between machines. Finally, finished goods inventory drops because inventory is produced only when there are orders in hand from customers (though finished goods inventories are also allowed to build if a company experiences high seasonal sales). Consequently, the cost of maintaining inventory declines, which in turn reduces the overhead costs associated with inventories that are charged to products. Some of these inventory-related costs are:

  ✓ Interest cost related to the debt that funds the inventory investment

  ✓ Cost of inventory that becomes obsolete over time

  ✓ Cost of rent for inventory storage facilities

  ✓ Cost of all equipment used in the warehouse

  ✓ Cost of warehouse utilities

  ✓ Cost of warehouse employees

  ✓ Cost of insurance needed to cover the possible loss of inventory

  ✓ Cost of taxes on the inventory

  According to several estimates the annual cost of inventory is 25% of the total inventory investment. By eliminating excessive storage of inventory a company experiences not only a decline in its inventory investment but also the elimination of all associated costs.

  Besides a reduction in the level of working capital and inventory-related costs, a company can also reduce its investment in capital assets. This occurs when a company with a few large machines replaces them with a larger number of much smaller, more easily configured machines. Then, equipment setup times become shorter, which in turn makes it profitable to have shorter production runs, thereby eliminating an excessive investment in inventory that would have been created by excessively long production runs. There is frequently a saving when such a change occurs, which releases cash for other uses while also reducing the amount of depreciation charged to overhead.

  A potentially significant one-time cost that many companies do not consider involves the cost layers in their inventory costing systems. When a JIT system is installed, there is an immediate focus on eliminating inventory of all types. If a company uses some kind of layering method to track the cost of its inventory, such as last-in-first-out or first-in-first-out, it will find itself burrowing down into costing layers that may have been undistributed for many years. Then, some unusually high or low costs may be charged off to the cost of goods sold when these inventory items are finally used up. For example, if the current market cost of a piston is ₹5,000/- but a company has some old (but serviceable) ones in stock from 20 years ago that cost ₹2,000, then only the ₹2,000 unit cost is charged to the cost of goods sold when these units are finally used as a result of clearing out the inventory. Because of the unusually low cost of goods sold, the gross margin is higher than usual until these early cost layers are eliminated. Because of the lower-of-cost-or-market rule (under which the cost of excessively expensive inventory must be reduced until it is not higher than the current market value), this problem tends to be less of an issue when early cost layers are *too high,* though the costs charged are still somewhat different from those for newer layers of inventory. Once all cost layers have been used up, the only costs the management sees being charged to the cost of goods sold are those currently charged by suppliers.

Thus, the cost reductions and reduced capital requirement of JIT systems have a significant impact on the levels of fixed assets, working capital, and inventory needed to run a business, which in turn reduces the associated overhead costs charged to products.

- **Product Prices:** when a company achieves a higher level of product quality, along with ability to deliver products on the dates required, customers may be willing to pay a premium. This is particularly true in industries where quality or delivery reliability is low. If customers are highly sensitive to these two factors, it may be possible to increase prices substantially. Alternatively, if these factors are not of great importance, or if customers place a higher degree of importance on other factors, then there will be no opportunity for a price increase.

  In industries where many companies are adopting JIT systems at the same time or have already installed them, an improvement in product quality and delivery times does not differentiate a company from its pee'Instead, since everyone else is offering the same level of quality and service, it just keeps a company from losing sales to its competitors. In such a situation it is more likely that all companies remaining in the industry will use their new-found lower costs to initiate a price war that will result in a drop in prices.

  Consequently, the impact of a JIT system on product pricing is primarily driven by customers'perceived need for higher product quality and reliable delivery times, as well as the presence of competitors with JIT system, the same installation, and operational base.

1.9.3 Costing Allocation Difference (JIT vs. Traditional System)

The chief difference between the types of cost allocations under JIT and traditional environment is that of converting most of the overhead costs to direct costs. The primary reason for this change is the machine cell. Because a machine cell is designed to produce either a single product or a single component that goes into a similar product line. Therefore all the costs generated by the machine cell can be charged directly to the only product it produces. When a company completely changes over to the use of machine cells in all locations, the costs related to all the cells can now be charged directly to products, which leave few costs of any kind to be allocated through a more traditional overhead cost pool. The result of this change results in more accurate product costs.

Specifically, the costs that can now be charged directly to product are:

- **Depreciation:** The depreciation cost of each machine in a machine cell can be charged directly to a product. It may be possible to depreciate a machine based on its actual use, rather than charging off a specific amount per month, since this allocation variation shifts costs to a product more accurately.
- **Electricity**: The power used by the machine in a cell can be separately metered and then charged directly to the products that pass through the cell. Any excess electricity cost charged to the facility as a whole still has to be charged to an overhead cost pool for allocation.
- **Material handling:** Most materials handling costs in a JIT system are eliminated since machine operators move parts around within their machine cells. Only costs for materials handling between cells should be charged to an overhead cost pool for allocation.
- **Operating supplies:** Supplies are used mostly within the machine cells, so the majority of items in this expense category can be separately tracked by individual cell and charged to products.
- **Repairs and maintenance:** Nearly all the maintenance costs a company incurs are for machinery and they are all grouped into machine cells. By having the maintenance staff, charge their time and materials to these cells, these costs can be charged straight to products. Only maintenance work on the facility is still charged to an overhead cost pool.
- **Supervision:** If supervision is by machine cell, the cost of the supervisor can be split among the cells supervised. However, the cost of general facility management as well as of any support staff must still be charged to an overhead cost pool.

As noted in several of the preceding items, a few remainder costs are still charged to an overhead cost pool for allocation. However, these represent a small percentage of the costs, with nearly everything now being allocable to machine cells. Only building occupancy costs, insurance, and taxes are still charged in full to an over-head cost pool. This is a vast improvement over the amount of money the traditional system allocates to products. A typical overhead allocation pool under the traditional system can easily include 75% of all costs incurred, whereas this figure can be dropped to less than 25% of total costs by switching to a JIT system. With such a higher proportion of direct costs associated with each product, managers then have much more relevant information about the true cost of each product manufactured.

1.9.4 Performance Measurements in a JIT System

Many of the performance measurement measures used under a traditional accounting system are not useful in a JIT environment, while new measures can be implemented that take advantage of the unique characteristics of this system.

- **One of the key measurements in a traditional system is machine utilization:** This is used to ensure that every asset a company purchases is being thoroughly utilized. It is particularly important in cases where there has been a large investment in automation or large, high-speed machinery, since these items are quite expensive and should be used to the utmost. However, making machine utilization a key measurement; forces production managers in the direction of manufacturing as much product as possible in order to show a high level of machine utilization, which can result in large amount of inventory piling up in the warehouse. This is not a desirable end result in a JIT environment, where producing only what is actually needed is the underlying rule. Also, machine cells in a JIT system tend to be smaller and less costly than the highly automated (and expensive) juggernauts used in more traditional systems, so there is less need to justify the investment in these smaller machines by proving that they have been heavily used. In short, machine utilization measurements can be discarded under JIT environment.

- **Another inappropriate measurement is any type of piece rate tracking for each employee:** This is a common measure in the textile industry, where employees are paid extra if they exceed certain production volume targets. However, a JIT system focuses on producing only what is needed, so an employee who has incentives to create vast piles of parts is producing contrary to the rules of the system. Accordingly, any piece rate system must be eliminated and replaced with measures that focus instead on the quality of output or the number of employee suggestions for improving the system, which are much more important outcomes in a JIT system.
- **Any type of direct labour efficiency tracking is highly inappropriate in a JIT system**: It is a key measurement in more traditional systems, where employee time and productivity are closely monitored and measured. However, a JIT system does not focus on how fast an employee works—only on the quality of the products manufactured. Also, labour variance measurements require considerable employee time tracking, which forces workers to fill in a time sheet, punch a clock, or use a barcoding system to track what they are doing and what job they are working on. All this labour tracking is a non-value-added activity, which is something a JIT system strives to avoid as an unncessary activity. Consequently, the cost accounting staff should advocate the complete elimination of all labour variance measurements.
- **Installing a JIT system does not mean that there should be a complete elimination of variance or operational measures:** There are still several measures that are highly relevant to operations. Some of them are :
  - ✓ Inventory turnover: Those who have installed JIT systems emphasize the extraordinarily high inventory turnover that they now experience, which is the case in most instances. The turnover levels of such well-known JIT companies as Toyota have been known to exceed 70 per year, as opposed to the levels of 2 to 10 per year that are more common for companies with other types of manufacturing systems. This measure is best subdivided into smaller parts, so that one can determine the turnover levels for raw materials, work in process, and finished goods.

- ✓ Setup time reduction: The average setup time per machine is of great importance as it can be measured periodically and plotted on a trend line. The shortest possible setup intervals are crucial for the success of short production runs, so this is a major JIT measurement. It is best to measure it by machine, rather than in the aggregate, since an aggregate measure does not reveal enough information about which equipments requires more setup time reduction work.
- ✓ Customer complaints: A JIT system is partly based on the premise that product quality will be superb. Consequently, any hint from customers that there are product problems should be greeted with the gravest concern and investigated immediately. The accumulation of customer complaints and their dissemination to management should be considered a major JIT measure.
- ✓ Scrap: Little waste should be generated by a JIT system, which means that materials scrap should be driven down to exceedingly low levels. The cost of scrap (especially when supported by a detailed list of items that were scrapped) is of particular concern as a JIT system is being implemented, since it helps to identify problem areas requiring further management attention.
- ✓ Cost of quality: One focus of JIT is on creating high-quality products, so it is reasonable to keep track of the full cost of quality (which comprises defect control costs, failure costs, and the cost of lost sales) on a trend line. Managers want to see the details behind this measure, so that they know where the largest quality costs still reside in the company and can then work to reduce them.
- ✓ Customer service: This measure really has several components—delivering products on the dates required by customers, shipping full orders to customers, and not having products returned because of poor quality. This measure can be summarized in a variety of ways or reported at the component level, but the main issue is to measure and post the information for all to see, so that the company focuses strongly on providing the highest possible degree of customer service.
- ✓ Ideas generated: A JIT system works best when employees pitch in with hundreds of suggestions for improvements that, when taken in total, result in a vastly improved, efficient operation. The amount of idea generation going on can be measured by the number of ideas per worker, the number of ideas suggested in total, the number of ideas implemented, or the proportion of ideas suggested that are implemented.

The common theme that unites all the JIT measures just listed is that they are not financial in nature (with the exception of the cost of quality)—they are operational measures that focus attention on the nuts-and-bolts details of creating and running a JIT system. A cost accountant involved in the calculation and reporting of these measures may feel that this is quite a departure from the more traditional cost variance measures, but the end result will be a much more efficient JIT process that churns out and delivers high-quality products.

1.9.5 Essential pre-requisites of a JIT system

- Low variety of goods
- Vendor reliability
- Good communication
- Demand stability
- TQM
- Defect free materials
- Preventive maintenance

1.9.6 Backflushing in a JIT System

Backflushing requires no data entry of any kind until a finished product is completed. At that time the total amount finished is entered into the computer system, which multiplies it by all the components listed in the bill of materials for each item produced. This yields a lengthy list of components that should have been used in the production process and which are subtracted from the beginning inventory balance to arrive at the amount of inventory that should now be left on hand. Given the large transaction volumes associated with JIT, this is an ideal solution to the problem.

However, there are some serious problems with backflushing that must be corrected before it will work properly. They are:

- **Production reporting:** The total production figure entered into the system must be absolutely correct, or else the wrong component types and quantities will be subtracted from stock. This is a particular problem when there is high turnover or a low level of training to the production staff that records this information, which leads to errors.
- **Scrap reporting:** All abnormal scrap must be diligently tracked and recorded; otherwise these materials will fall outside the blackflushing system and will not be charged to inventory. Since scrap can occur anywhere in a production process, a lack of attention by any of the production staff can result in an inaccurate inventory. Once again, high production turnover or a low level of employee training increases this problem.
- **Lot tracing:** Lot tracing is impossible under the backflushing system. It is required when a manufacturer need to keep records of which production lots were used to create a product in case all the items in a lot must be recalled. Only a picking system can adequately record this information. Some computer system allows picking and backflushing system to coexist, so that pick transactions for lot tracing purpose can still be entered in the computer. Lot tracing may then still be possible if the right software is available; however, this feature is generally present only on high-end systems.
- **Inventory accuracy:** The inventory balance may be too high at all times because the backflushing transaction that relievs inventory usually does so only once a day, during which time other inventory is sent to the production process; this makes it difficult to maintain an accurate set of inventory records in the warehouse.

Of all the issues noted here, the worst is a situation where the production staff is clearly incapable of providing sufficiently accurate scrap or production reporting for the backflushing system. If there is an easily traceable cause, such as less capable workers on a particular shift, moving a few reliable employees into these positions can provide immediate relief from the problem. It may even be possible to have an experienced shift supervisor to collect this information. However, where this is not possible for whatever reason, computer system users experience *backflushing garbage in, garbage out (GIGO)*—entering inaccurate information rapidly eliminates any degree of accuracy in the inventory records, requiring many physical inventory counts to correct the problem. Consequently, the success of a backflushing system is directly related to a company's willingness to invest in a well-paid, experienced well-educated production staff that undergoes little turnover.

1.9.7 Concept of 'Just-in-Time'followed by 'Mahindra & Mahindra'

Mahindra & Mahindra (M&M)

M&M wanted to implement JIT at their main plant in Nasik as they were aware of the fact that JIT approach will help them to operate with minimal levels of inventory. Their business objective was to make all our suppliers active participants" in the production process. They wanted that the suppliers should be "enabled" to know of any change in the whole production process and at the same time contribute actively. This was necessary to reduce the time-to-respond to a situation and help "just-in-time" approach in the production process.

Objective:

Make all the suppliers active participants in the production process.

Suppliers should be able to know of any change in the whole production process and at the same time contribute actively.

Update to best practices for supply strategies for 400 vendors, 150 vehicles per day and 1,100 parts.

Improvement of the replenishment efficiency.

Reduction of stock at the assembly line favoring a flexible manufacturing.

VSS Service:

Concept planning for JIT and supply chain including definition of load units and their arrangement at the assembly line, definition of the replenishment trigger concept, design of stores and handling equipment and review of the method of supply from vendors.

Implementation of the proposed concept.

Solution:

Modular standard metal containers and totes based on Indian truck dimensions. Load units ergonomically presented to the workers.

25 JIT parts identified (supplied in sequence), two-tier shelving system for totes with dynamic allocation and picking, containerized supply from local vendors with round pick up.

Reduced personnel and replenishment lead time; improved manufacturing flexibility.

Benefits:

By making the suppliers participant in the 'just-in-time" method of production, they could maintain the least inventory level.

Suppliers could see real time the status of the supplies, bill settlement and host of other parameters.

All active participants of a process, for instance, the process from a supplier to the dealer can handle change management with the help of a particular solution and a defined process.

Set up times are significantly reduced in the warehouse. Cutting down the set up time to be more productive allowed the company to improve their bottom line to look more efficient.

Having employee focused on specific areas of the system allowed them to process goods faster instead of having them vulnerable to fatigue from doing too many jobs at once and simplifies the tasks at hand.

Increase emphasis on the supplier relationships.

1.9.8 Illustrations

Illustration 1

Give Backflush Costing Journal Entries in respect of following transactions:

(i) *Raw Material were purchased ₹ 3,20,000*

(ii) *Material placed into production*

(iii) *Actual Direct Labour Cost ₹ 50,000*

(iv) *Actual Overhead Cost ₹ 4,50,000*

(v) *Conversion Cost applied ₹ 4,70,000*

(vi) *All Units were completed & Sold*

(vii) *Variance is Recognized.*

Solution

For Raw Material Purchased: Raw Material in Process A/c To Accounts Payable	Dr	3,20,000	 3,20,000
For Material Placed into Production: No Entry			
For Actual Direct Labour Cost: Combined with Overhead			
For Actual Overhead Cost : Conversion Cost Control To Payroll To Accounts Payable	Dr	5,00,000	 50,000 4,50,00
For Application of Overheads: No Entry			
For Completion of Units:			

Finished Goods	Dr	7,90,000	
To Raw Material in Process A/c			3,20,000
To Conversion Control Account			4,70,000
For Units Sold:			
Cost of Goods Sold	Dr	7,90,000	
To Finished Goods			7,90,000
For Recongininition of Variance:			
Cost of Goods Sold	Dr	30,000	
To Conversion Control Account			30,000

1.10 Manufacturing Resources Planning (MRP I&II)

1.10.1 It is a part of production operation system. Management has to develop a lot of strategies for production plan. In early 1960's a material acquisition plan was first introduced known as Material Requirement Plan (MRP-I). MRP-2 is latest all-round development of that plan. Benefit of MRP is 'Detailed forecast of the inventory position is highlighted period by period'.

CIMA defines **MRP (material requirements planning)** as *"System that converts a production schedule into a listing of the materials and components required to meet that schedule, so that adequate stock levels are maintained and items are available when needed".*

A brief history of MRP –1: Material requirement planning is a computerized production scheduling system which takes the forward schedule of final product requirements (the master production schedule) and translates it progressively into the numbers of sub-assemblies, components and raw materials required at each stage of the manufacturing cycle.

It is a management information system providing a basis for production decisions when what is manufactured has a composite structure and when lead items are important features. Obviously, the ability of the system to deliver what is required in the correct place at the correct time will be dependent on the quality of information which is put into the computer model.

1.10.2 Aims of material requirement planning:

- Determine for final products namely, what should be produced and at what time.
- Ascertaining the required units of production of sub-assemblies.
- Determining the requirement for materials based on an up-to-date bill of materials file (BOM).
- Computing inventories, WIP, batch sizes and manufacturing and packaging lead times.
- Controlling inventory by ordering bought-in components and raw materials in relation to the orders received or forecast rather than the more usual practice of ordering from stock-level indicators.

1.10.3 Data requirements to operate material requirement planning system:

- **The master Production schedule:** This schedule specifies the quantity of each finished unit of products to be produced, and the time at which each unit will be required.
- **The Bill of material file:** The bill of material file specifies the sub-assemblies, components and materials required for each finished good.
- **The inventory file:** This file maintains details of items in hand for each sub-assembly, components and materials required for each finished goods.
- **The routing file:** This file specifies the sequence of operations required to manufacture components, sub-assemblies and finished goods.
- **The master parts file:** This file contains information on the production time of sub-assemblies and components produced internally and lead times for externally acquired items.

1.10.4 Method of operation of material requirement planning system:

A material requirements planning (MRP) system is a computer based inventory information system which is used to plan and control raw material and component parts inventories.

Like all computer-based information system, MRP systems can be divided in two stages:

- **Pre-requisite information and system input:**

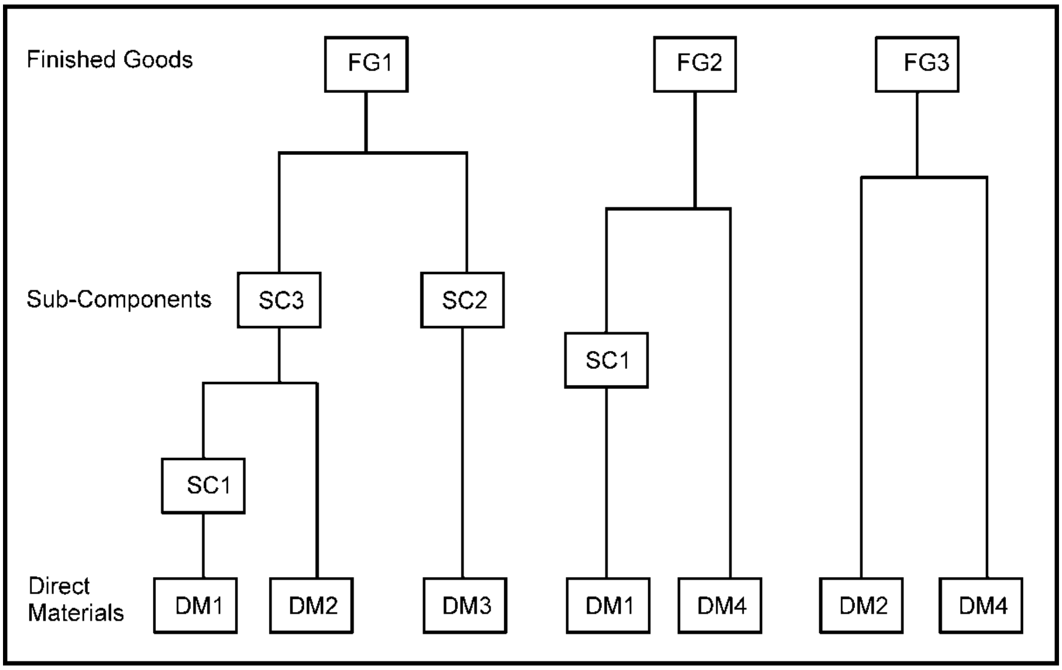
  The master production schedule (MPS) file states the production goal, generally for a week time, in terms of desired units of production. MRP system first focuses on the forecasted units of production and timing of finished goods demand and determines the demand for materials, components and sub-assemblies at each stages of production. This makes MRP a push system in which once the scheduled production starts, the output of each department is pushed through the system to the next department for processing or into inventory to be retrieved later.

  The bill of materials (BOM) file contains information about how the production of the finished goods is undertaken. A bill of material structure is used:

  ✓ To assess all of the raw materials and component parts required to complete a product and

  ✓ To describe the multiple levels of assembly or manufacturing necessary to complete a unit of finished product.

  In a figure given below a typical BOM structure file is presented for three and products FG1, FG2 and FG3. The MRP system breaks the requirements for each product by working into its primary sub-components (SC)/sub-assemblies (SA), and these in turn are further separated into second, third and so on levels of sub-components, until at the lower level in the hierarchy only purchased items (i.e. backward each end products direct material DM) exist. It is apparent from the figure below that four direct material (DM1, DM2, DM3 and DM4) are purchased for finished goods. For both FG1 and FG2 the materials are used to manufacture the components that are assembled into the end product. For FG3 no intermediary components are produced:



The inventory records files of the MRP system defines current levels of finished goods, raw materials, and component parts inventory at the beginning of some planning period. During the planning period, the organization may receive units of raw materials, components parts, sub-assemblies, and even finished goods inventory from suppliers, vendors, and subcontractors' These planned inventory receipts and delivery lead times are included in the inventory records file so that their addition can be appropriately considered in the time bucket of their arrival.

- **System Processing and Output:** The MRP system decides the demand for materials, components and sub assemblies at each stage of production.

  Once the scheduled production starts, the output of each department is pushed through the MRP system to the next department.

  From the data input, the MRP system knows:

  ✓ What it is expected to produce (through the MPS file)?

  ✓ How it should produce it (through the BOM file)? and with

  ✓ What it has to produce it (through the inventory records file)?

  This programme starts with the finished goods demand (from the MRPs) andconverts the demand requirements backward in time to schedule the desired production of the finished goods from raw materials and component parts with 'time phased'adjustments for lead time requirements. This process is called **Requirements Explosion**.

1.10.5 Pre-requisites for successful operation of MRP:

- **Strict adherence to the schedule:** The successful operation of MRP system requires a strict adherence to the latest production and purchasing schedules. Workers must be educated to understand the importance of schedule adherence, and controls should be in place to ensure this adherence.
- **Accurate data base:** Data accuracy is vital to the system. If a plan is based on inaccurate data it may be impossible to adhere to the schedule. For example, if the bill of materials file is not updated to reflect any changes in product composition it will be impossible to adhere to the schedule.

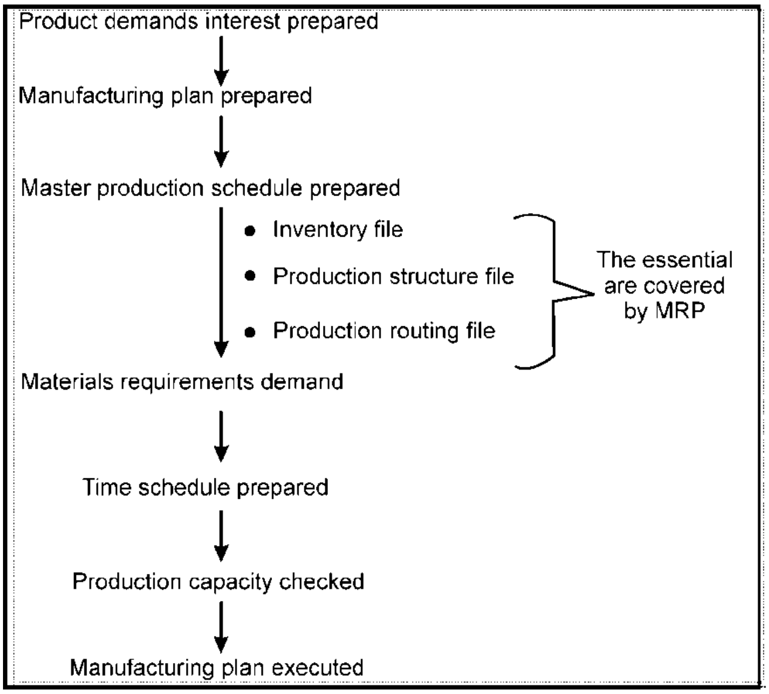
1.10.6 MRP- II

When the scope of MRP-1 is developed further that includes:

- Planning of raw material
- Planning of component & sub- assemblies
- Compute the other resources e.g. machine or labour capacity
- To create a fully integrated plan for management

Then it is known as Manufacturing Resources Planning (MRP – 2)

MRPII (also written MRP-2) adds the MRP schedule into a capacity planning system and then builds the information into a production schedule. It is also seen as a link between strategic planning and manufacturing control. The sequence of events is as follows:



From that document, a manufacturing, plan is developed based upon inputs from purchasing& production. Adjustments may be necessary to allow for production rates. Possible inventory levels in seasonal trades & the size of the workforce. The manufacturing plan leads into a detailed master production schedule which is akin to the original philosophy of MRP already outlined.

If correctly applied, MRPII provides a common data base for the different function units such as manufacturing, purchasing and finance within a firm.

1.11 Synchronous Manufacturing

This concept of 'synchronousmanufacturing' was started in 1984. It has been defined as: an all-encompassing manufacturing management philosophy that includes a consistent set of principles, procedures, and techniques where every action is evaluated in terms of the common global goal of the organisation.

A set of seven 'principles' are associated with synchronous manufacturing:

- Do not focus on balance idle capacities; focus on synchronizing the production flow.
- The marginal value of time at a bottleneck resource is equal to the throughput rate of the products processed by the bottleneck.
- The marginal value of time at a non-bottleneck resource is negligible.
- The level of utilization of a non-bottleneck resource is controlled by other constraints within the system.

- Resources must be utilized, not simply activated.
- A transfer batch may not, and many times should not, be equal to the process batch.
- A process batch should be variable both along its route and over time.

According to synchronous manufacturing principles 2 and 3, the return on improvements at a bottleneck resource is very high. But the return on improvement made at non-bottlenecks is marginal at best.

The synchronous manufacturing philosophy required managers to focus on those areas of operations where there exist potential global improvements.

1.12 Business Process Re-engineering

Business process re-engineering involves examining business processes and making substantial changes in the day to day operation of the organisation. It involves the redesign of work by changing the activities.

A business process consists of a collection of activities that are linked together in a coordinated & Sequential manner to achieve goal & objective.

For example, material handling might be classed as

- Scheduling production,
- Storing materials,
- Processing purchase orders,
- Inspecting materials and
- Paying suppliers.

The aim of business process re-engineering is to improve the key business process in an organisation by focusing on

- Simplification,
- Cost reduction,
- Improved quality and
- Enhanced customer satisfaction.

1.13 Theory of Constraints

1.13.1 During the 1980s Goldratt and Cox (1989) advocated a new approach to production management called optimized production technology (OPT). OPT is based on the principle that profits are expanded by increasing the throughput of the plant. The OPT approach determines what prevents throughput being higher by distinguishing between bottleneck and non-bottleneck resources. This approach advocates that bottleneck resources/ activities should be fully utilized while non bottleneck resources/activities should not be utilized to 100% of their capacity since it would result in increase in inventory.

OPT is based on the principle that profits are expanded by increasing throughput of the plant i.e. rate at which raw material are turned into sales. The most widely recognized management accounting system developed for this purpose is known as Throughput accounting (TA). The concept behind the system was first formulated and developed by Goldratt and Core (1986) in USA. Goldratt developed the concept and eventually gave it the name the Theory of Constraints (TOC). The theory was picked up and inducted into an accounting system in the UK where it is known as Throughput Accounting (TA).

Throughput Accounting (TA) is a method of performance measurement which relates production and other costs to throughput. Throughput accounting product costs relate to usage of key resources by various products.

Throughput is influenced by:

- Selling price
- Directpurchaseprice
- Usage of direct materials
- Volume of throughput.

Constraints on throughput might include:

- The existence of anuncompetitive selling price
- The need to deliver on time to particular customers
- The lack of product quality and reliability
- The lack of reliable materials suppliers
- The existence of shortage of production resources.

It becomes management's task to eliminate these constraints. Shortage of resources is usually termed bottlenecks, and their elimination often only moves a problem from one location to another. Thus the careful planning to minimize and eliminate all bottlenecks becomes very important.

Throughput *term defined, in work by Goldratt, 'as sales minus material and component costs. Similar to contribution except material is considered the only variable cost'. Goldratt argues that*

labour costs should be treated as fixed'. In Goldratt's analysis 'operating expense is all non-material costs' and 'inventory cost is defined as the cost of assets employed'.

Throughput Accounting (TA) *- Variable cost accounting presentation based on the definition of throughput (sales minus material and component costs). Sometimes referred to as super variable costing because only material costs are treated as variable.*

Throughput per Bottleneck Minute- *Method of ranking products that share the same (bottleneck) facility. Very similar to the use of contribution per unit of limiting factor.*

Throughput Ratios *- Several ratios were defined by Galloway and Waldron based on the* *definition of throughput. The TA (throughput accounting) ratio is:*

$$\frac{\text{Throughput per bottleneck minute}}{\text{Factory cost per bottleneck minute}}$$

[Note: Galloway and Waldron define factory cost in the same way that Goldratt defines operating expense. See throughput]

If the TA ratio is greater than 1 the product in question is "profitable" because, if all capacity were devoted to that product, the throughput generated would exceed the total factory cost. If there was a bottleneck products could be ranked by a variant of the TA ratio (although the ranking is the same as that derived by the use of throughput per bottleneck minute). Other performance ratios suggested include:

$$\frac{\text{throughput}}{\text{labour cost}} \textit{ and } \frac{\text{throughput}}{\text{material cost}}$$

***Theory of constraints (TOC)-** Procedure based on identifying bottlenecks (constraints), maximising their use, subordinating other facilities to the demands of the bottleneck facilities, alleviating bottlenecks and re-evaluating the whole system.*

The theory of constraint focuses its attention on constraints and bottlenecks within the organisation which hinder speedy production. The main concept is to maximize the rate of manufacturing output i.e. the throughput of the organisation. This requires examining the bottlenecks and constraints which are defined as:

A bottleneck is an activity within the organisation where the demand for that resource is more than its capacity to supply.

A constraint is a situational factor which makes the achievement of objectives/throughput more difficult than it would otherwise be. Constraints may take several forms such as lack of skilled employees, lack of customer orders or the need to achieve a high level of quality product output.

Using above definition, therefore, a bottleneck is always a constraint but a constraints need not be a bottleneck. Let the customers due date performance i.e. meeting the delivery schedule for customers orders is the major constraint in the organisation. The bottleneck in such a case may be certain machine in the factory.

Throughput thus related directly to the ability to cope with the constraint and to manage the bottleneck. This focus on throughput forced management to examine both the constraints and the bottleneck in order to increase throughput.

The theory of constraints (TOC) describes methods to maximize operating income under bottleneck situation. The three measurements:

- **Throughput Contribution** equal to Sale **-** Direct Materials Cost of the goods sold.
- **Investments** equal to Sum of materials costs in direct materials, work-in-process, and finished goods inventories along with R & D costs and costs of equipment and buildings.
- **Operating costs** equal to all costs of operations (other than direct materials) incurred to earn throughput contribution. Operating costs include salaries and wages, rent, utilities

and depreciation.

The objective of TOC is to increase throughput contribution while decreasing investments and operating costs. TOC considers a short – run time and assumes that operating costs are fixed costs.The steps in managing bottleneck operations are:

- Identify that the bottleneck operation determine throughput contribution of the entire system.
- Locate the bottleneck operations by identifying operations with huge quantities of inventory coming up to be worked on.
- Maintain the bottleneck operation busy and subordinate all non bottleneck operations to the bottleneck operation.
- Take necessary steps to increase the efficiency and capacity of the bottleneck operation.

To understand TOC, Dr. Goldratt gives a simple example. Corporate may be analogous to a chain. Different links are connected one after another to form the chain. Various divisions, departments, products or rules are analogous to link. The strength of the chain is the strength of its weakest link. The weakest link restricts the chain's capability in transmitting a greater force. Similar is the case for corporate. Every system contains at least one constraint, which prevents the system from attaining very high level of performance. Therefore, TOC emphasizes the need to identify constraints that prevent the system from achieving infinite profits, which is the goal of the corporate.

1.13.2 Concept of 'Theory of constraints (TOC)'– Process Improvement – An Example:

Sunshine Pte Ltd produces parts for automotive. Its primary measure of productivity is labour absorption under the assumption that if more work is being done to create inventory, profits will increase. However, using this measure resulted in actions to increase inventory and build stock products rather than fill actual customer orders.

Process improvements (like Lean Sigma initiatives) were implemented to reduce costs. Efforts were made to decrease the labour involved in producing parts. This was done for all operations. Many non-constraints became faster, producing even more work than the constraints could handle. Even though labour went down, inventory increased and it became more difficult to fulfill orders on time and to properly prioritize manufacturing jobs.

When management learned about throughput, it shifted its focus from absorbing costs into inventory to increasing how quickly work could be completed. Emphasis was given to improving constraints. By investing $89,000 in the facility and adding 3 additional workers to the day shift, output increased by 83%. Under traditional Cost Accounting, these expenses would not have been justified because local output efficiency would have declined on a per labour hour basis. However, the cost was minimal compared to the increase in throughput.

1.13.3 Illustrations

Illustration1

A company produces 3 products A, B and C.The following information is available for a period.

	Production		
	A	B	C
Contribution(Sales – Direct Materials)	₹ 24	₹ 20	₹ 12
Machine hours required per unit:			
Machine 1	12	4	2
Machine 2	18	6	3
Machine 3	6	2	1
Estimated sales demand	200	200	200

It is given that machine capacity is limited to 3,200 hours for each machine, you are required to analyze the above information and apply TOC process to remove the constraint.

Solution

Note-1:

Production		A	B	C	Total	Machine Capacity	Throughput Accounting ratio
Demand (units)		200	200	200			
Hrs. required in Dept.							
Machine	1	2,400	800	400	3,600	3,200	112.5%
	2	3,600	1,200	600	5,400	3,200	168.75%
	3	1,200	400	200	1,800	3,200	56.25%

Machine 2 is the bottleneck

Note-2:

Through put contribution & rank

		A	B	C
(a)	Throughput Contribution	24	20	12
(b)	MR/unit in Machine 2	18	6	3
(c)	Contribution/hr. Machine –2	1.33	3.33	4
	Rank	III	II	I

Identification of product mix

Production	Machine hoursused	Bal.machine hours available
Rank – I - 200 units of product C	600 (200*3)	2,600
Rank – II - 200 units of product B	1,200 (200*6)	1,400
Rank III- 77 units of Product A	1,386 (77*18)	14

☞ Students should try Illustration No.2 only after understanding the concept of "Linear Programming"

Illustration2

A company manufactures two products. Each product passes through two departments A and B before it becomes a finished product. The data for the year are as under:

	Products	***X***	***Y***
(i)	*Maximum Sales Potential (in units)*	*7,400*	*10,000*
(ii)	*Product unit data:*		
	S.P. p.u	*₹ 90*	*₹ 80*
	Machine hours p.u.		
	Department A hours @ ₹40/hr	*0.50*	*0.30*
	Department B hours @ ₹60/hr.	*0.40*	*0.45*

(iii) *Maximum capacity of Department A is 3,400 hours and Department B is 3,640 hours.*

(iv) *Maximum quantity of direct materials available is 17,000 kgs. Each product requires 2 kg. of direct materials. The purchase price of direct materials is ₹5/kg.*

(a) *You are required to find optimum product mix.*

(b) *In view of the aforesaid production capacity constraints, the company has decided to produce only one of the two products during the year. Which of the two products should be produced and sold in the year to maximise profit? State the number of units of that product and relevant contribution.*

Solution:

(a) Calculation of Optimum Production Mix

Evaluation of Limiting factor:

Particulars	Material	Hours in Department A	Hours in Department B
Required :X Y	14,800 kgs. 20,000 kgs.	3,700 hours 3,000 hours	2,960 hours 4,500 hours
Total Requirement	34,800 kgs.	6,700 hours	7,460 hours
Available resources	17,000 kgs.	3,400 hours	3,640 hours
Shortage	17,800 kgs.	3,300 hours	3,820 hours

Hence all the three resources are limiting factors.

Statement of Rank

Particulars	Product X	Product Y
Sales	90	80
Less: Direct Material Dept. A Dept. B	10 20 24	10 12 27
Contribution p.u.	36	31
Contribution per kg. of raw material	18	15.5
Rank	I	II
Contribution /hr. of Dept. A	72	103.33
Rank	II	I
Contribution /hr. of Dept. B	90	68.89
Rank	I	II

Formulation:

Let x_1 and x_2 donate quantities of product 'x' and product 'y' respectively.

$Z_{max} = 36x_1 + 31x_2$ – Total Fixed Cost

Subject to:

(i) For material, $2x_1 + 2x_2 \leq 17{,}000$

(ii) For Dept. A, $0.5x_1 + 0.3x_2 \leq 3{,}400$

(iii) For Dept. B, $0.4x_1 + 0.45x_2 \leq 3{,}640$

(iv) Demand constraint, $x_1 \leq 7{,}400$ and $x_2 \leq 10{,}000$

So, different combinations of product mix include,

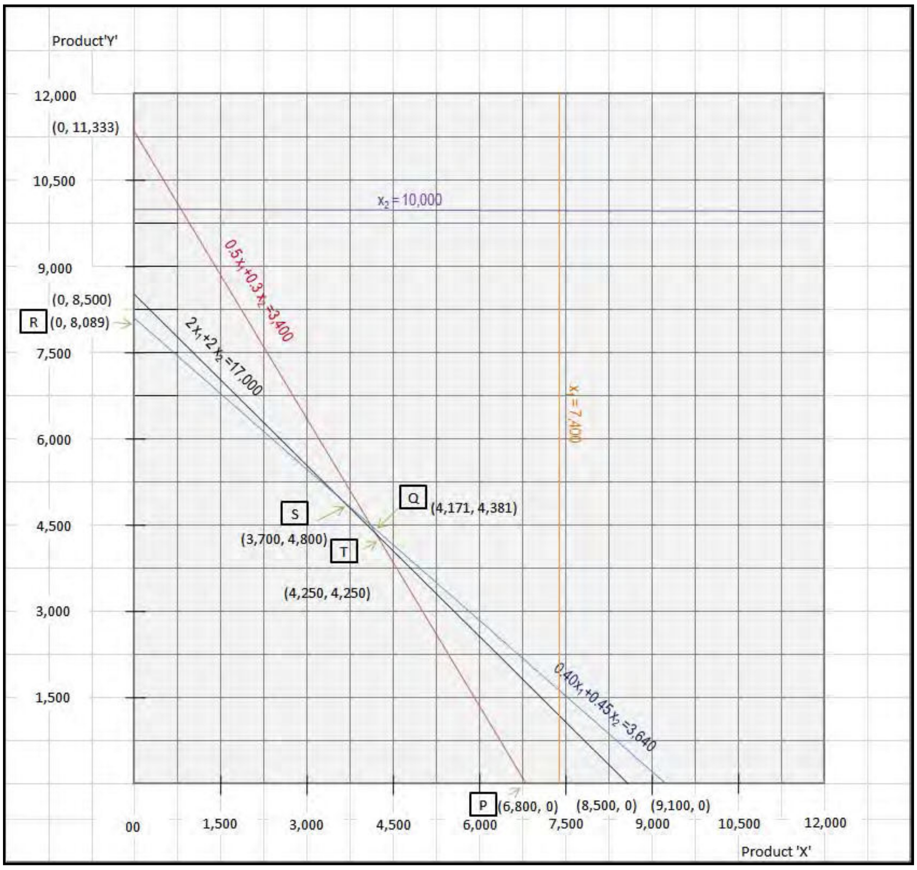
Combination	x_1	x_2	Total Contribution (in ₹)	Rank
P	6,800	0	2,44,800	IV
Q*	4,171	4,381	2,85,967	-
R	0	8,089	2,50,759	III
S	3,700	4,800	2,82,000	II
T	4,250	4,250	2,84,750	I

*Combination Q (4,171, 4,381) is not possible as it is satisfying three conditions out of above four conditions. To produce combination Q (4,171, 4,381), requirement of the material will be 17,104 Kgs. (2 Kg x 4,171 units + 2 Kg x 4,381 units). However, material is available 17,000 Kgs. Accordingly this combination is not possible.

Therefore, optimum product mix = X 4,250 units and Y 4,250 units.

Points to draw $2x_1 + 2x_2 = 17{,}000$	Points to draw $0.5x_1 + 0.3x_2 = 3{,}400$
If $x_1 = 0$ $2x_2 = 17{,}000$	If $x_1 = 0$ $0.3x_2 = 3{,}400$

$x_2 = 17{,}000/2$ $= 8{,}500$ If $x_2 = 0$ $2x_1 = 17{,}000$ $x_1 = 17{,}000/2$ $= 8{,}500$ **(x_1, x_2) - (0, 8,500); (8,500, 0)**	$x_2 = 11{,}333$ If $x_2 = 0$ $0.5x_1 = 3{,}400$ $x_1 = 6{,}800$ **(x_1, x_2) - (0, 11,333); (6,800, 0)**
Points to draw $0.4x_1 + 0.45x_2 = 3{,}640$ If $x_1 = 0$ $0.45x_2 = 3{,}640$ $x_2 = 8{,}089$ If $x_2 = 0$ $0.4x_1 = 3{,}640$ $x_1 = 3{,}640/0.4$ $= 9{,}100$ **(x_1, x_2) - (0, 8,089); (9,100, 0)**	**Intersection Point (Q)** $0.5x_1 + 0.3x_2 = 3{,}400$ (Equation1) $0.4x_1 + 0.45x_2 = 3{,}640$ (Equation2) Or $\cancel{2x_1} + 1.2x_2 = 13{,}600$ [(Equation1) x 4] $\cancel{2x_1} + 2.25x_2 = 18{,}200$ [(Equation1) x 5] – – – $-1.05x_2 = -4{,}600$ $x_2 = 4{,}381$ On putting value of x_2 in any one of the above equation, the value of $x_1 = 4{,}171$ **Point Q – (4,171, 4,381)**
Intersection Point (T) $0.5x_1 + 0.3x_2 = 3{,}400$ (Equation1) $2x_1 + 2x_2 = 17{,}000$ Or $\cancel{2x_1} + 1.2x_2 = 13{,}600$ [(Equation1) x 4] $\cancel{2x_1} + 2x_2 = 17{,}000$ – – – $-0.8x_2 = -3{,}400$ $x_2 = 4{,}250$ On putting value of x_2 in any one of the above equation, the value of $x_1 = 4{,}250$ **Point T – (4,250, 4,250)**	**Intersection Point (S)** $0.4x_1 + 0.45x_2 = 3{,}640$ (Equation1) $2x_1 + 2x_2 = 17{,}000$ Or $\cancel{2x_1} + 2.25x_2 = 18{,}200$ [(Equation1) x 5] $\cancel{2x_1} + 2x_2 = 17{,}000$ – – – $0.25x_2 = 1{,}200$ $x_2 = 4{,}800$ On putting value of x_2 in any one of the above equation, the value of $x_1 = 3{,}700$ **Point S – (3,700, 4,800)**



(b) Statement showing product with higher contribution

Product	Maximum Demand (a)	Maximum Production by Dept A (b)	Maximum Production by Dept B (c)	Maximum Production with available materials (d)	Feasible Maximum production (lower of a, b, c and d)	Contrib-ution
X	7,400	6,800	9,100	8,500	6,800	2,44,800
Y	10,000	11,333	8,089	8,500	8,089	2,50,759

Therefore, Product Y should be produced at 8,089 units resulting in a contribution of ₹2,50,759.

Illustration3

The following data is to be used to answer questions (a), (b) and (c) below.

HG Ltd manufactures four products.The unit cost, selling price and bottleneck resource details *per unit are as follows:*

	Product W ₹	*Product X* ₹	*Product Y* ₹	*Product Z* ₹
Selling price	*56*	*67*	*89*	*96*
Materials	*22*	*31*	*38*	*46*
Labour	*15*	*20*	*18*	*24*
Variable overhead	*12*	*15*	*18*	*15*
Fixed overhead	*4*	*2*	*8*	*7*
	Minutes	***Minutes***	***Minutes***	***Minutes***
Bottleneck resource time	*10*	*10*	*15*	*15*

(a) Assuming the labour is a unit variable cost, if the products are ranked according to their contribution, the most profitable product is

(A) W (B) X (C) Y (D) Z

(b) Assuming that labour is a unit variable cost, if budgeted unit sales are in the ratio W:2; X:3; Y:3; Z:4 and monthly fixed costs are budgeted to be ₹15,000, the number of units of W that would be sold at the budgeted breakeven point is nearest to

(A) 106 units (B) 142 units. (C) 212 units (D) 283 units

(c) If the company adopted throughput accounting and the products were ranked according to 'product return per minute', the highest ranked product would be

(A) W (B) X (C) Y (D) Z

Solution: (a)

	W	**X**	**Y**	**Z**
Sales price	56	67	89	96
Less: Material	22	31	38	46
Labour	15	20	18	24
Variable overhead	12	15	18	15
Contribution	7	1	15	11
Factor (a/b) Rank	III	IV	I	II

Thus, if the products are ranked according to their contribution, the most profitable product is Y.

(b)

Product	**Contribution/unit**	**Mix**	**Total**
W	7	2	14
X	1	3	3
Y	15	3	45
Z	11	4	44
		12	106

Fixed cost = 15,000

BEP = 15,000/(106/12) = 1,698units

Sales of W = 1,698 x 2/12 = 283 units

Sales of X = 1,698 x 3/12= 425 units

Sales of Y = 1,698 x 3/12 = 424 units

Sales of Z = 1,698 x 4/12 = 566 units

(c)

	W	X	Y	Z
Sales	56	67	89	96
Less: Material	22	31	38	46
(a) Through put Contribution p.u.	34	36	51	50
(b) Minutes /unit	10	10	15	15
(c) Through Contribution/min (a/b)	3.4	3.6	3.4	3.33
Rank	II	I	II	III

Illustration 4

A company makes two products A and B, its machines can work on only one product at a time. The two products are worked on in two departments by different grades of labour. The labour requirements for the two products are as follows:

Minutes per unit of product

	A	*B*
Department 1	*12*	*16*
Department 2	*20*	*15*

There is currently a shortage of labourand the maximum times available each day in Department 1 and Department 2 are 480 minutes and 840 minutes respectively. The current selling prices and costs for the two products are shown below:

	A	*B*
Selling price	*50.00*	*65.00*
Direct materials	*10.00*	*15.00*
Direct labour	*10.40*	*6.20*
Variable overhead	*6.40*	*9.20*
Fixed overhead	*12.80*	*18.40*
Profit p.u.	*10.40*	*16.20*

The company needs to know the optimum output levels. All outputs are sold.

(i) Calculate the maximum number of product that can be produced each day and identify the limiting factor

(ii) Using a throughput approach, calculate the"throughput – maximizing" output each day, at the contribution at this level of output.

Solution:

(i)

If only product A is produced,

Department	1	2
Maximum Units that can be produced	(480/12) = 40	840/20 = 42

Therefore, Department 1 is the bottleneck department and thus maximum feasible production is 40 units.

Contribution p.u of A = (Profit + Fixed Cost)p.u = 23.20

Total Contribution = 23.20 x 40 = ₹928

If only product B is produced,

Department	1	2
Maximum Units that can be produced	(480/16) = 30	840/15 = 56

Therefore, Department 1 is the bottleneck department and thus maximum feasible production is 30 units.

Contribution p.u = 34.60

Total Contribution = 34.60 x 30 = ₹1,038

Hence product B should be produced in order to record higher contribution.

(ii)

Product	Units	Throughput Contribution/unit	Total contribution
A	40	40	1,600
B	30	50	1,500

Hence, production/day = 40 units of A.

Summary

- ISO 8402-1986 standard defines quality as "the totality of features and characteristics of a product or service that bears its ability to satisfy stated or implied needs.
- TQM is composed of three paradigms: Total, Quality & Management. Total Quality Management (TQM) is a management strategy aimed at embedding awareness of quality in all organizational processes. TQM requires that the company maintain this quality standard in all aspects of its business. This requires ensuring that things are done right the first time and that defects and waste are eliminated from operations.
- The plan - do - study - act (PDSA) cycle describes the activities a company needs to perform in order to incorporate continuous improvement in its operation. This cycle, is also referred to as the Shewhart cycle or the Deming wheel.

- The Six Cs for successful implementation of a Total Quality Management (TQM) process are Commitment;, Culture, Continuous improvement, Co-operation, Customer focus, Control.
- Activity Based Costing is an accounting methodology that assigns costs to activities rather than products or services. This enables resources & overhead costs to be more accurately assigned to products & services that consume them.
- Unit level activities, Batch level activities, Product level activities and Facility level activities are the categories of activities helps to determine the type of activity cost driver required.
- The use of ABC as a costing tool to manage costs at activity level is known as Activity Based Cost Management (ABM). ABM is a discipline that focuses on the efficient and effective management of activities as the route to continuously improving the value received by customers. ABM utilizes cost information gathered through ABC.
- The value-added activities are those activities which are indispensible in order to complete the process.
- NVA activity represents work that is not valued by the external or internal customer. NVA activities do not improve the quality or function of a product or service, but they can adversely affect costs and prices.
- Activity-based budgeting is a process of planning and controlling the expected activities for the organisation to derive a cost-effective budget that meets forecast workload and agreed strategic goals.
- Key elements of ABB are type of work/activity to be performed, Quantity of work/activity to be performed and Cost of work/activity to be performed.
- Target Costing can be defined as "a structured approach to determining the cost at which a proposed product with specified functionality and quality must be produced, to generate a desired level of profitability at its anticipated selling price".
- Value engineering involves searching for opportunities to modify the design of each component or part of a product to reduce cost, but without reducing functionality or quality of the product.
- Value analysis entails studying the activities that are involved in producing the product to detect non-value-adding activities that may be eliminated or minimized to save costs, but without reducing the functionality or quality of the product.
- Kaizen costing is technique, which focuses on the reduction of waste in the production process, thereby further lowering costs below the initial targets specified during the design phase,
- Skimming Pricing Strategy and Penetration Pricing Strategy are Pricing Strategies for new products entering the market.
- The life cycle of a product consists of four phases viz., Introduction; Growth; Maturity;

Saturation and Decline.

- Industry Value Chain refers to the series of activities, which add value to the product supplied to the industry. The industry value chain starts with the value-creating processes of suppliers, who provide the basic raw materials and components. It continues with the value creating processes of different classes of buyers or end-use consumers, and culminates in the disposal and recycling of materials.
- Value Analysis is a systematic interdisciplinary examination of factors affecting the cost of a product or service in order to devise means of achieving the specified purpose at the required standard of quality and reliability at the target cost.
- Differentiation Advantage occurs when customers perceive that a business unit's product offering (defined to include all attributes relevant to the buying decision) is of higher quality, involves fewer risks and/or outperforms competing product offerings.
- Low-Cost Advantage, A firm enjoys a relative cost advantage if its total costs are lower than the market average.
- Structural cost drivers consist of organisational factors that determine the economic structure driving the cost of a firm's products. These cost drivers reflect a firm's long-term decisions, which position the firm in its industry and marketplace.
- Executional cost drivers capture a firm's operational decisions on how best to employ its resources to achieve its goals and objectives. These cost drivers are determined by management policy, style and culture.
- Vertical linkage analysis is a much broader application of internal cost and differentiation analysis that includes all upstream and downstream value-creating processes throughout the industry. Vertical linkage analysis considers all links from the source of raw materials to the disposal and/or recycling of the product.
- Non-availability of data, identification of stages, ascertainment of cost, revenues and assets, identification of cost drivers and resistance from employees are limitations of Value Chain Analysis.
- Cost Reduction is the achievement of real and permanent reduction in unit cost of products manufactured.
- Cost Control involves a comparison of actual with the standards or budgets, to regulate the actual costs.
- A just in time approach is a collection of ideas that streamline a company's production process activities to such an extent that wastage of all kinds viz., of time, material, and labour is systematically driven out of the process.
- Benefit of MRP is 'Detailed forecast of the inventory position is highlighted period by period'.
- MRP systems can be divided in two stages 'Pre-requisite information and system input' and System Processing and Output.

- Pre-requisites for successful operation of MRP are 'Strict adherence to the schedule' and 'Accurate data base'
- Synchronous manufacturing has been defined as: an all-encompassing manufacturing management philosophy that includes a consistent set of principles, procedures, and techniques where every action is evaluated in terms of the common global goal of the organisation.
- A business process consists of a collection of activities that are linked together in a coordinated & Sequential manner to achieve goal & objective.
- Throughput Accounting (TA) is a method of performance measurement which relates production and other costs to throughput. Throughput accounting product costs relate to usage of key resources by various products.
- The theory of constraints (TOC) describes methods to maximize operating income under bottleneck situation, The three measurements:

  Throughput Contribution equal to Sale - Direct Materials Cost of the goods sold.

  Investments equal to Sum of materials costs in direct materials, work-in-process, and finished goods inventories along with R & D costs and costs of equipment and buildings.

  Operating costs equal to all costs of operations (other than direct materials) incurred to earn throughput contribution. Operating costs include salaries and wages, rent, utilities and depreciation.

Source

Management Accounting For Business , By Colin Drury

Total Quality Management; By L. SuganthiAnand A. Samuel

Website of British Standards Institution’s Standards in Action

2

Decision Making Using Cost Concepts and CVP Analysis

LEARNING OBJECTIVES

After studying this unit you will be able to :

- Understand the meaning and prerequisites of relevant costs.
- Understand different cost concepts used in decision making
- Learn and apply the various cost concepts in management decision making.
- Apply the Incremental/Differential cost techniques in managerial decisions.
- Ascertain the ways of optimising the investment plan.
- Make use of cash flow technique regarding decision relating to investment alternatives.
- Understand the Marginal cost concepts
- Understand the meaning of Cost-Volume-Profit analysis and its application in managerial decision making.

2.1 Introduction

Cost has different meaning in different settings and the kind of cost concepts used in a particular situation depends upon the circumstances/requirement of each case. The costs reported by financial accountants are actual costs. For the purpose of decision making and control, costs are distinguished on the basis of their relevance to the different type of decisions and control functions. For business decision making purposes, relevant costs rather than actual costs are considered. Relevant costs constitute a practical basis of decision making which is different from historical cost approach.

2.2 Different Cost Concepts

To make appropriate decision a management accountant needs to understand different functions related to costs. Though mainly cost concepts for decision making can be broadly classified as relevant cost and non-relevant cost. However, it is very essential to understand different cost concepts separately as to make the process of decision making easier and to make correct decision making for a business organization.

2.2.1 Relevant cost and Non-relevant cost: Relevant Costs may be understood as expected future costs which are different for every alternative course of action being planned. As per

CIMA terminology relevant costs are those which will be affected by the decision being taken. All relevant costs should be considered in management decision-making.

Non-relevant costs will remain unaltered regardless of the decision being taken. Examples of non relevant costs are fixed overhead which remain constant regardless of the decision being taken, committed costs i.e. expenditure that will be incurred in future due to present obligation.

Following two conditions need to be satisfied for a cost to be called a relevant cost:-

1. **Occur in the Future -** every decision deals with selecting a course of action based on its expected future results
2. **Differ among the alternative courses of action** -costs and revenues that do not differ will not matter and will have no bearing on the decision being made.

For example, while considering a proposal for plant replacement by discarding the existing plant, the original cost and the present depreciated book value of the old plant are irrelevant as they have no impact on the decision for replacement just going to be taken place. However the expected sales value of the discarded plant is relevant, as it just goes to reduce the amount of investment to be made in the new plant and so it has an influence on the decision. Moreover, outcome of the investment is also taken into consideration for decision making.

Relevant cost analysis helps in drawing the attention of managers to those elements of cost which are relevant for the decision.

The following examples pin-point those costs which are not relevant to a decision at hand. These irrelevant costs do not play any role in the decision making:

(i) ***Historical or sunk costs are irrelevant as they do not play any role in the decision making process.*** But they are the best basis for predicting future costs. For example, if old and obsolete spare parts worth ₹5,00,000 are to be scrapped and sold for ₹15,000, the original cost of ₹5,00,000 is irrelevant to the decision.

(ii) ***Even among future costs, those variable costs which will not differ under various alternatives are irrelevant.*** For example, a company proposes to re-arrange plant facilities and estimates its future costs under two alternative choices, as under:

Particulars	Do not re-arrange	Re-arrange
	(₹)	(₹)
Direct materials cost/unit	10.00	10.00
Direct labour cost/unit	5.00	4.00

In the above example, the direct material cost remains constant under both the alternatives, hence it is irrelevant to the decision "as to whether plant facilities are to be re-arranged or not". Only direct labour cost which differs under the two alternatives is relevant. Since there is a saving of ₹1/- per unit in the second alternative, the company is advised to go in for re-arrangement of plant facilities.

(iii) ***If fixed expenses remains un-changed under different alternatives such expenses are irrelevant to the decision at hand.*** Consider, for example, the following data given for a hypothetical firm:

Expected sales	50,000 units
Variable costs	₹2.50 per unit
Fixed costs	₹1.50 per unit
Selling price	₹5.00 per unit

The firm expects to get a special export order for 10,000 units at a price of ₹3.75 per unit. Advise whether the export order should be accepted or not.

In order to advise the firm, we may analyse the figures as under:

Particulars	**Sales at 50,000 units**	**Sales at 60,000 units**	**Difference**
	(₹)	(₹)	(₹)
Sales values	2,50,000	2,87,500	+37,500
Less : Variable costs	1,25,000	1,50,000	–25,000
Contribution margin	1,25,000	1,37,500	+12,500
Less : Fixed expenses	75,000	75,000	—
Net profit	50,000	62,500	+12,500

The unit total cost is ₹4.00 (₹2.50 variable plus ₹1.50 fixed). If we use this unit total cost in taking a decision to accept the sale of additional 10,000 units, our decision will be wrong because the additional unit will incur a loss of ₹0.25 (₹3.75 – ₹4.00). If, however, we analyse the costs, we find that fixed expenses are irrelevant to the decision and, hence by excluding them we find that the new order is profitable.

Fixed costs should, however, be considered as relevant if they are expected to be altered by the decision at hand. Suppose, in the above example, the plant capacity is only 50,000 units and additional 10,000 units can only be manufactured by expanding capacity which entails additional fixed expenses of ₹50,000. This increase in fixed expenses is relevant to the decision and will be compared with the incremental contribution of ₹12,500. This will alter the earlier choice.

(iv) ***Quite often question arises whether the book value of equipment is relevant or not.*** Three points as described below emerge in such circumstances:

(a) Book value of old equipment is irrelevant because it is a past cost.

(b) The disposal value of equipment is relevant because it adds to the cash inflow arising from the decision.

(c) Cost of new equipment is relevant because cash outflow arises by the decision to buy the new equipment.

Consider the following example as an illustration of the principle. A firm is considering the replacement of an existing machine whose written down value is ₹4,000 and has four years life to run. The data are analysed as under:

Particulars	For a period of four years		
	Keep	Replace	Difference
	(₹)	(₹)	(₹)
Sales : (A)	40,000	40,000	—
Costs	32,000	22,400	+ 9,600
Depreciation:			
Old machine	4,000		+ 4,000
New machine		6,000	– 6,000
Write off of old machine		4,000	– 4,000
Disposal of the old machine		– 400	+ 400
Total expenses : (B)	36,000	32,000	+ 4,000
Operating income : {(A) – (B)}	4,000	8,000	+ 4,000

The above analysis shows that the replacement of the machine will be advantageous by ₹ 4,000.

It may be noted that in the above example, the written down value of the old machine has been written off in replacing old machine as loss because it is sunk cost. Thus it appears in both the proposals and cancels out and proves that it is irrelevant to the decision. Since the disposal value goes to reduce costs, or increases revenue and the depreciation cost of the new equipment affects such outflow, both these expenses are relevant to the decision.

In decisions involving the retention or replacement of a machine, the relevant cost concept may help in arriving at the proper decision. However, such decisions are best taken through discounted cash flow analysis.

This is because the relevant cost concept ignores the fact that under the two alternatives, cash inflows and outflows will accrue at different points of time. In the case sited above, we have taken data for four years. However, if we replace the machine, the new machine will run for more than four years. Thus, the two machines have different number of years to serve and they give rise to different cash flows. The best course is to work out the cash flows and discount them at proper rate. This will give correct result.

2.2.2 Opportunity costs: The opportunity cost of the value of opportunity foregone is taken into consideration when alternatives are compared. Opportunity Cost is the value of the next best alternative. In other words, it is the opportunity cost lost by diversion of an input factor from one use to another. It is the measure of the benefit of opportunity foregone. It is defined in the CIMA Terminology as '*the value of the benefit sacrificed when one course of action is chosen, in preference to an alternative. The opportunity cost is represented by the forgone potential benefit from the best rejected course of action.*'

The opportunity cost is helpful to managers in evaluating the various alternatives available when multiple inputs can be employed for multiple uses. These inputs may nevertheless have a cost and this is measured by the sacrifice made by the alternative action in course of choosing another alternatives.

Examples of opportunity cost:

(a) The opportunity cost of using a machine to produce a particular product is the earnings foregone that would have been possible if the machine was used to produce other products.

(b) The opportunity cost of funds invested in a business is the interest that could have been earned by investing the funds in bank deposit.

(c) The opportunity cost of one's time is the salary which he would have earned by his profession.

2.2.3 Notional Cost: As per CIMA terminology *'Cost used in product evaluation, decision making and performance measurement to reflect the use of resources that have no actual (observable) cost'*. For example, notional interest for internally generated funds ornotional rent for use of space.

Notional costs are relevant for the decision making only if company is actually forgoing benefits by employing its resources to alternative course of action. For example, notional interest on internally generated fund is treated as relevant notional cost only if company could earn interest from it.

2.2.4 Avoidable Costs: Avoidable costs are cost that can be skipped if the decision on activity in consideration will not been taken up. As per CIMA terminology avoidable costs are the *'Specific cost of an activity or sector of a business that would be avoided if the activity or sector did notexist'*. For example, if a company wants to discontinue a product line to manufacture, then the cost that can be saved due to discontinuation of a product line is called avoidable cost.

2.2.5 Historical Costs: Cost that has been already incurred in the past is called historical cost. Amount invested in purchase of machinery, for example is historical cost and is not relevant for decision making.

2.2.6 Sunk Cost: Costs which do not change under given circumstance and do not play any role in decision making process are known as sunk costs. These are the costs which have been incurred by a decision made in past and cannot be changed by any decision made in the future. *CIMA defined sunk cost as 'Cost that has been irreversibly incurred or committed and cannot therefore be considered relevant to a decision. Sunk costs may also be termed irrecoverable costs'*. All sunk costs are irrelevant for decision making but all irrelevant cost may not be sunk cost.

2.2.7 Out of Pocket cost: This is that portion of costs which involves payments to outsiders i.e. it gives rise to cash expenditure as opposed to such costs as depreciation, which do not involve any cash expenditure. Such costs are relevant for price fixation during recession or when make or buy decisions are to be made. These costs include cost incurred under some heads for which separate cost accounting records are not maintained and they are sometimes termed as miscellaneous cost.

2.2.8 Discretionary cost: These are the costs which can be changed with the discretion of a manager or appropriate decision making authority. *CIMA defines discretionary costs as 'Cost whose amount within a time period is determined by a decision taken by the appropriate budget fluctuations in the levels of activity (output or turnover)'*. Examples of discretionary costs are maintenance, research and development, employees training, advertisement etc.

2.2.9 Committed cost: Committed costs are the cost which can not be changed during the budgeted period. *As per CIMA terminology 'Cost arising from prior decisions, which cannot, in the short run, be changed. Committed cost incurrence often stems from strategic decisions concerning capacity with resulting expenditure on plant and facilities. Initial control of committed costs at the decision point is through investment appraisal techniques'*. Examples of committed costs are depreciation of assets, lease rentals, other contractual costs etc.

2.2.10Shut down cost: When an organization suspends its manufacturing, certain fixed expenses can be avoided and certain extra fixed expenses may be incurred depending upon the nature of the industry. By closing down the manufacturing, the organization will save variable cost of production as well as some discretionary fixed costs. This particular discretionary cost is known as shut-down cost.

2.2.11 Engineered costs: Engineered costs results from a defined mathematical relationship with the cost object and resources consumed to produce an output. For example,to produce one unit of a leather shoe requires known amount of leather and time to be spent in stitching is also known. The leather,labour and machine hours are therefore engineered costs.

2.2.12 Inventoriable costs: Coststhat are considered as part of merchandise and considered as asset when these are incurred and these costs become cost of goods sold when the final output is sold. In other words inventoriable costs are cost of purchase plus cost expended to make final product in saleable condition. For a manufacturer, cost of raw material issued to production plus labour cost plus manufacturing overheads are inventoriable cost.

2.2.13 Period cost: Period costs are all costs other than cost of goods sold and treated as expense in income statement. CIMA defines it as *'Cost relating to a time period rather than to the output of products or services'*.A period cost is charged to expense in the period incurred. This type of cost is not included within the cost of goods sold on the income statement.This type of expenses is generally included within the selling and administrative expenses. Example of period costs are: depreciation expenses, commission, advertisement expenses, sales expenses etc.

2.2.14 Differential cost, Incremental cost and Incremental revenue: Differential cost (which may be incremental or decremental cost) is the difference in total cost that will arise from the selection of one alternative instead of another. It involves the estimation of the impact of decision alternatives on costs and revenues. The two basic concepts which go together with this type of cost analysis are *incremental revenue* and *incremental costs*. *Incremental revenue* is the change in the total income resulting from a decision. *Incremental costs* represent a change in the total costs resulting from a decision. Such a change in cost is not necessarily variable in nature.

2.2.15 Limiting factor or key factor: A limiting factor is any factor that is in scarce supply and without that further activities cannot be performed i.e. it limits the organizations activity. CIMA defines limiting factor or key factor as 'Anything which limits the activity of an entity. An entity seeks to optimise the benefit it obtains from the limiting factor. Examples are a shortage of supply of a resource or a restriction on sales demand at a particular price'. Key factor may be anything i.e. materials, labour, machine time, sales quantity etc.

2.3 Application of Cost Concepts in Decision Making

2.3.1 Decision on Acceptance of New Offer

Illustration1

A company has been making a machine to order for a customer, but the customer has since gone into liquidation, and there is no prospect that any money will be obtained from the winding up of the company.

Costs incurred to-date in manufacturing the machine are ₹ 50,000 and progress payments of ₹15,000 have been received from the customer prior to the liquidation.

The sales department has found another company willing to buy the machine for ₹34,000 once it has been completed.

To complete the work, the following costs would be incurred:

(a) Materials-these have been bought at a cost of ₹6,000. They have no other use, and if the machine is not finished, they would be sold as scrap for ₹2,000.

(b) Further labour costs would be ₹8,000. Labour is in short supply, and if the machine is not finished, the work force would be switched to another job, which would earn ₹30,000 in revenue, and incur direct costs (not including direct labour) of ₹12,000 and absorbed fixed overhead of ₹8,000.

(c) Consultancy fees, ₹4,000. If the work is not completed, the consultant's contract would be cancelled at a cost of ₹1,500.

(d) General overheads of ₹ 8,000 would be added to the cost of the additional work.

Should the new customer's offer be accepted? Prepare a statement showing the economics of the proposition.

Solution

Working notes:

(i) Costs incurred in the past are sunk costs and revenue received in the past is also not relevant because they do not have a bearing on a decision at hand. Hence costs of ₹50,000 incurred to date in manufacturing the machine and progress payment (revenue) of ₹15,000 received are irrelevant and should be ignored.

(ii) The price paid in the past for the material is irrelevant. The only relevant cost of materials affecting the decision is the opportunity cost in the form of revenue from scrap which would be foregone i.e.₹2,000.

(iii) *Labour costs:*

	(₹)
Contribution from the use of labour at another job foregone is opportunity cost and is relevant (₹30,000 – ₹8,000 – ₹12,000)	10,000

(iv) Differential (Incremental) cost of consultancyfor completing the work

Cost of completing the work	4,000
Less: Cost of cancelling the contract	1,500
Incremental cost of completing the work	2,500

(v) Absorbed overheads and general overheads are allocated costs and should be ignored.

Statement showing the economics of the proposition

(Only relevant costs considered)	(₹)	(₹)
Revenue from completing work : (A)		34,000
Less: Relevant cost of:		
Material – Opportunity cost	2,000	
Labour – Cost to be incurred	8,000	
Opportunity cost	10,000	
Incremental cost of consultancy	2,500	
Cost of completing work : (B)		22,500
Extra profit to be earned by accepting the offer of the new customer to complete the work : {(A) – (B)}		11,500

Since the acceptance of the offer would yield an extra profit of ₹11,500, the offer should be accepted.

Alternative solution on cash flow basis :

Statement showing economics of the proposition

	When machine is completed	**When machine is not completed**	**Incremental cash flow**
	(₹)	(₹)	(₹)
Cash inflow from sale of machine	34,000	—	34,000
Cash inflow from :			
Sale of material as scrap	—	2,000	–2,000
Use of labour at another job			

₹ 30,000 – (₹ 8,000 + ₹ 12,000)	—	10,000	–10,000
Total cash inflow : (A)	34,000	12,000	22,000
Cash outflow on :			
Labour	8,000	—	–8,000
Consultancy fees	4,000	1,500	–2,500
Total cash outflow : (B)	12,000	1,500	–10,500
Net cash inflow {(A) – (B)}	22,000	10,500	11,500

Completion of machine would result in an incremental cash inflow of ₹11,500; hence the machine should be completed.

2.3.2 Cost Sheet of a product with relevant cost

Illustration 2

Tiptop Textiles manufactures a wide range of fashion fabrics. The company is considering whether to add a further product the "Superb" to the range. A market research survey recently undertaken at a cost of ₹50,000 suggests that demand for the "Superb" will last for only one year, during which 50,000 units could be sold at ₹18 per unit. Production and sale of "Superb" would take place evenly throughout the years. The following information is available regarding the cost of manufacturing "Superb".

Raw Materials: *Each "Superb" would require 3 types of raw material Posh, Flash and Splash. Quantities required, current stock levels and cost of each raw material are shown below. Posh is used regularly by the company and stocks are replaced as they are used. The current stock of Flash is the result of overbuying for an earlier contract. The material is not used regularly by Tiptop Textiles and any stock that was not used to manufacture "Superb" would be sold. The company does not carry a stock of Splash and the units required would be specially purchased.*

Raw Material	***Quantity required per unit of Superb (metres)***	***Current stock level (metres)***	***Costs per metre of raw material***		
			Original Cost	***Current replace-ment cost***	***Current resale value***
			(₹)	***(₹)***	***(₹)***
Posh	*1.00*	*1,00,000*	*2.10*	*2.50*	*1.80*
Flash	*2.00*	*60,000*	*3.30*	*2.80*	*1.10*
Splash	*0.5*	*0*	*—*	*5.50*	*5.00*

Labour : *Production of each "Superb" would require a quarter of an hour of skilled labour and two hours of unskilled labour. Current wage rates are ₹3 per hour for skilled labour and ₹2 per hour for unskilled labour. In addition, one foreman would be required to devote all his working time for one year in supervision of the production of Superb. He is currently paid an annual salary of ₹15,000. Tiptop Textiles is currently finding it very difficult to get skilled labour. The skilled workers needed to manufacture "Superb" would be transferred from another job, on* *which they are earning a contribution surplus of ₹1.50 per labour hour, comprising sales revenue ₹10.00 less skilled labour wages of ₹3.00 and other variable cost of ₹5.50. It would not be possible to employ additional skilled labour during the coming year. If "superb" are not manufactured, the company expects to have available 2,00,000 surplus unskilled labour hours during the coming year. Because the company intends to expand in the future, it has decided not to terminate the services of any unskilled worker in the foreseeable future. The foreman is due to retire immediately on an annual pension of ₹6,000 payable by the company. He has been prevailed upon to stay on for a further year and to defer his pension for one year in return for his annual salary.*

Machinery: *Two machines would be required to manufacture "Superb" MT 4 and MT 7. Details of each machine are as under:*

		Start of the year (₹)	***End of the year*** (₹)
MT 4	*Replacement cost* *Resale value*	*80,000* *60,000*	*65,000* *47,000*
MT 7	*Replacement cost* *Resale value*	*13,000* *11,000*	*9,000* *8,000*

Straight line depreciation has been charged on each machine for each year of its life. Tiptop Textiles owns a number of MT 4 machines, which are used regularly on various products. Each MT 4 is replaced as soon as it reaches the end of its useful life. MT 7 machines are no longer used and the one which would be used for "Superb" is the only one the company now has. If it was not used to produce "Superb", it would be sold immediately.

Overheads: *A predetermined rate of recovery for overhead is in operation and the fixed overheads are recovered fully from the regular production at ₹3.50 per labour hour. Variable overhead costs for "Superb" are estimated at ₹1.20 per unit produced.*

For the decision-making, incremental costs based on relevant costs and opportunity costs are usually computed.

You are required to compute such a cost sheet for "Superb" with all details of materials, labour, overhead etc., substantiating the figures with necessary explanations.

Solution

Details of relevant costs with explanations:

(i) Market Research Survey expenses of ₹50,000 is sunk cost and hence not relevant for the decision on hand.

(ii) *Raw materials:*

(a) Posh is used regularly and stocks are replaced as they are used. Therefore, its current replacement cost of ₹2.50 is relevant.

Posh: 50,000 metres (50,000 units x 1 mtr) × ₹2.50 =₹1,25,000

(b) 1,00,000metres (50,000 units x 2 mtr) of Flash are required for the output of "Superb". There are already 60,000 metres in stock as a result of overbuying for an earlier contract purchased @ ₹3.30 per metre, and 40,000 metres additionally would be purchased at the current replacement cost of ₹2.80 per metre. If "Superb" were not produced, the company would have sold 60,000 metres of Flash at ₹1.10. This is an opportunity foregone and relevant. Hence

Flash : –

		(₹)
Incremental cost		
40,000 metres × ₹2.80	=	1,12,000
Opportunity cost		
60,000 metres × ₹1.10	=	66,000
1,78,000		

(c) 25,000 metres (50,000 units x 0.5 mtr) of splash would be specially purchased for the output

Splash 25,000 metres×₹5.50 = ₹1,37,500

(iii) *Labour:*

To manufacture 50,000 units of "Superb"
Skilled labour required: 50,000 × ¼ = 12,500 hours, and
Unskilled labour required: 50,000 × 2 = 1,00,000 hours.

Wage rate for skilled labour is ₹3 per hour. If "Superb" were not manufactured and the skilled labour were not transferred, they would have given a clean contribution of ₹1.50 per hour. This is the cost of an opportunity foregone:

Therefore,

Cost of skilled labour:	(₹)
Cost of deployment (12,500 × ₹3)	37,500
Add: Opportunity cost (12,500 × ₹1.50)	18,750
	56,250

Unskilled labour:

No work has suffered and no extra cost is involved hence cost of unskilled labour: Zero

Foreman:

	(₹)
Annual salary	15,000
Less: Pension saved	6,000
Effective cost	9,000

(iv) *Machinery:*

MT 4 machines are used and replaced regularly. The difference of the replacement cost between start and end of the year is relevant.

Hence, MT 4 cost of using: ₹15,000

MT 7 machine is not in vogue and will be sold now or in near future. The fall in its resale value represents the relevant cost.

Hence, cost of using MT 7: ₹11,000 – ₹8,000 = ₹ 3,000

(v) *Overheads:*

Fixed overheads have been recovered fully from existing production. So its rate of recovery is not relevant.

Variable overheads: 50,000 × ₹1.20 =₹60,000

Now we can prepare the cost sheet.

Cost sheet for 50,000 units of "Superb"

	(₹)	(₹)
Raw material:		
Posh	1,25,000	
Flash	1,78,000	
Splash	1,37,500	4,40,500
Labour:		
Skilled	56,250	
Unskilled	0	
Foreman	9,000	65,250
Machinery costs		
MT4	15,000	
MT7	3,000	18,000
Variable overheads		60,000
Total cost		5,83,750
Profit (₹9,00,000 – ₹ 5,83,750)		3,16,250
Sales revenue (50,000 × ₹18)		9,00,000

2.3.3 Decisionmaking on choosing profitable course of action

Illustration3

The Officers' Recreation Club of a large public sector undertaking has a cinema theatre for the exclusive use of themselves and their families. It is a bit difficult to get good motion pictures for show and so pictures are booked as and when available.

The theatre has been showing the picture "Blood Bath" for the past two weeks. This picture which is strictly for Adults only has been great hit and the Manager of the theatre is convinced *that the attendance will continue to be above normal for another two weeks, if the show of "Blood Bath" is extended. However, another popular movie, eagerly looked forward to by both adults and children alike, - "Appu on the Airbus" is booked for the next two weeks. Even if "Blood Bath" is extended, the theatre has to pay the regular rental on "Appu on the Airbus" as well.*

Normal attendance at the theatre is 2,000 patrons per week, approximately one-fourth of whom are children under the age of 12. Attendance for "Blood Bath" has been 50% greater than the normal total. The manager believes that this would taper off during a second two weeks, 25% below that of the first two weeks during the third week and 33.1/3% below that of the first two weeks during the fourth week. Attendance for "Appu on the Airbus" would be expected to be normal throughout its run, regardless of the duration.

All runs at the theatre are shown at the regular price of ₹ 2 for adults and ₹ 1.20 for children under 12. The rental charge for "Blood Bath" is ₹ 900 for one week or ₹ 1,500 for two weeks. For "Appu on the Airbus" it is ₹750 for one week or ₹ 1,200 for two weeks. All other operating costs are fixed ₹ 4,200 per week, except for the cost of potato wafers and cakes which average 60% of their selling price. Sales of potato wafers and cakes regularly average ₹1.20 per patron, regardless of age.

The Manager can arrange to show "Blood Bath" for one week and "Appu on the Airbus" for the following week or he can extend the show of "Blood Bath" for two weeks; or else he can show "Appu on the Airbus" for two weeks, as originally booked.

Show by computation, the most profitable course of action he has to pursue.

Solution

THE OFFICERS' RECREATION CLUB
Comparative predicted income for two weeks

	Three decision alternatives		
	Show "Blood Bath" for two weeks	**Show "Blood Bath" for one week and "Appu on the Airbus"for the following week**	**Show "Appu on the Airbus" for two weeks**
Attendance:			
Adults:			
First week	2,250	2,250	1,500
Second week	2,000	1,500	1,500
	4,250	3,750	3,000
Children:			
First week	—	—	500
Second week	—	500	500
Total attendance	4,250	4,250	4,000

Revenue:	(₹)	(₹)	(₹)
Sale of Tickets:			
Adults @ ₹2/-	8,500	7,500	6,000
Children @ ₹1.20	—	600	1,200
Sale of Potato Wafers & Cakes			
@ ₹1.20 per patron	5,100	5,100	4,800
Total revenue: (A)	13,600	13,200	12,000
Costs *(only relevant):*			
Hire charges of			
"Blood Bath"	1,500	900	—
Cost of Potato Wafers & Cakes			
(60% of their selling price)	3,060	3,060	2,880
Total relevant cost: (B)	4,560	3,960	2,880
Profit: {(A) – (B)}	9,040	9,240	9,120

It is seen from the above statement that the most profitable course of action is to show each film for one week. Hence, the manager should arrange to show "Blood Bath" for one week and "Appu on the Airbus" for the following week.

Note: The hire charges for "Appu on the Airbus" and the fixed operating costs of ₹4,200 per week are irrelevant to this analysis as these are committed fixed costs.

2.3.4 Decision making on pricing against a special order

Illustration 4

(a) *A machine which originally cost ₹12,000 has an estimated life of 10 years and is depreciated at the rate of ₹1,200 per year. It has been unused for some time, however, as expected production orders did not materialise. A special order has now been received which would require the use of the machine for two months.*

The current net realisable value of the machine is ₹8,000. If it is used for the job, its value is expected to fall to ₹7,500. The net book value of the machine is ₹8,400. Routine maintenance of the machine currently costs ₹40 per month. With use, the cost of maintenance and repairs would increase to ₹60 per month.

What would be the relevant cost of using the machine for the order so that it can be charged as the minimum price for the order?

(b) *X Ltd. has been approached by a customer who would like a special job to be done for him and is willing to pay ₹22,000 for it. The job would require the following materials:*

Material	Total units required	Units already in stock	Book value of units in stock ₹/unit	Realisable value ₹/unit	Replacement cost ₹/unit
A	1,000	0	—	—	6

B	*1,000*	*600*	*2*	*2.5*	*5*
C	*1,000*	*700*	*3*	*2.5*	*4*
D	*200*	*200*	*4*	*6*	*9*

(i) Material B is used regularly by X Ltd. and if stocks are required for this job, they would need to be replaced to meet other production demand.

(ii) Materials C and D are in stocks as a result of previous excess purchase and they have restricted use. No other use could be found for material C but material D could be used in another job as substitute for 300 units of material E which currently costs ₹5 per unit (of which the company has no units in stock at the moment).

What are the relevant costs of material, in deciding whether or not to accept the contract? Assume all other expenses on this contract to be specially incurred besides the relevant cost of material is ₹550.

Solution

(a) Relevant costs of using the machine for the order

		(₹)
(i)	Loss in the net realisable value of machine by using it on the order(₹8,000 – ₹7,500)	500
(ii)	Additional maintenance and repair for two months, i.e., (₹ 60 – ₹40) × 2	40
	Minimum price	540

Notes (a) (i) Books value of ₹8,400 is irrelevant for decision.

(ii) Net realisable value of the machine fall from ₹8,000 to ₹7,500. This loss of ₹500 is relevant for decision, because it is influenced exclusively by the decision.

(iii) ₹7,500 will be realised after months at least. Therefore, time value of ₹7,500 for two months atleast. Therefore, present value of future realisable value of ₹7,500 should be found out and this present value should be deducted from ₹8,000. This will be the correct relevant cost in place of ₹500 shown above in the absence of discounting factor.

(b) (i) Material A is not yet owned. It would have to be purchased in full at the replacement cost of ₹6.00 per unit. Relevant cost is therefore 1,000 units at the replacement Cost.

(ii) Material B is used by the company regularly. There is already existing a stock of 600 units. If these are used in the contract, a further 400 units would have to be purchased.

Relevant cost is therefore 1,000 units at the replacement Cost.

(iii) Material C: 1,000 units of material C are required. 700 units are already in stock. If it is used for the contract, a further 300 units will have to be purchased at a replacement cost of ₹4.00 each. The existing stock of 700 units will not be replaced. If they are used for the contract, they cannot be sold @ ₹ 2.50 each unit. The realisable value of these units 700 units @ ₹2.50 per unit represent opportunity cost.

(iv) Material D is already in stock and will not be replaced. There is an opportunity cost of using D in the contract. It has following two uses:

It can be sold to fetch ₹1,200 i.e., 200 × ₹6

It can also be used for E, which would cost ₹1,500 i.e., 300 × ₹5.

Since substitution is more useful, ₹1,500 is the opportunity cost.

(c) Summary of relevant costs:

		(₹)
Material A	1,000 units× ₹6	6,000
Material B	1,000 units× ₹5	5,000
Material C	700 units× ₹2.5	1,750
	300 units× ₹4	1,200
Material D	300 units× ₹5	1,500
Other expenses		550
Total relevant cost		16,000

(d) Contract should be accepted since offer is of ₹22,000 in relation to relevant cost of ₹16,000.

2.3.5 Budgeting overhead cost and pricing decision

Illustration 5

Forward Foundry Ltd., is feeling the effects of a general recession in the industry. Its budget for the coming year is based on an output of only 500 tonnes of castings a month, which is less than half of its capacity. The prices of castings vary with the composition of the metal and the shape of the mould, but they average ₹175 a tonne. The following details are from the monthly Production Cost Budget at the 500 tonne level:

	Core Making (₹)	***Melting and Pouring*** (₹)	***Moulding*** (₹)	***Cleaning and Grinding*** (₹)
Labour	*10,000*	*16,000*	*6,000*	*4,500*
Variable overhead	*3,000*	*1,000*	*1,000*	*1,000*
Fixed overhead	*5,000*	*9,000*	*2,000*	*1,000*
Total	*18,000*	*26,000*	*9,000*	*6,500*
Labour and Overhead per direct labour hr.	*9.00*	*6.50*	*6.00*	*5.20*

Operating at this level has brought the company to the brink of break-even. It is feared that if the lack of work continues, the company may have to lay off some of the most highly skilled workers whom it would be difficult to get back when the volume picks up later on. No wonder, the Works Manager at this juncture, welcomes an order for 90,000 castings, each weighing about 40 lb. to be delivered on a regular schedule during the next six months. As the immediate concern of the Works Manager is to keep his work force together, occupied, he does not want to lose the order and is ready to recommend a quote on a no profit no loss basis.

Materials required would cost ₹1 per casting after deducting scrap credits. The direct labour hours per casting required for each department would be:

Core making	*0.09*
Melting and pouring	*0.15*
Moulding	*0.06*
Cleaning and grinding	*0.06*

Variable overhead would bear a normal relationship to labour cost in the melting and pouring department and in the moulding department. In core making, cleaning and grinding, however, the extra labour requirements would not be accompanied by proportionate increases in variable overhead. Variable overhead would increase by ₹1.20 for every additional labour hour in core making and by 30 paise for every additional labour hour in cleaning and grinding. Standard wage rates are in operation in each department and no labour variances are anticipated.

To handle an order as large as this, certain increases in fixed factory overhead would be necessary amounting to ₹1,000 a month for all departments put together. Production for this order would be spread evenly over the six months period.

You are required to:

(a) Prepare a revised monthly labour and overhead cost budget, reflecting the addition of this order,

(b) Determine the lowest price at which quotation can be given for 90,000 castings without incurring a loss.

Solution

(a) **FORWARD FOUNDRY LTD.**

Revised Monthly Labour and Overhead Cost Budget

(After the acceptance of an order for 90,000 castings)

	DEPARTMENTS				
	Core Making (₹)	**Melting and pouring (₹)**	**Moulding (₹)**	**Cleaning andGrinding (₹)**	**Total (₹)**
Labour	16,750	25,000	9,600	7,740	59,090

Variable overhead	4,620	1,563	1,600	1,270	9,053
Fixed overhead	5,000	9,000	2,000	1,000	17,000
	26,370	35,563	13,200	10,010	85,143
Incremental fixed factory overhead for all departments					1,000
Total labour and overhead cost					86,143

Working notes :

(i) Current labour hours per month in each department are obtained by dividing the total labour and overheads by the figure of labour and overheads per direct labour hour as follows:

Core Making	Melting and Pouring	Moulding	Cleaning and Grinding
₹ 18,000 / 9 hrs. = 2,000 hrs.	₹ 26,000 / 6.50 hrs. = 4,000 hrs.	₹ 9,000 / 6 hrs. = 1,500 hrs.	₹ 6,500 / 5.2 hrs. = 1,250 hrs.

(ii) 90,000 castings spread over 6 months give a production of 15,000 castings per month. Incremental labour hours per month are got by multiplying the 15,000 castings by direct labour hour per casting as under:

Core Making	Melting and pouring	Moulding	Cleaning and Grinding
15,000 × 0.09 = 1,350 hrs.	15,000 × 0.15 = 2,250 hrs.	15,000 × 0.06 = 900 hrs.	15,000 × 0.06 = 900 hrs.

(iii) *Wage rate per hour is found by dividing labour cost by direct labour hours as under*:

Core Making	Melting and Pouring	Moulding	Cleaning and Grinding
₹10,000 / 2,000 hrs. = ₹5	₹16,000 / 4,000 hrs. = ₹4	₹6,000 / 1,500 hrs. = ₹4	₹4,500 / 1,250 hrs. = ₹3.60

(iv) *Revised monthly labour cost:*

In Core Making	:	₹10,000 + (1,350 × ₹5)	= ₹16,750
In Melting and Pouring	:	₹16,000 + (2,250 × ₹4)	= ₹25,000
In Moulding	:	₹6000 + (900 × ₹4)	= ₹9,600
In Cleaning & Grinding	:	₹4,500 + (900 × ₹3.60)	= ₹7,740

(v) Revised monthly variable overhead cost:

In Core Making, existing charges ₹3,000 plus ₹1.20 × 1,350 (incremental hours)

= ₹3,000 + ₹1,620 =₹4,620

In the Melting and Pouring Department, it is 1/16 of labour cost. Hence revised variable overhead cost

= ₹25,000 × 1/16 =₹1,563

In Moulding Department, it is 1/6 of labour cost. Hence revised variable overhead cost

= ₹9,600 × 1/6 =₹1,600

In Cleaning and Grinding, existing charges ₹1,000 plus ₹0.30 × 900 (incremental hours)

= ₹1,000 + ₹270 = ₹1,270

(b) Determination of the lowest price at which quotation can be given for 90,000 castings without incurring a loss:

	(₹)	(₹)
Materials cost: 15,000 castings per month @ ₹1 each		15,000
Labour and overhead cost:		
Revised budget (above)	86,143	
Less: Current budget		
(₹18,000 + ₹26,000 + ₹9,000 + ₹6,500)	59,500	26,643
Total incremental cost for 15,000 castings		41,643

Lowest price at which quotation can be given for 90,000 castings:

$$\frac{₹41,643}{15,000 \text{ castings}} \times 90,000 \text{ castings} = ₹2,49,858$$

2.3.6 Decision makingwith Opportunity Cost and Sunk Cost

Illustration6

Estimated direct material requirements of a business concern viz., ABC Ltd. for the year 2011-12 are 1,00,000 units. Unit cost for orders below 1,20,000 units is ₹10. When size of order equals 1,20,000 units or more the concern received a discount of 2% on the above quoted per unit price. Keeping in view the following two alternatives:

(i) Buy 1,20,000 units at the start of the year;

(ii) Buy 10,000 units per month.

Calculate the opportunity cost, if the concern has the facility of investing surplus funds in government bonds at the rate of 10% interest.

Solution

Average investment in inventory under the given two alternatives is:

(i) (1,20,000 units × ₹9.80)/2 = ₹ 5,88,000

(ii) (10,000 units × ₹10)/2 = ₹ 50,000

Difference between the average investment in inventory under:

*Alternative*s (i) and (ii) is (₹5,88,000 – ₹50,000) = ₹5,38,000

The concern can invest ₹5,38,000 at 10 percent and can earn ₹53,800 as interest annually. The sum of ₹53,800 is an opportunity foregone if alternative (i) is chosen. Hence ₹53,800 is the opportunity cost of the 1,20,000 units purchase order.

Note: ₹53,800 would not ordinarily be recorded in the accounting system as it is a foregone cost.

Illustration7

A company produces a certain waste which can be sold at a salvage price of ₹0.90 per kg. The company wants to process the waste product further at a labour and overhead cost of ₹0.75 per kg. and sell it at a higher price of ₹1.60 per Kg. Here the sale value of processed waste has no meaning unless we take into account the opportunity cost, viz, the disposal value of waste product. While analysing the profitability of processing the waste further, the salvage value of waste should, therefore, be taken into consideration as opportunity cost as under:

	Waste sold (₹)	**Waste processed** (₹)
Income per kg. : (A)	0.90	1.60
Labour& overheads	—	0.75
Opportunity cost of waste	—	0.90
Total cost : (B)	—	1.65
Net gain (loss) : {(A)–(B)}	0.90	(0.05)

Solution

It is not advisable to process the waste further since it incurs a loss of 5 paise per kg, after taking into account the opportunity cost of waste. Thus the opportunity cost represents the maximum contribution foregone by using the limited resources for a particular purpose.

Illustration 8

An owner of a plot of land has three proposals as under:

A. *Sell the plot now for a net income of ₹1,00,000*

B. *Rent out the land at an annual net rental of ₹8,000 for 25 years and thereafter sell it for a value of ₹1,50,000.*

C. *Spend ₹10,00,000 in construction of building now and thereafter rent out the building at a net annual rental of ₹1,10,000 for 25 years. Thereafter sell the building for ₹3,00,000.*

Taking the rate of return at 10% advise as to which of the three alternatives is the most profitable course of action.

Solution

Taking the rate of return at 10% the result may be tabulated as under:

	A Sell now the land (₹)	B Rent out building (₹)	C Construct (rent out) (₹)
0 (Initial year)	1,00,000	Nil	–10,00,000
1 to 25 years	–	2,00,000	27,50,000
After 25 years	–	1,50,000	3,00,000
Net cash inflow	1,00,000	3,50,000	20,50,000
Net present value of cash inflow @ 10%	1,00,000	86,416*	26,070**

* 8000 x PVIFA(10%,25) + 1,50,000 x PVIF(10%,25)

8000 x 9.077 + 1,50,000 x 0.092 = 86,416

** (1,10,000 x 9.077 + 3,00,000 x 0.092) – 10,00,000 =26,070

The opportunity costs of three alternatives are shown explicitly. The first alternative, namely, to sell now yields the highest net present value and hence it is acceptable.

2.3.7 Decision making on acceptance of an offer

Illustration9

ZED Ltd. operates two shops. Product A is manufactured in Shop - 1 and customers' jobs against specific orders are being carried out in Shop - 2. Its annual statement of income is:

	Shop - 1 (Product -A) (₹)	*Shop - 2 (Job works)* (₹)	*Total* (₹)
Sales/Income	*1,25,000*	*2,50,000*	*3,75,000*
Material	*40,000*	*50,000*	*90,000*
Wages	*45,000*	*1,00,000*	*1,45,000*
Depreciation	*18,000*	*31,500*	*49,500*
Power	*2,000*	*3,500*	*5,500*
Rent	*5,000*	*30,000*	*35,000*
Heat and light	*500*	*3,000*	*3,500*
Other expenses	*4,500*	*2,000*	*6,500*
Total costs	*1,15,000*	*2,20,000*	*3,35,000*
Net Income	*10,000*	*30,000*	*40,000*

The depreciation charges are for machines used in the shops. The rent and heat and light are apportioned between the shops on the basis of floor area occupied. All other costs are current expenses identified with the output in a particular shop.

A valued customer has given a job to manufacture 5,000 units of X for Shop - 2. As the company is already working at its full capacity, it will have to reduce the output of product - A *by 50%, to accept the said job. The customer is willing to pay ₹25 per unit of X. The material and labour will cost ₹10 and ₹18 respectively per unit. Power will be consumed on the job just equal to the power saved on account of reduction of output of A. In addition the company will have to incur additional overheads of ₹10,000.*

You are required to compute the following in respect of this job:

(a) Differential cost;

(b) Full cost;

(c) Opportunity cost; and

(d) Sunk cost

Advise whether the company should accept the job

Solution

(a) Differential cost of the job:

	Increase (₹)	Decrease (₹)
Material cost	50,000	20,000
Labour cost	90,000	22,500
Additional overheads	10,000	—
Other expenses	—	2,250
Total	1,50,000	44,750

Net differential cost of the jobs: ₹1,05,250

(₹1,50,000 – ₹44,750)

Note: Depreciation, rent, heat and light and power are not going to affect the costs.

(b) Full cost of the jobs:

Cost as above at (a) (i.e. increased costs)	1,50,000
Depreciation	9,000
Power	1,000
Rent	2,500
Heat & light	250
	1,62,750

(c) Opportunity cost of taking the order:

	(₹)	(₹)
Sale of product A		62,500
Less:		
Material	20,000	
Labour	22,500	
Power	1,000	

Other expenses	2,250	45,750
		16,750

(d) Sunk cost of the jobs:

	(₹)
Depreciation	9,000
Power*	1,000
Rent	2,500
Heat & light	250
	12,750

*If a student treats power as a relevant cost, in that case it would not appear here.

Advice regarding the jobs:

ZED Ltd., should not accept the job as there will be a cash disadvantage of ₹ 42,750/- as computed below:

	(₹)	(₹)
Incremental revenue		
5,000 units @ ₹25	1,25,000	
Less: Sale of Product A	62,500	62,500
Differential cost(a)		1,05,250
Cash disadvantage		42,750

Illustration 10

The Aylett and Co., Ltd has been offered a contract, if accepted would significantly increase next year's activity levels. The contract requires the production of 20,000 kg. of product X and specifies a contract price of ₹100 per kg. The resources used in the production of each kg. of X include the following:

Resources per kg. of Product X

Labour	*Grade -1*	*2 hours*
	Grade -2	*6 hours*
Materials	*A*	*2 units*
B	*1 litre*	

Grade 1 labour is highly skilled and although it is currently under utilised in the firm it is Aylett's policy to continue to pay grade 1 labour in full. Acceptance of the contract would reduce the idletime of grade 1 labour. Idle time payments are treated as non-production overheads.

Grade 2 is unskilled labour with a high turnover and may be considered a variable cost.

The costs to Aylett of each type of labour are:

Grade 1 ₹ 4 per hour

Grade 2 ₹ 2 per hour

The materials required to fulfil the contract would be drawn from those materials already in stock. Materials A is widely used within the firm and any usage for this contract willnecessiate replacement. Materials B was purchased to fulfil an expected order that was not received, if material B is not used for the contract, it will be sold. For accounting purposes FIFO is used. The various values and costs for A and B are:

	A Per Unit (₹)	**B Per Litre (₹)**
Book value	*8*	*30*
Replacement cost	*10*	*32*
Net realisable value	*9*	*25*

A single recovery rate for fixed factory overheads is used throughout the firm even though some fixed production overheads could be attributed to single products or Departments. The overhead is recovered per productive labour hour and initial estimates of next year's activity, which excludes the current contract, show fixed production overheads of ₹6,00,000 and productive labour hours of 3,00,000. Acceptance of the contract would increase fixed production overheads by ₹2,28,000. Variable production overheads are accurately estimated at ₹3/- per productive hour.

Acceptance of the contract would be expected to encroach on the sale and production of another product, Y which is also made by Aylett Ltd. It is estimated that sales of Y, would then decreases by 5,000 units in the next year only. However this forecast reduction in sales of Y would enable attributable fixed factory overheads of ₹58,000 to be avoided. Information on Y is as follows:

	Per unit
Sales price	*₹ 70*
Labour - Grade 2	*4 hours*
Materials -relevant variable costs	*₹ 12*

All activity undertaken by Aylett is job costed using full, absorption, costing in order to derive a profit figure for each contract if the contract for X is accepted it will be treated as a separate job for routine costing purposes. The decision to accept or reject the contract will be taken in sufficient time to enable its estimated, effects to be incorporated in the next year's budgets and also in the calculations carried out to derive the overhead recovery rate to be used in the forthcoming year.

Required:

(a) Advise Aylett on the desirability of the contract

(b) Show how the contract, if accepted, will be reported on by the routine job costing system used by Aylett.

(c) Briefly explain the reasons for any differences between the figures used in (a) and (b) above.

Solution

(a) Statement of profit on the basis of historical costing system

		(₹)	(₹)
Sales: 20,000 kg @ ₹100 : (A)			20,00,000
Less: Costs			
Material A:	20,000 kg × 2 units @ ₹8	3,20,000	
Material B:	20,000 kg × 1 litre @ ₹30	6,00,000	
Labour grade 1 :	20,000 kg × 2 hrs. @ ₹4	1,60,000	
Labour grade 2 :	20,000 kg × 6 hrs. @ ₹2	2,40,000	
Variable overheads:	20,000 kg × 8 hrs. @ ₹3	4,80,000	
Fixed overheads:	20,000 kg × 8 hrs. @ ₹1.75	2,80,000	
Total cost : (B)			20,80,000
Loss : {(A) – (B)}			80,000

Working note:

The fixed overhead rate/hr is computed as below:	(₹)
Total fixed overheads for 3,00,000 hrs	6,00,000
Add: Additional fixed overheads for 1,60,000 hrs.	2,28,000
Less: Fixed costs saved due to the reduction of production of Y for 20,000 hrs. (5,000 units × 4 hours)	58,000
Total fixed overheads	7,70,000

Total number of hours:

3,00,000 hrs. + 1,60,000 hrs. – 20,000 hrs. = 4,40,000 hrs.

Therefore: Fixed overhead rate/hr

$$\frac{\text{(Total fixed overheads)}}{\text{(Total number of hours)}} = \frac{₹7,70,000}{440,000 \text{ hrs}}$$

Conclusion: On the basis of Historical Costing approach (similar to cost sheet workings) the offer should be rejected as it incurs loss

(b) Statement of profit under relevant costing system

	(₹)	(₹)
Sales: 20,000 kg @ ₹100 : (A)		20,00,000
Less : Costs		
Material A: 20,000 kg × 2 units × ₹10	4,00,000	
(Refer to working note: A)		
Material B: 20,000 kg × 1 litre × ₹25	5,00,000	
(Refer to working note: B)		
Labour grade 1	Sunk cost	

(Refer to working note: C)		
Labour grade 2 : 20,000 kg × 6 hrs × ₹2	2,40,000	
(Refer to working note: D)		
Variable overheads: 20,000 kg × 8 hrs. × ₹3	4,80,000	
(Refer to working note: E)		
Fixed costs	2,28,000	
(Refer to working note: F)		
Loss of profit due to Y (opportunity cost)	1,32,000	
(Refer to working note: (G)		
Total costs : (B)		19,80,000
Profit : {(A) – (B)}		20,000

Working notes :

A: Material A & B required for this contract are already available in stores. Therefore, the original purchase price is considered as sunk cost. Since A is a regularly used item the present stock is not meant for this job. Therefore the requirement will have to be purchased therefore, the replacement cost is considered as relevant.

B: Material B will be sold if not used for this contract. Therefore, the resale value should be considered. (Loss of cash inflow is treated as cash outflow).

C: Grade 1 labour is at present under utilised. Acceptance of the contract will only reduce the idle time, the wages will be paid on time basis. Therefore the cost should be treated as sunk cost and is not relevant for decision making.

D: Grade 2 labour is considered as a variable cost, i.e. out of pocket cost and hence relevant for decision making.

E: Variable overheads are out of pocket costs are therefore relevant for decision making.

F. Fixed cost already incurred is a sunk cost whereas fixed costs to be incurred is relevant. Hence, only the additional fixed costs are considered.

G: Because of accepting of this contract, production and sale of Y to the extent of 5,000 units (which is included in the original budget) is affected resulting in a loss of cash flow of ₹1,32,000, which should be treated as an opportunity cost.

Computation of opportunity cost:

	(₹)	(₹)
Sales: 5,000 units @ ₹70		3,50,000
Material: 5,000 units × ₹12	60,000	
Labour: 5,000 units × 4 hours × ₹2	40,000	
Variable overhead: 5,000 units × 4 hours × ₹3	60,000	
Fixed overheads	58,000	2,18,000
Opportunity costs		1,32,000

(c) Reasons: Under the relevant cost approach only the out of pocket costs have been considered. The adjustment A to G can be classified into either actual cash outflows or loss of cash inflows which are also treated as cash outflows.

Conclusion: We should accept the contract as it will improve the overall profitability.

Illustration 11

Ranka Builders has been offered a contract by Excel Ltd. to build for it five special Guest Houses for use by top management. Each Guest House will be an independent one. The contract will be for a period of one year and the offer price is ₹1 crore. In addition, Excel Ltd. will also provide 2 grounds of land, free of cost for the purpose of construction. The Chief Accountant of Ranka Builders has prepared an estimate on the basis of which he has advised that the contract should not be accepted at the price offered. His estimate was as follows:

		(₹) ***in Lacs***
Land (3 Grounds at ₹20 lakhs each)		*60*
Drawings and Design		*7*
Registration		*10*
Materials :	*Cement and Sand*	*6*
	Bricks and Tiles	*4*
	Steel	*10*
	Others (including interior decoration)	*10*
Labour	*– Skilled*	*12*
	– Unskilled	*8*
	– Supervisor's Salary	*5*
Overheads	*General*	*12*
	Depreciation	*6*
	Total Cost	*150*

The Accountant also provides the following information:

Land: The total requirement of land is 3 grounds costing ₹20 lakhs per ground. Excel Ltd. will provide 2 grounds free of cost.

Drawing and Design: These have already been prepared and 50% of the cost has already been incurred.

Materials:

(i) Cement and sand are already in stock and are in regular use. If used for this contract, they have to be replaced at a cost of ₹8 lacs.

(ii) Bricks and tiles represent purchases made several months before for a different contract. They could be sold readily for a net ₹5 lakhs after meeting all further expenses.

(iii) Others: Materials worth ₹2 lakhs relating to interior decoration are in stock for which no alternative use is expected in the near future. However they can be sold for ₹1 lac.

Labour:

(i) Skilled workers will be transferred to this project from another project. The Project Manager claimed that if the men were returned to him, he could have earned the company an additional ₹ 2 lakhs in terms of profits.

(ii) The supervisor undertakes various tasks in the sites and his pay and continuity of employment will not be affected by the new contract. If the contract is taken, he will devote half of his time.

Overheads:

(i) The equipment that would be used on the contract was bought one year before for ₹ 30 lakhs and is expected to last for five years. It can also be used on other contracts and the current replacement price will be ₹ 32 lakhs and in a year's time it will be ₹ 25 lacs.

(ii) The general overheads includes both specific and absorbed overheads. If the contract is not undertaken, ₹ 4 lakhs of the same can be avoided.

Ranka Builders has also on hand another project, which would not be executed if the contract from Excel Ltd. were to be accepted. The estimated profit on that project is ₹ 10 lacs.

In the light of information given above, you are required to indicate with reasons whether the contract from Excel Ltd. should be accepted or not.

Solution

M/s. Ranka Builder's
Statement of relevant costs on theAcceptance of contract from Excel Ltd.

(Figures in lakhs of ₹)

Sl. No.	Particulars	Basis for the cost to be relevant	Relevant cost if contract is accepted (₹)	Irrelevant cost if the contract isaccepted (₹)
1.	Land cost	*(working note 1)*	20	
2.	Drawings and design		–	7 (Sunk cost)
3.	Registration	Incremental	10	–
4.	*Materials:*			
	Cement and sand	Replacement	8	
	Bricks and tiles	Opportunity	5	
	Steel	Incremental	10	
	Others	*(working note 2)*	9	
5.	*Labour:*			
	Skilled	Opportunity	2	

	Unskilled	Incremental	8	
	Supervisor's salary			5 (Sunk cost)
6.	*Overheads:*			
	General	Relevant (avoidable)	4	
	Depreciation			6 (Sunk cost)
	Replacement cost of machine	*(working note 3)*	7	
7.	Estimated profit foregone on other project	Opportunity foregone	10	
	Total cost		93	

Decision: Since the offer price of contract is ₹ 1 crore and its total relevant cost is ₹ 93 lacs; these figures clearly show that the offer should be accepted.

Working notes:

		(₹in lakhs)
1.	Total cost of 3 grounds of land	60
	Cost of 2 grounds of land will beborne by Excel Ltd.	40
	Cost of 1 ground of land will be borne by M/s. Ranka Builders	20

2. Others material cost of ₹ 10 lakhs includes material worth ₹ 2 lakhs relating to interior decoration, which is a sunk cost, this material can be sold for ₹ 1 lakh (which is a relevant opportunity cost) and ₹ 8 lakhs material is an incremental cost. Hence total relevant cost of others material is ₹ 9 lakhs (₹ 8 lakhs incremental + ₹ 1 lakh opportunity cost).

3. Since the equipment can also be used on this contract its current replacement price is ₹ 32 lakhs and after one year its cost will be ₹ 25 lakhs. Therefore the relevant opportunity cost of machine is : ₹ 7 lakhs (₹ 32 lakhs –₹ 25 lakhs).

Illustration12

Intervero Ltd., a small engineering company, operates a job order costing system. It has been invited to tender for a comparatively large job, which is outside the range of its normal activities, and, since there is surplus capacity, the management are keen to quote as low a price as possible. It is decided that the opportunity should be treated in isolation without any regard to the possibility of its leading to further work of a similar nature (although such a possibility does exist). A low price will not have repercussions on Intervero's regular work.

The estimating department has spent 100 hours on work in connection with the quotation and they have incurred travelling expense of ₹550 in connection with a visit to the prospective customers' factory. The following cost estimates has been prepared on the basis of their study.

Cost Estimate *Inquiry 205 H 81*

	(₹)	(₹)
Direct material and components:		
2,000 units of A at ₹ 25 per unit	*50,000*	
200 units of B at ₹ 10 per unit	*2,000*	
Other material and components to be bought is (specified)	*12,500*	
		64,500
Direct labour:		
700 hrs. of skilled labour at ₹ 3.50 per hour		*2,450*
1,500 hrs. of unskilled labour at ₹ 2 per hour		*3,000*
Overhead:		
Department P- 200 hrs. at ₹ 25 per hour		*5,000*
Department Q- 400 hrs. at ₹ 20 per hour		*8,000*
Estimating Department:		
100 hours at ₹ 5 per hour		*500*
Travelling expenses		*550*
Planning Department:		
300 hours at ₹ 5 per hour		*1,500*
		85,500

The following information has been brought together:

Material A: This is a regular stock item. The stock holding is more than sufficient for this job. The material currently held has an average cost of ₹ 25 per unit but the current replacement cost is ₹ 20 per unit.

Material B: A stock of 4,000 units of B is currently held in the stores. This material is slow moving and the stock is the residue of a batch bought seven years ago at a cost of ₹ 10 per unit. B currently costs ₹ 24 per unit but the resale value is only ₹ 18 per unit. A foreman has pointed out that B could be used as a substitute for another type of regularly used raw material which costs ₹ 20 per unit.

Direct Labour: The work force is paid on a time basis. The company has adopted no redundancy policies which mean that skilled workers are frequently moved to jobs which do not make proper use of their skills. The wages included in the cost estimate are for the mix of labour which the job ideally requires. It seems likely, if the job is obtained, that most of the 2,200 hours of direct labour will be performed by skilled staff receiving ₹ 3.50 per hour.

Overhead: Department P : It is a department of Intervero Ltd., that is working at full capacity. The department is treated as a profit centre and it uses a transfer price of ₹ 25 per hour for charging out its processing time to other departments. This charge is calculated as follows:

	(₹)
Estimated variable cost per machine hour	*10*
Fixed departmental overhead	*8*
Departmental profit	*7*
	25

Department P's facilities are frequently hired out to other firms and a charge of ₹ 30 per hour is made. There is a steady demand from outside customers for the use of these facilities.

Overhead :Department Q : Department Q uses a transfer price of ₹ 20 for charging out machine processing time to other Departments. This charge is calculated as follows :

	(₹)
Estimated variable cost per machine hour	*8*
Fixed departmental overhead	*9*
Departmental profit	*3*
	20

Estimating department: This department charges out its time to specific jobs using a rate of ₹ 5/- per hour. The average wage rate within the department is ₹ 2.50 per hour but the higher rate is justified as being necessary to cover departmental overheads and the work done on unsuccessful quotations.

Planning department: This department also uses a charging out rate which is intended to cover all departmental costs.

The offer received for the above contract is ₹ 70,000.

You are required to restate the cost estimate by using an opportunity cost approach. Make any assumptions that you deem to be necessary and briefly justify each of the figures that you give.

Solution

Statement of cost of Intervero Ltd.

	(₹)
Direct material:	
A: 2,000 units @ ₹20	40,000
(Refer to working note 1)	
B: 200 units @ ₹20	4,000
(Refer to working note 2)	
Other Material and components	12,500
Direct labour (skilled)	sunk
Direct labour (Unskilled)	sunk
(Refer to working note 3)	

Overheads:	
Dept. P: 200 hrs. @ ₹30	6,000
(Refer to working note 4)	
Dept. Q: 400 hrs. @ ₹8	3,200
Estimating department	sunk
Travelling expenses	sunk
Planning department	sunk
Total cost	65,700

Working notes:

1. Material A is a regularly used item. Therefore, the present stock is not meant for this contract. Hence, the replacement cost is considered.
2. Material B will be sold if not used for this contract. Therefore, the resale value (₹20 or ₹ 18 whichever is more should be considered).
3. The entire job will be performed by skilled workers (unskilled jobs also). This implies skilled workers are idle at present and since the company follows no redundancy policy and their wages will be paid on time basis whether the contract is accepted or not. Hence, they are treated as a sunk cost.
4. Department P is a profit centre and is working at full capacity at present and has a steady demand from outside for its facilities at ₹30 per hour. Therefore, we should consider this ₹ 30/hr. as relevant (out of pocket cost ₹10 + opportunity cost of ₹20).
5. Department Q: Only the out of pocket costs being the variable overheads of ₹8 hr. should be considered.
6. All other costs mentioned are sunk costs.

   Conclusion: We should accept this offer.

2.3.8 Decision making on different option of services

Illustration13

A Ltd. produces and markets a range of consumer durable appliances. It ensures after-sales service through X Ltd. The big appliances are serviced at customer's residence while small appliances are serviced at workshop of X Ltd.

The material supplied to X Ltd. is charged at cost at 10%. X Ltd. charges customers at 25% over the above price. For labour, the company receives 10% of the rate fixed for work done under the after-sales service agreement and 15% of the rate fixed in case of jobs not covered under the agreement from X Ltd. 60% by value of the total work undertaken by X Ltd. was for big appliances and rest accounted for small appliances during the previous year.

The company decides to carry out all or some of the work itself and has chosen one area in the first instance. During the previous year the company earned a profit of ₹2,16,000 as detailed below from X Ltd. for the area chosen :

	Material (₹)	**Labour (₹)**
Under after-sales service agreement	*60,000*	*1,00,000*
For jobs not covered under the agreement	*20,000*	*36,000*

The company forecasts same volume of work in that area for the ensuing period. The following three options are under consideration of the management:

(1) To set up a local service centre to provide service for small appliances only. The existing system is to continue for big appliances.

(2) To set up a local service centre to provide service for big appliances only. The existing system is to continue for small appliances.

(3) To set up a local service centre to provide service to all appliances. The existing system then stands withdrawn.

The relevant costs for carrying out jobs under the above options are as under:

(₹'000)

	Option -1	**Option-2**	**Option-3**
Heat, rent, light etc.	*125*	*50*	*150*
Management costs	*108*	*83*	*150*
Service staff costs	*230*	*440*	*750*
Transport costs	*25*	*220*	*230*

You are required to find out the most profitable option.

Solution

Statement showing value of total work undertaken by X Ltd. at customer's price

	(₹ '000)
Material cost (for appliances covered under agreement)	825
[Refer to working note 1(i)]	
Material cost (for appliances not covered under agreement)	275
[(Refer to working note 2(i)]	
Labour cost (for appliances covered under agreement)	1,000
[Refer to working note 1(ii)]	
Labour cost (for appliances not covered under agreement)	240
[Refer to working note 2(ii)]	
Total receipts	2,340

Break up of receipts:

Big appliances	60%	1,404
Small appliances	40%	936

Profitability Statement

(₹'000)

	Option 1	Option 2	Option 3
Income:			
Big appliances	129.6 (60% × ₹216)	1,404	1,404
Small appliances	936	86.4 (40% × ₹216)	936
Total receipts : (A)	1,065.6	1,490.4	2,340
Costs :			
Material	320 40% × (825 + 275) 137.5%	480 60% × (825 + 275) 137.5%	800 (825 + 275) 137.5%
Heat, rent, light etc.,	125	50	150
Management costs	108	83	150
Service staff costs	230	440	750
Transport costs	25	220	230
Total costs : (B)	808	1,273	2,080
Profit : [(A) – ((B)]	257.6	217.4	260

Recommendation:

Option 3 is most profitable one.

Working Notes :

1. *Material and labour cost (for appliances under after sales agreement) :*

(₹)

(i) Cost of material per unit charged to customer's by X Ltd.

(₹100 + 10% (₹100+ 25% × ₹110) 137.50

Cost of material charged to customer's by X Ltd.

$\left(\frac{₹\ 60,000}{10} \times 137.50\right)$ 8,25,000

(ii) Cost of labour charged to customers by X Ltd.

$\left(\frac{₹\ 1,00,000}{10} \times 100\right)$ 10,00,000

2. *Material and Labour cost (for appliances not covered under sales agreement) :*

(₹)

(i) Cost of material charged to customers by X Ltd.

$\left(\frac{₹\ 20,000}{10} \times 137.50\right)$ 2,75,000

(ii) Cost of labourer charged to customers by X Ltd.

$\left(\frac{₹\ 36,000}{15}\right) \times 100$ 2,40,000

2.3.9 Decision on use of alternative resources

Illustration14

B Ltd. is a company that has in stock, materials of type XY that cost ₹ 75,000, but that are now obsolete and have a scrap value of only ₹ 21,000. Other than selling the material for scrap, there are only two alternative uses for them.

Alternative 1 – Converting the obsolete materials into a specialised product, which would require the following additional work and materials –

Material A	*600 units*
Material B	*1,000 units*
Direct Labour	
5,000 hours unskilled	
5,000 hours semi-skilled	
5,000 hours highly skilled	
Extra selling and delivery expenses	*₹27,000*
Extra advertising	*₹18,000*

The conversion would produce 900 units of saleable product and these could be sold for ₹300 per unit.

Material A is already in stock and is widely used within the firm. Although present stocks, together with orders already planned, will be sufficient to facilitate normal activity and extra material used by adopting this alternative will necessitate such materials being replaced immediately. Material B is also in stock, but it is unlikely that any additional supplies can be obtained for some considerable time because of an industrial dispute. At the present time material B is normally used in the production of product Z, which sells at ₹ 390 per unit and incurs total variable (cost excluding Material B) of ₹ 210 per unit. Each unit of product Z uses four units of Material B. The details of Materials A and B are as follows:

	Material A (₹)	*Material B* (₹)
Acquisition cost at the time of purchase	*100 per unit*	*₹10 per unit*
Net realizable value	*85 per unit*	*₹18 per unit*
Replacement cost	*90 per unit*	*—*

Alternative 2 – Adopting the obsolete materials for use as a substitute for a sub-assembly that is regularly used within the firm. Details of the extra work and materials required are as follows:

Material C	*1,000 units*
Direct Labour	
4,000 hours unskilled	
1,000 hours semi-skilled	
4,000 hours highly skilled	

1,200 units of the sub-assembly are regularly used per quarter at a cost of ₹900 per unit. The adaptation of material XY would reduce the quantity of the sub-assembly purchased from outside the firm to 900 units for the next quarter only. However, since the volume purchased would be reduced, some discount would be lost and the price of those purchased from outside would increase to ₹1,050 per unit for that quarter.

Material C is not available externally though 1,000 units required would be available from stocks, it would be produced as extra production. The standard cost per unit of Material C would be as follows :

	(₹)
Direct labour 6 hours unskilled labour	*18*
Raw materials	*13*
Variable overhead : 6 hours at ₹1	*6*
Fixed overhead : 6 hours at ₹3	*18*
	55

The wage rates and overhead recovery rates for B Ltd are :

Variable overhead	*₹1 per direct labour hour*
Fixed overhead	*₹3 per direct labour hour*
Unskilled labour	*₹3 per direct labour hour*
Semi-skilled labour	*₹4 per direct labour hour*
Highly skilled labour	*₹5 per direct labour hour*

The unskilled labour is employed on a casual basis and sufficient labour can be acquired to exactly meet the production requirements. Semi-skilled labour is part of the permanent labour force, but the company has temporary excess supply of this type of labour at the present time.

Highly skilled labour is in short supply and cannot be increased significantly in the short-term, this labour is presently engaged in meeting the demand for product L, which requires 4 hours of highly skilled labour. The contribution from the sale of one unit of product L is ₹ 24.

Given the above information, you are required to present cost information advising whether the stocks of Material XY should be sold, converted into a specialised product (Alternative 1) or adopted for use as a substitute for a sub-assembly (Alternative 2).

Solution

Alternative 1 – (Conversion versus immediate sale)

	(₹)	(₹)	(₹)
Sales revenue 900 units at ₹ 300 per unit			2,70,000
(*Refer to working note 1*)			
Less : Relevant costs			
Material XY opportunity cost		21,000	
(*Refer to working note 2*)			
Material A – 600 units @ ₹ 90 per unit		54,000	
(*Refer to working note 3*)			
Material B – 1,000 units @ ₹ 45 per unit		45,000	
(*Refer to working note 4*)			
Direct Labour :			
Unskilled - 5,000 hours @ ₹ 3 per hour	15,000		
Semi-skilled	NIL		
Highly skilled - 5,000 hours @ ₹ 11	55,000	70,000	
(*Refer to working note 5*)			
Variable overheads 15,000 hours @ ₹ 1		15,000	
(*Refer to working note 6*)			
Extra selling and delivery expenses	27,000		
Extra advertising	18,000	45,000	
Fixed overheads		NIL	2,50,000
(To remain same, not relevant)			
Excess of relevant revenues			20,000

Alternative 2 – (Adaptation versus immediate sale)

Saving on purchase of sub-assembly :			
Normal spending - 1,200 units @ ₹ 900 per unit		10,80,000	
Less : Revised spending - 900 units @ ₹ 1,050 per unit		9,45,000	1,35,000
(*Refer to working note 7*)			
Less : Relevant costs			

Material XY opportunity cost		21,000	
(*Refer to working note 2*)			
Material C - 1,000 units @ ₹ 37		37,000	
(*Refer to working note 8*)			
Direct labour			
Unskilled – 4,000 hours @ ₹ 3 per hour	12,000		
Semi-skilled	NIL		
Highly skilled – 4,000 hours @ ₹ 11 per hour	44,000	56,000	
(*Refer to working note 5*)			
Variable overheads – 9,000 hours @ ₹ 1/- per hour		9,000	
(*Refer to working note 6*)			
Fixed overheads		NIL	1,23,000
(*To remain same not relevant*)			
Net relevant savings			12,000

Evaluation :

The evaluation of two alternatives clearly shows that Alternative 1, yields higher net revenue of ₹ 8,000 (₹ 20,000 – ₹ 12,000). Hence because of higher net revenue of Alternative 1, it is advisable to convert material XY into a specialised product.

Working notes :

1. There will be an additional sales revenue of ₹ 2,70,000 if Alternative 1 is chosen.
2. Acceptance of either Alternative 1 or 2 will mean a loss of revenue of ₹ 21,000 from the sale of the obsolete material XY and hence it is an opportunity cost for both of the alternatives. The original purchase cost of ₹ 75,000 is a sunk cost and thus not relevant.
3. Acceptance of Alternative 1 will mean that material A must be replaced at an additional cost of ₹ 54,000.
4. Acceptance of Alternative 1 will mean diversion of material B from the production of product Z. The excess of relevant revenues over relevant cost for product Z is ₹ 180 (₹ 390 – ₹ 210) and each unit of product Z uses four units of material B. The lost contribution (excluding the cost of material B which is incurred for both alternatives) will therefore be ₹ 45 for each unit of material B, that is used for converting the obsolete materials into a specialised product.
5. Unskilled labour can be matched exactly to the company's production requirements. Hence acceptance of either alternative 1 or 2 will cause the company to incur additional unskilled labour cost at ₹ 3 for each hour. It is assumed that the semi-skilled labour will be able to meet the extra requirements of either alternatives at no extra cost to the company. Hence, cost of semi–skilled labour will not be relevant. Skilled labour is in short supply and can only be obtained by reducing the production of product L, resulting

in a loss of contribution of ₹ 24 (given) or ₹ 6 per hour of skilled labour. Hence the relevant labour cost will be ₹ 6 (contribution lost per hour) + ₹ 5 (hourly rate of skilled labour) i.e. ₹ 11 per hour.

6. It is assumed that for each direct labour of input, variable overhead will increase by ₹ 1, hence for each alternative using additional direct labour hours, variable overheads will increase.
7. The cost of purchasing the sub-assembly will be reduced by ₹ 1,35,000 if the second alternative is chosen and so these savings are relevant to the decision.
8. The company will incur additional variable costs of ₹ 37 for each unit of material C that is manufactured, so the fixed overheads for material C viz. ₹ 18/- per unit is not a relevant cost.

2.4 Application of Incremental/Differential Cost Techniques in Managerial Decisions

It is a technique used for arriving at managerial decisions in which only cost and income differences between alternative courses of action are taken into consideration. This technique is applicable to situations where fixed costs alter. This technique emphasizes on comparing the incremental costs with incremental revenues for taking a managerial decision. So long as the incremental revenue is greater than incremental costs, the decision should be in favour of the proposal.

The areas in which the above techniques of cost analysis can be used for making managerial decisions are:

(i) Whether to process a product further or not.

(ii) Dropping or adding a product line.

(iii) Making the best use of the investment made.

(iv) Acceptance of an additional order from a special customer at lower than existing price.

(v) Opening of new sales territory and branch.

(vii) Make or Buy decisions.

(viii) Submitting tenders

(ix) Lease or buy decisions

(x) Equipment replacement decision.

2.4.1 Decision on further processing of a product

Whether to process a product further or not: Many companies manufacture products which can be sold or subjected to further processing. It is also possible that waste emanating from one operation of a factory can be sold as such or sold after further processing in the company's plant.

Examples of such companies are: meat processing, manufacture of copper or aluminium, etc. In such cases, the matter for consideration is whether the incremental revenue arising from the processing of the product further is sufficient to cover the incremental cost involved in such additional processing and still leave a contribution towards profit.

Illustration15

A company produces product 'A' which is at present being sold at ₹7 each; the monthly production is 25,000 units. The company can, as an alternative make product 'F', by using one unit of 'A', as raw material, in each unit of 'F'. Product 'F' can be sold at ₹10/- each. The company pays sales commission at 10 percent on sales value. Capacity to manufacture 'F' is available to the extent of 12,500 units per month without additional capital cost. Taking the following additional information advise whether or not the company should go in for the manufacture of product 'F'.

Product	***'A' (₹)***	***'F' (₹)***
Raw material cost	*1.50*	*0.50*
Labour& overheads	*2.10*	*1.40*

The figures given for 'F' represent expenses incurred in addition to those incurred for the manufacture of 'A'. The supervision charges for manufacturing 'F' will be ₹4,000 per month extra.

Solution

Particulars	**Product 'A' 12,500 units**		**Product 'F' 12,500 units**		**Incremental Revenue/ Cost (₹)**
	Per unit (₹)	**Total (₹)**	**Per unit (₹)**	**Total (₹)**	
Sales revenue	7.00	87,500	10.00	1,25,000	37,500
Less: Commission @ 10%	0.70	8,750	1.00	12,500	3,750
Net revenue : (i)	6.30	78,750	9.00	1,12,500	33,750
Raw materials	1.50	18,750	2.00	25,000	6,250
Labour& overheads	2.10	26,250	3.50	43,750	17,500
Additional fixed expenses			0.32	4,000	4,000
Total cost : (ii)	3.60	45,000	5.82	72,750	27,750
Profit : (i) – (ii)	2.70	33,750	3.18	39,750	6,000

The above table shows that by resorting to further processing of 12,500 units the company can earn an additional profit of ₹6,000 per month and hence the proposal is recommended.

Note: In the above problem it is likely that the company, instead of utilising its capacity to make product 'F' may go in for a further increase in production of product 'A' to the extent possible. In such circumstances, the incremental profit of the second alternative should be compared with the incremental profit as obtained above.

Illustration16

X Ltd. is in the Food Processing Industry. In one of its processes, three joint products are manufactured. Traditionally, the company has apportioned costs incurred up to the Joint Products' pre-separation point on the basis of weight of outputs of the product.

You have been recently appointed Cost Accountant, and have been investigating process cost and accounting procedure.

You are required to prepare statements for management to show:

(a) The profit or loss of each product as ascertained using weight basis of apportioning pre-separation joint costs.

(b) The optimal contribution which could be obtained from the manufacture of these products.

The following process data for December are given. Cost incurred up to separation point are ₹96,000 :

	Product A	*Product B*	*Product C*
Cost incurred after separation point (₹)	*20,000*	*12,000*	*8,000*
Selling price per Ton of completed product (₹)	*500*	*800*	*600*
Estimated, if sold at separation point (₹)	*250*	*700*	*450*
Output (Tons)	*100*	*60*	*80*

The cost of any unused capacity after the separation point should be ignored.

Solution

(a) Statement showing profit or loss of each product using weights as the basis of apportioning pre-separation joint costs

	Total (₹)	**Product A** (₹)	**Product B** (₹)	**Product C** (₹)
Cost upto the point of separation in the ratio of (100: 60: 80) or (5:3:4)	96,000	40,000	24,000	32,000
Cost after separation point	40,000	20,000	12,000	8,000
Total costs	1,36,000	60,000	36,000	40,000
Sales revenue	1,46,000	50,000	48,000	48,000
Profit/(Loss)	10,000	(10,000)	12,000	8,000

(b) To ascertain optimal contribution, first, the incremental profit or loss from each product, if products are further processed after separation point is to be computed. The statement below shows the incremental profit or loss after further processing:

	Product A	Product B	Product C
Output (Tons)	100	60	80
	(₹)	(₹)	(₹)
Incremental revenue			
from further processing	25,000	6,000	12,000
	(₹500–₹250)×100	(₹800–₹700)×60	(₹600–₹450)×80
Less: Incremental cost	20,000	12,000	8,000
Incremental profit (loss)	5,000	(6,000)	4,000

It can be seen from the above statement that there will be a loss of ₹6,000 if further processing of product B is done after the separation point. It is, therefore, recommended that Product B should be sold at separation point. The optimal contribution based on this recommendation will be as follows:

Statement showing optimal contribution

	Product A	Product B	Product C	Total
Output (Tons)	100	60	80	
	(₹)	(₹)	(₹)	(₹)
Sales revenue	50,000	42,000	48,000	1,40,000
	(100×₹500)	(60×₹700)	(80×₹600)	
Less: Post separation cost	20,000	—	8,000	28,000
Contribution	30,000	42,000	40,000	1,12,000

2.4.2 Dropping or adding a product line: Often firms manufacturing a number of products may find that one or more of its products are not worthwhile for producing from the profitability point of view. Since the objective of any business organisation is to maximise its profits, the firm can in such cases consider the economies of dropping such unprofitable products, and adding a more profitable product(s). In such cases, the firm may have two alternatives as under:

(a) To drop the unprofitableproduct and to leave the capacity unutilised.

(b) To drop the unprofitable product and to utilise the capacity for the manufacture of a more remunerative product.

If we consider alternative (a) above, what is more important is the amount of fixed expenses apportioned to the product which is going to be discontinued. If the capacity relating to the product in question is going to be left unutilised, the contribution which the product is making towards recovery of fixed expenses will not be forthcoming. Thus continuance of a product line which is adjudged as unprofitable, on the basis of absorption costing may make matters worse. The use of contribution approach will help the firm to take a sound decision on such occasions. As some of the fixed expenses can be reduced by dropping a product line, such fixed costs become relevant while making a decision about the discontinuance of product line.

If we take alternative (b) the fixed expenses apportioned to the product line discontinued will remain the same for being absorbed by the new product. The comparison is now between discontinuance of the product line and introduction of another remunerative product line. Since the fixed expenses are not going to be reduced in this alternative, the product which yields the highest contribution is preferred because it will maximise the overall profitability of the firm.

Illustration 17

A firm produces three products, the details of sales, contribution margin and the fixed costs apportioned to the products are as under:

	Products		
	A	***B***	***C***
Sales values (₹)	*10,00,000*	*8,00,000*	*2,00,000*
Contribution (₹)	*40%*	*30%*	*25%*
Fixed expenses (₹)	*3,40,000*	*1,80,000*	*90,000*

Fixed expenses attributable to product 'C' are ₹40,000.

Product C incurs a loss and hence the management wishes to consider two alternatives:

(i) Discontinue product 'C' to save the loss.

(ii) Discontinue product 'C' and utilise the capacity to manufacture product 'D'. In this case the sales value of product D is ₹2,00,000 and the contribution is 35%.

Which alternative should be adopted?

Solution

(i) Let us tabulate the results of the firm as under:

Profitability of the firm:

	Products			**Total**
	A	**B**	**C**	
Sales value (₹)	10,00,000	8,00,000	2,00,000	20,00,000
Contribution (%)	40%	30%	25%	34.5%*
Contribution (₹)	4,00,000	2,40,000	50,000	6,90,000
Fixed costs (₹)	3,40,000	1,80,000	90,000	6,10,000
Net profit (loss) (₹)	60,000	60,000	(40,000)	80,000

$$\frac{₹\ 6,90,000}{₹\ 20,00,000} \times 100 = 34.5\%\ \text{Average}$$

Under alternative (i), a comparison between the total profitability and the profitability without product 'C' is relevant, shown as under:

Particulars		Products A,B &C	Products A&B only	Difference
Sales value	(₹)	20,00,000	18,00,000	2,00,000
Contribution	(₹)	6,90,000	6,40,000	50,000
Fixed expenses	(₹)	6,10,000	5,70,000	40,000
Net profit	(₹)	80,000	70,000	10,000

It may be observed from the above table that discontinuance of product 'C' reduced the total profit from ₹80,000 to ₹70,000. In other words, product 'C' was yielding a contribution sufficient to cover more than the incremental fixed costs of ₹40,000 attributable to its manufacture.

(ii) Under alternative (ii), comparison between the profitability of product 'C' and product 'D' is relevant as under:

Particulars		Product'C'	Product'D'	Difference
Sales value	(₹)	2,00,000	2,00,000	_
Contribution	(%)	25%	35%	
Contribution	(₹)	50,000	70,000	20,000

Since product D yields a higher contribution, the total profitability will increase by ₹20,000. The fixed expenses remain constant. This is illustrated below:

Product		A	B	D	Total
Sales value	(₹)	10,00,000	8,00,000	2,00,000	20,00,000
	(%)	40%	30%	35%	35.5%*
Contribution	(₹)	4,00,000	2,40,000	70,000	7,10,000
Fixed expenses	(₹)	3,40,000	1,80,000	90,000	6,10,000
Net profit (loss)	(₹)	60,000	60,000	(20,000)	1,00,000

$$\frac{{}^{*}₹\ 7{,}10{,}000}{₹\ 20{,}00{,}000} \times 100 = 35.5\%\ \text{Average}$$

2.4.3 Optimization of profit under a limiting factor condition

Illustration 18

Something More Ltd. is considering adding products to its product line. After a lot of deliberations between the sales and production personnel, it is decided that Products P, Q and R would be the most desirable additions to the company's product range on account of the technical competency, marketing potential and production flexibility as regards these products. In fact P, Q and R can all be made on the same kind of plant as that already in use and therefore as regards production, all products can be readily interchanged. However, it is considered necessary to build further plant facilities to cater for this additional production.

In this connection the following data are relevant:

Products	*P* (₹)	*Q* (₹)	*R* (₹)
Direct materials per unit	*100*	*120*	*90*
Direct labour per unit	*50*	*70*	*90*
Variable overheads per unit	*50*	*130*	*100*
Selling price per unit	*350*	*420*	*370*
Demand in units per cost period	*200*	*125*	*750*
(on the basis of the above selling price)			
Machine hours required per unit of production	*15*	*5*	*3*

It is felt that initially extra plant facilities can be built to operate at the following five different levels of activity viz. 1,800, 2,300, 2,800, 3,300 and 3,800 machines hours per cost period. The fixed overhead costs for a cost period relevant to these five different levels of activity are estimated at ₹15,000, ₹20,000, ₹26,000, ₹33,000 and ₹39,000 respectively.

You are required to advise, with supporting figures, the product or products to be manufactured and in what quantities at each of the five contemplated levels of activity that would seem most desirable to be pursued for maximisation of profits.

Solution

Something More Ltd.

Statement showing the production of R, Q and P in order of priority determined on the basis of contribution per machine hour, to maximise the profit at various levels of activity

Product Levels of Activity (Machine Hours)	P (Max. demand = 200 units Machine hours for max. demand = 3,000 hours)		Q (Max. demand = 125 units Machine hours For max. demand = 625 hours)		R (Max. demand = 750 units Machine hours For max demand = 2,250 hours)		Total Contri-bution	Fixed Cost	Net Profit
	Units	Contribution	Units	Contribution	Units	Contribution			
		(₹)		(₹)		(₹)	(₹)	(₹)	(₹)
1,800	–	–	–	–	600	54,000	54,000	15,000	39,000
2,300	–	–	10	1,000	750	67,500	68,500	20,000	48,500
2,800	–	–	110	11,000	750	67,500	78,500	26,000	**52,500**
3,300	28*	4,200	125	12,500	750	67,500	84,200	33,000	51,200
3,800	61*	9,150	125	12,500	750	67,500	89,150	39,000	50,150

* 28 and 61 units of product P consumes 420 and 915 machine hours respectively whereas actually 425 and 925 machine hours are available. As incomplete units can be sold only in the next period after they are completed, for these computations only complete units have been considered.

The most desirable level of activity to be pursued is at 2,800 machine hours to produce and sell 750 units of R and 110 units of Q. At this activity level profit is maximum, i.e. ₹52,500.

Working note

Computation of contribution per unit of product and per machine hour

	Products		
	P ***(₹)***	***Q*** ***(₹)***	***R*** ***(₹)***
Selling price: (A)	350	420	370
Less : Variable costs			
Direct materials	100	120	90
Direct labour	50	70	90
Variable overheads	50	130	100
Total variable costs: (B)	200	320	280
Contribution per unit: (A–B)	150	100	90
Machine hours required per unit	15	5	3
Contribution per machine hour	10	20	30

For maximizing profits, the products giving the maximum contribution per machine hour should be concentrated upon as machine hours available are the foremost constraint.

Hence the order of priority for producing the product should be R, Q, and P.

2.4.4 Optimising Investment Plan: The investment decisions, also termed as capital budgeting decisions involve current outlays in return for a stream of benefits in future year. When multiple alternatives of investment opportunities having similar risks are available, we can use incremental cost and revenue approach to find out the optimum investment plan. In this case we increase the total investment by increment till such time the incremental rate of return falls below the cut off rate given. The cut off rate is defined as the minimum rate of return expected from the investment.

Illustration 19

A company has ₹2,00,000 to invest and it can earn 14% by investing the sum in company deposits. It also has got an opportunity to invest this sum in five projects as under:

	Projects				
	A ***(₹)***	***B*** ***(₹)***	***C*** ***(₹)***	***D*** ***(₹)***	***E*** ***(₹)***
Total investment	*20,000*	*60,000*	*1,00,000*	*1,60,000*	*2,00,000*
Annual net profit	*1,000*	*8,400*	*15,600*	*24,240*	*28,640*
Rate of return	*5%*	*14%*	*15.6%*	*15.15%*	*14.32%*

Find the optimum investment plan.

Solution

The optimum investment plan by using the incremental approach is as under:

	Projects			
	B (₹)	C (₹)	D (₹)	E (₹)
Incremental investment	40,000	40,000	60,000	40,000
Incremental net profit	7,400	7,200	8,640	4,400
Rate of return on incremental investment	18.5%	18%	14.4%	11%

Looking at the figures above, we find that project D is the most profitable one because if we take project E, the rate of return on the incremental investment will fall below the cut off rate of 14% and hence not acceptable. The following table will prove the facts. It may be observed here that the total income is the highest figure if the funds are invested in project D and the balance available in other company deposits at 14%.

	Project				
	A	B	C	D	E
	(₹)	(₹)	(₹)	(₹)	(₹)
Total funds available	2,00,000	2,00,000	2,00,000	2,00,000	2,00,000
Funds invested in project	20,000	60,000	1,00,000	1,60,000	2,00,000
Balance invested in deposits	1,80,000	1,40,000	1,00,000	40,000	—
Income from project	1,000	8,400	15,600	24,240	28,640
Income from deposits @ 14%	25,200	19,600	14,000	5,600	—
Total income	26,200	28,000	29,600	29,840	28,640

2.4.5 Decision making using Cash flow technique: This technique is useful in making decision regarding investment alternatives.

In this technique of decision making only differential cash flows are considered. Cash flow refers to the actual movement of cash in and out of an organisation. When cash is received it is called 'cash inflows' or positive cash flow, and when cash is paid out, it is called 'cash outflows' or negative cash flow. The difference between the two flows is termed as the net cash flow'. This technique of decision making can be applied in those circumstances in which the prime differences can be measured in terms of the future cash flows of the various alternatives under consideration. An alternative which gives most favourable net cash flow is selected. The application of the cash flow technique involves the following steps:

(i) Compute the future cash inflows and the future cash outflows of each alternative under consideration.

(ii) Ascertain the overall net cash flow.

(iii) Select the alternative that has the most favourable net cash flow i.e. the highest net cash flow.

Illustration 20

A firm needs component in an assembly operation. If it wants to do the manufacturing itself, it would need to buy a machine for ₹ 4 lakhs which will last for 4 years with no salvage value. Manufacturing costs in each of the 4 years would be ₹ 6 lakhs, ₹ 7 lakhs, ₹ 8 lakhs, and ₹ 10 lakhs respectively. If the firm had to buy the components from a supplier, the cost would be ₹ 9 lakhs, ₹ 10 lakhs, ₹ 11 lakhs and ₹ 14 lakhs respectively in each of the four years. However, the machine would occupy floor space which would have been used for another machine. This latter machine would be hired at no cost to manufacture an item, the sale of which would produce net cash flows in each of the four years of ₹ 2 lakhs. It is impossible to find room for both the machines and there are no other external effects. The cost of capital is 10% and the present value factor for each of the four years is 0.909, 0.826, 0.751 and 0.683 respectively.

Should the firm make the components or buy from outside?

Solution

Evaluation of Make or Buy proposal
(All figures are in lakhs of rupees)

Year	P.V. factors at 10%	When the component is manufactured		When the component is bought from an outside supplier	
		Cash outflow*	Present value	Cash outflow (Buying cost)	Present value
		(₹)	(₹)	(₹)	(₹)
(a)	(b)	(c)	(d) = (b) × (c)	(e)	(f) = (b) × (e)
0	1.000	4	4.000	—	—
1	0.909	8	7.272	9	8.181
2	0.826	9	7.434	10	8.260
3	0.751	10	7.510	11	8.261
4	0.683	12	8.196	14	9.562
Total			34.412		34.264

(Capital cost +manufacturing cost+ opportunity cost)

$$\begin{array}{l}\text{Saving in cash outflow} \\ \textit{(when bought from outside)}\end{array} = \left[\begin{array}{l}\text{Total present value of} \\ \text{cash outflow, when the} \\ \text{component is manufac-} \\ \text{tured internally}\end{array}\right] - \left[\begin{array}{l}\text{Total present value of} \\ \text{cash outflow, when the} \\ \text{component is bought} \\ \text{from outside}\end{array}\right]$$

= ₹ 34.412 – ₹ 34.264

= ₹ 0.148 (lakhs)

Conclusion : Since there is a saving of ₹ 0.148 (lakhs) in buying the component from outside, therefore, we should stick to this decision.

Note: The loss of ₹ 2 lakhs cash inflow for each of the four years due to the inability of the firm to operate another machine if it manufactures the component has been treated as an opportunity cost.

2.5 Shut Down & Divestment Decision

Very often it becomes necessary for a firm to temporarily close down the factory due to trade recession with a view to reopening it in the future. In such cases, the decision should be based on the marginal cost analysis. If the products are making a contribution towards fixed expenses or in other words if selling price is above the marginal cost, it is preferable to continue because the losses are minimized. By suspending the manufacture, certain fixed expenses can be avoided and certain extra fixed expenses may be incurred depending up on the nature of the industry, say, for example, extra cost incurred in protecting the machinery. So the decision is based on as to whether the contribution is more than the difference between the fixed expenses incurred in normal operation and the fixed expenses incurred when the plant is shut down. In other words, the shut down point is calculated by using the formula:

Shut down point = (Total fixed cost – Shut down costs) ÷ Contribution per unit

In case of decision rendering closure or shut down we consider the following points:

1. Current profit situation has to be maintained, so by analyzing the proposal of shut down or outsourcing if the current income is reduced then shut down will not be allowed unless the product or factory has reached at the end of its life cycle.
2. Avoidance of short term loss may cause the loss of brand value in the market as well as loss of efficient employees. So in that case the business may be continued for better profit after the recession period.
3. In case of outsourcing proposal we can also apply the differential cost concept i.e. saving in cost must be greater than or equal to out-sourcing fees payment. Here saving in cost i.e. cash inflow is computed from the concept of relevant cost i.e. by closing down we are saving variable cost of production, and discretionary fixed cost or shut down cost. This cash inflow further may be classified into two parts:
   1. Saving in variable cost per time period i.e. Cash Inflow (CIF) per annum.
   2. One time cash inflow following shut down i.e. sale of machine, sale of current stock of material etc.

∴ Total CIF or saving following shut down = one time CIF + CIF p.a. × life span of the proposal of outsourcing.

If this total is greater than total outsourcing fees then shutdown will take place other production will be continue.

Illustration- 21

Fixed expenses at 50% activity	*₹15,000*
Fixed expenses when the factory is shut down	*₹10,000*
Additional expenses in closing down	*₹1,000*

Production at 50% activity = 5,000 units

Contribution per unit ₹ 1

Solution

		(₹)
A.	If the plant is shut down the sunk costs or fixed expenses	11,000
B.	If it is working at 50% activity the fixed expenses	15,000
C.	Additional fixed expenses: [(B-A)]	4,000
D.	Contribution (5,000 units ₹ 1 p.u.)	5,000

By working at 50% activity the firm is able to recover the additional fixed expenses of ₹ 4,000 and earn an extra contribution of ₹1,000 towards shut down expenses. Hence it is advisable to continue production in the factory instead of closing it down.

Illustration 22

Alfa Engineering Works Ltd. had the following annual budget for the year ending on 30th June

Production capacity	***60%***	***80%***
Costs		*(₹ in lakhs)*
Direct materials	*9.60*	*12.80*
Direct Labour	*7.20*	*9.60*
Factory expenses	*7.56*	*8.04*
Administrative expenses	*3.72*	*3.88*
Selling & distribution expenses	*4.08*	*4.32*
Total cost	*32.16*	*38.64*
Profit	*4.86*	*10.72*
Sales	*37.02*	*49.36*

Owing to adverse trading conditions, the company has been operating during July/Septemberat 40% capacity, realizing budgeted selling prices.

Owning to acute competition, it has become inevitable to reduce prices by 35% even to maintain the sales at the existing level. The directors are considering whether or not their factory should be closed down until the trade recession has passed. A market research consultant has advised that in about a year's time there is every indication that sales will increase to 75% of normal capacity and that the revenues to be produced for the full year at that volume could be expected to ₹ 40 lakhs

If the directors decide to close down the factory for a year it is estimated that:

(a) The present fixed costs would be reduced to ₹6 lakhs p.a.

(b) Closing down costs (redundancy payments, etc.) would amount to ₹2 lakhs.

(c) Necessary maintenance of plant would cost ₹50,000 p.a.; and

(d) On re-opening the factory, the cost of overhauling the plant, training and engagement of new personal would amount to ₹80,000.

Prepare a report for the directors, making your recommendations.

Solution

The Directors,
Alfa Engineering Works Ltd. New Delhi

Date……………………

Dear Sir,

As desired, we have analysed the cost implications of the decision of temporary closure of the trade recession. We find that if the factory is run at 40% capacity and with reduced selling prices, the loss likely to be incurred in one full year (the estimated period of recession), would be around ₹ 7.17 lakhs as detailed below:

	(₹) **In lakhs**
Direct materials	6.40
Direct labour	4.80
Factory expenses	7.08
Administrative expenses	3.56
Selling & distribution expenses	3.84
	25.68
Loss	(7.17)
Sales $\left(₹\ 36.02 \text{ lakhs} \times \frac{40}{60} \times \frac{75}{100}\right)$	18.51

If the factory is closed, the following costs will be incurred:

	(₹) **in lakhs**
Fixed costs	6.00
Closing down costs	2.00
Maintenance costs	0.50
Cost of overhauling the plant, training and engagement of staff	0.80
	9.30

It is obvious from the above, that despite the fact that running at 40% capacity would imply a loss of ₹ 7.17 lakhs, it is better not to close down the factory since in that case the loss would be higher.

In our views, even if running the factory entailed a somewhat bigger loss as compared to the loss incurred by closing it down temporarily, it may be better to keep the factory in operation. This is because a closure, even if temporary, results in the loss of regular and old customers, suppliers and skilled personal. This, coupled with a loss of goodwill in the market, may give rise to substantial losses at the time of restarting the factory. We trust that the above analysis would be helpful to you in reaching an appropriate decision in the matter. We shall be glad to be of any further assistance that may be required in this regard.

Yours faithfully
X and Co.
Chartered Accountants.

Working Note:

		Factory Expenses (₹ Lakhs)	**Admn. expenses (₹ Lakhs)**	**Selling expenses (₹ Lakhs)**
(i)	Amount at 60%	7.56	3.72	4.08
(ii)	Amount at 80%	8.04	3.88	4.32
(iii)	Change for 20%	0.48	0.16	0.24
(iv)	Amount at 40% (i)-(iii)	7.08	3.56	3.84

Illustration23

Universe Ltd. manufactures 20,000 units of 'X' in a year at its normal production capacity. The unit cost as to variable costs and fixed costs at this level are ₹13 and ₹4 respectively.

Due to trade depression, it is expected that only 2,000 units of 'X' can be sold during the next year. The management plans to shut-down the plant. The fixed costs for the next year then is expected to be reduced to ₹ 33,000. Additional costs of plant shut-down are expected at ₹12,000. Should the plant be shut-down? What is the shut-down point?

Solution

Note: The decisions regarding the plant to shut-down and the calculation of shut-down point requires the figure of selling price per unit of the units sold. As the statement of the question fails to indicate the selling price (per unit) therefore one is free to assume it.

Let's assume the selling price per unit be ₹ 20

Statement of cost for taking a decision about shutdown of plant

	Plant is operated (₹)	Plant is shut down (₹)
Variable cost	26,000 (2,000 units × ₹13)	—
Fixed costs	80,000 (20,000 units × ₹ 4)	33,000 (Inescapable cost)
Additional shut down cost		12,000
Total cost	1,06,000	45,000
State of loss:		
Sales	40,000 (2,000 units ₹ 20)	-
Less: Total cost (as above)	1,06,000	45,000
Loss	(66,000) (if continued)	(45,000) (if shut-down)

Recommendation: A comparison of loss figures indicated as above points out, that, loss is reduced if the plant is shut-down. In fact by doing so the concern's loss would be reduced by ₹ 21,000 (₹66,000 - ₹ 45,000).

Calculation of shut down point:

$$\text{Shut-down point} = \frac{₹80{,}000 - ₹45{,}000}{₹20 - ₹13}$$

= 5,000 units

Illustration 24

A paint manufacturing company manufactures 2,00,000 per annum medium-sized tins of "Spray Lac Paints" when working at normal capacity. It incurs the following costs of manufacturing per unit :

	(₹)
Direct material	*7.80*
Direct labour	*2.10*
Variable overhead	*2.50*
Fixed overhead	*4.00*
Product cost (per unit)	*16.40*

Each unit (tin) of the product is sold for ₹ 21 with variable selling and administrative expenses of 60 paise per tin.

During the next quarter only 10,000 units can be produced and sold. Management plans to shut down the plant estimating that the fixed manufacturing cost can be reduced to ₹ 74,000 for the quarter.

When the plant is operating, the fixed overheads are incurred at a uniform rate throughout the year. Additional costs of plant shut-down for the quarter are estimated at ₹14,000.

You are required:

(a) To express your opinion, along with the calculations, as to whether the plant should be shut down during the quarter, and

(b) To calculate the shut down point for quarter in units of products (i.e., in terms of number of tins).

Solution

(a) Working note:

Contribution per tin:

Selling price – Variable cost of the product

= ₹ 21 – ₹ (7.80 + 2.10 + 2.50 + 0.60)

= ₹ 8 per tin

Calculation of loss, if plant is operated

	(₹)
Total contribution on 10,000 tins @ ₹ 8 each	80,000
Less: Total fixed cost for three months	
(2,00,000 × ₹ 4) × $\frac{3}{12}$	2,00,000
Expected loss on operation	(1,20,000)

Calculation of loss (shut down costs) if plant is shut down

	(₹)
Unavoidable fixed cost	74,000
Additional cost of shut-down	14,000
Total loss on shut down	(88,000)

It is clear from the above, that if plant is operated, loss would be ₹ 1,20,000 which exceeds the loss by ₹ 32,000 (₹ 1,20,000 – ₹ 88,000) when the plant is shut-down. Therefore, the management should shut-down the plant during the quarter.

(b) Calculation of shut-down point

Avoidable fixed cost for the period (or fixed cost which will not be incurred if the plant is shut down) :

= Total fixed cost for the period – (Unavoidable fixed costs + Additional shut down costs)

= ₹ 2,00,000 – (₹ 74,000 + ₹ 14,000)

= ₹ 2,00,000 – ₹ 88,000

= ₹ 1,12,000

Shut-down point $= \dfrac{\text{Avoidable fixed cost}}{\text{Contribution per unit}}$

$= \dfrac{₹\ 1{,}12{,}000}{₹\ 8} = 14{,}000$ tins

or

Shut-down point $= \dfrac{\text{Total fixed cost – Shut down costs}}{\text{Contribution per unit}}$

$= \dfrac{₹\ 2{,}00{,}000 - ₹\ 88{,}000}{₹\ 8} = 14{,}000$ tins

Other considerations in shut down decisions:

Cost is not the only criterion for deciding in favour of shut down. Non-cost factors should also be taken into consideration. Few non-cost factors are:

1. Interest of the workers – If the workers are discharged it may become difficult to get skilled workers later, on re-opening of the factory. Also shut down may create problem for the workers which may far exceed the cost benefits of the shut down.
2. Once the firm is closed down competitors may establish their products and thus it may be difficult to introduce the product in the market again.
3. The plant may become obsolete or depreciate at a larger rate when not in operation. Thus, heavy capital expenditure may have to be incurred on re-opening.

Divestment Strategy

Divestment involves a strategy of selling off or shedding business operations to divert the resources, so released, for other purposes. Selling off a business segment or product division is one of the frequent forms of divestment strategy. It may also include selling off or giving up the control over a subsidiary where by the wholly owned subsidiaries may be floated as independently quoted companies.

Reasons for Divestment

1. In case of a firm having an opportunity to get more profitable product or segment but has resource constraint, it may selling off its unprofitable or less profitable division and utilise the recourse so released. Cost Benefit Analysis & Capital Budgeting Method is the useful tool for analysing this type of situation.
2. In case of purchase of new business, it may be found that some of the part of the acquired business is not up to the mark. In such type of situation disposal of the unwanted part of the business is more desirable than hold it.
3. In case where any business segment or product or subsidiary is pull down the profit of the whole organisation, it is better to cut down of that operation of the product or business segment or subsidiary.

4. If managing of the organisation is very constrained, it is good to dispose off the unwanted and undesirable activity of the organisation, which involves large management skill. So that it can concentrated on the core activities of the organisation.

5. In the situation where the firm suffering from loss, selling off or divestment policy is one suitable option to exit in the current position and to go for turnaround strategy.

2.6 Other Decision Makings

Illustration 25

An item of finished goods inventory that cost ₹200 per unit to make is facing the danger of becoming obsolete. There are two following alternative ways of disposing it of:

Sell the stock to P for ₹200; or to Q for ₹216. Q's place is twice as far away as that of P though due to favourable transport conditions the delivery time will be the same. The cost accountant has prescribed the following cost estimates for delivery:

P - Petrol and oil ₹10, wages ₹12, share of licence, insurance and depreciation (based on mileage) ₹14.

Q - Petrol and oil ₹20, wages ₹12, share of licence, insurance and depreciation ₹28.

You are required to recommend whether the stock should be sold to P or Q.

Solution

	P (₹)	**Q** (₹)
Cash inflow (Sales)	200	216
Cash outflow (Petrol, oil and wages)	22	32
Net cash flow	178	184

Since the most favourable net cash flow occurs in the case of sale to Q, hence it is recommended that the item be sold to Q.

Notes: 1. In the above analysis ₹14 and ₹28 the share of licence, insurance and depreciation have not been included because there is no actual cash outflow in respect of these costs. Otherwise also these costs are not going to affect the cashflow whether sale is made to P or Q or even to neither of them.

2. It may be noted that the net cashflow does not measure the profits of the projects. It only indicates which project is more profitable.

Note: *Students should note that the discounted cash flow technique has been discussed in detail in the book of Strategic Financial Management. Students should refer to the aforesaid book to refresh their knowledge.*

Illustration 26

The Z company owns and operates a chain of 25 stores. Budgeted data for the Garden stores are as follows

	(₹)
Annual sales	*4,25,000*
Annual cost of goods sold and other operating expenses	*3,82,000*
Annual building ownership costs (not included above)	*20,000*

The company can lease the building to a large flower shop for ₹ 4,000 per month. Decide whether to continue operations of this store or to lease using:

(i) The total project (or comparative statement) approach.
(ii) The incremental (or relevant cost) approach.
(iii) The opportunity cost approach.

Solution

(i) Comparative statement showing the profitability of two alternatives

	Continue operation (₹)	***Lease the building*** (₹)
Annual sales	4,25,000	48,000 (@₹4,000 p.m.)
Less: Cost of goods sold (excluding ownership costs)	3,82,000	—
Building ownership costs	20,000	20,000
Net income	23,000	28,000

Net income is ₹ 28,000 if the building is leased out and thus leasing is a profitable proposition.

(ii) Incremental or relevant cost approach

Building ownership costs are not relevant as there is no change in these costs under both the alternatives. Therefore, the correct approach will be to consider the incremental cash inflows from the continuing operation.

	(₹)
Net cash flow from continuing the operation (₹ 4,25,000 – ₹ 3,82,000)	43,000
Less: Income from leasing	48,000
Incremental loss from continuing operations	5,000

Therefore, company should not continue the operation

(iii) The opportunity cost approach

	(₹)
Total sale revenue	4,25,000
Less: Cost of goods sold	(3,82,000)
Opportunity cost of leasing	(48,000)
Loss due to continuing operation	5,000

Therefore, the company should lease out the building.

Illustration 27

Carcare Corporation has just today paid for and installed a special machine for polishing cars at one of its prestigious outlets. It is the first day of company's fiscal year. The machine costs ₹ 20,000. Its annual operating costs total ₹ 15,000 exclusive of depreciation. The machine will have a four year useful life and a zero terminal disposal value.

After the machine has been used for one day, a machine salesman walks in. He offers a different machine that promises to do the same job at a yearly operating cost of ₹ 9,000, exclusive of depreciation. The new machine will cost ₹ 24,000 in cash, duly installed. The "old" machine is unique and can be sold outright for only ₹ 10,000 minus ₹ 2,000 removal cost. The new machine, like the old one, will have a four-year useful life and zero terminal disposal velue.

Sales, all in cash, will be ₹ 1,50,000 annually and other cash costs will be ₹ 1,10,000 annually, regardless of this decision.

For simplicity, ignore income taxes, interest and present value considerations.

Required :

(a) Prepare a statement of cash receipts and disbursements for each of the four years under both alternatives. What is the cumulative difference in cash flows for the four years taken together ?

(b) Prepare Income Statements for each of the four years under both alternatives. Assume straight-line depreciation. What is the cumulative difference in operating income for the four years taken together ?

(c) What are the irrelevant items in your presentations in requirements (a) and (b) ? Why are they irrelevant ?

(d) Suppose the cost of the "old" machine was ₹ 10,00,000 rather than ₹ 20,000. Nevertheless, the old machine can be sold outright for only ₹ 10,000 minus ₹ 2,000 removal cost. Would the net differences in requirements (a) and (b) change ? Explain.

(e) "To avoid a loss, we should keep the old machine." What is the role of book value in decisions about replacement of machines ?

Solution

(a) Statement of Cash Receipts, Disbursements and cumulative difference in Cash flows for four years taken together under both alternatives.

(₹ in thousands)

Alternatives	Keep old machine			Buy new machine			
	Year 1	2nd, 3rd & 4th year each	All 4 years	Year 1	2nd, 3rd & 4th year each	All 4 years	Cumulative difference in cash flows for four years taken together
Receipts :							
Sales revenue	150	150	600	150	150	600	

Sales of old equipment	—	—	—	8	—	8	
Total receipts : (A)	150	150	600	158	150	608	
Disbursements :							
Annual operating cost	15	15	60	9	9	36	
Other cash costs	110	110	440	110	110	440	
Purchase cost of "old" machine	20	—	20	20	—	20	
Purchase of "new" machine	—	—	—	24	—	24	
Total disbursements : (B)	145	125	520	163	119	520	
Net cash in-flows : (A) – (B)	5	25	80	(5)	31	88	08

(b) Statement of income for each of the four years and cumulative difference in operating income

(₹ in thousands)

Alternatives	Keep old machine		Buy new machine			
	1st. 2nd 3rd & 4th year each (₹)	All 4 years (₹)	Year 1 (₹)	2nd, 3rd & 4th year each (₹)	All 4 years (₹)	Cumulative difference in operating income
Income :						
Sales revenue	150	600	150	150	600	
Total revenue : (A)	150	600	150	150	600	
Costs :						
Annual operating cost	15	60	9	9	36	
Other cash costs	110	440	110	110	440	
Depreciation	5	20	6	6	24	
(*Refer to working note 1*)						
Loss on the disposal of oldmachine	—	—	12	—	12	
(*Refer to working note 2*)						
Total costs : (B)	130	520	137	125	512	
Operating income : (A) – (B)	20	80	13	25	88	08

(c) The purchasecost of old machine ₹ 20,000; the sales revenue ₹ 1,50,000 and other cash costs of ₹ 1,10,000 are irrelevant items for the presentation in requirements (a) and (b) above. These items are irrelevant because their amount are common to both the alternatives.

(d) The net difference in requirements under (a) and (b) will not change if the cost of 'old' machine becomes ₹ 10,00,000 instead of ₹ 20,000. This is so because the cost of old machine is common for both the alternatives.

(e) In the decision about the replacement of machine the book value of the machine is irrelevant because it is a past (historical) cost. All past costs are down the drain. Nothing can change what has already happened. As apparent from (a) and (b) above; we can completely ignore the cost of old machine i.e. ₹ 20,000 and still have a correct analysis.

Working Notes :

1. *Depreciation (under straight line method):*

		Old machine	New machine
(i)	Cost of machine (₹)	20,000	24,000
(ii)	Terminal disposal value (₹)	Zero	Zero
(iii)	Useful life	4	4
	Depreciation $\left[\frac{\text{(i)} - \text{(ii)}}{\text{(iii)}}\right]$ ₹	5,000	6,000

2. *Loss on the disposal of old machine :*

	(₹)	(₹)
Purchase price of old machine		20,000
Disposal value	10,000	
Less: Removal cost	2,000	8,000
Loss		12,000

2.7 Introduction to Marginal Costing

According to CIMA, Marginal costing is the system in which variable costs are charged to cost units and fixed costs of the period are written off in full against the aggregate contribution.

Marginal costing is not a distinct method of costing like job costing, process costing, operating costing, etc. but a special technique used for managerial decision making. Marginal costing is used to provide a basis for the interpretation of cost data to measure the profitability of different products, processes and cost centre in the course of decision making. It can, therefore, be used in conjunction with the different methods of costing such as job costing, process costing, etc., or even with other technique such as standard costing or budgetary control.

In marginal costing, cost ascertainment in made on the basis of the nature of cost. It gives consideration to behaviour of costs. In other words, the technique has developed from a particular concept and expression of the nature and behaviour of costs and their effect upon the profitability of an undertaking.

Marginal Costing (and Absorption costing) is covered in CA-Integrated Proficiency Competence Course (IPCC). Here an overview has been included to explain extension of Marginal Costing i.e. C-V-P Analysis.

2.8 Introduction of Cost-Volume-Profit (CVP) Analysis

Cost-Volume-Profit Analysis (as the name suggests) is the analysis of three variable viz., cost, volume and profit. Such an analysis explores the relationship existing amongst costs, revenue, activity levels and the resulting profit. It aims at measuring variations of cost with volume. In the profit planning of a business, cost-volume-profit (C-V-P) relationship is the most significant factor.

Features of CVP Analysis:

- It is a technique for studying the relationship between cost volume and profit.
- Profit of an undertaking depends upon a large number of factors. But the most important of these factors are the cost of manufacture, volume of sales and selling price of products.
- In words of Herman C. Heiser, “the most significant single factor in profit plannibg of the average business is the relationship between volume of business, cost and profits”.
- The CVP relationship is an important tool used for profit planning of a business.

Some important marginal cost concepts are as follows

1. Profit Statement under Marginal Costing

Statement of Profit	(₹)	(₹)
Revenue/Sales		xxx
Less: Variable cost of production		
Material	xx	
Labour	xx	
Direct Expenses	xx	
Variable overheads	xx	
	xx	
Add: Opening Stock of Finished goods (at MC)	xx	
Less: Closing Stock of Finished goods (at MC)	xx	
Variable cost of goods sold	xxx	
Add: Variable Selling overhead	xx	
Variable cost of sales		xx
Contribution		xx
Less: All types of Fixed cost		xx
(Committed, Discretionary costs)		xx

2. Sales - Variable Cost = Contribution = Fixed Cost ± Profit/ (loss)
3. Profit Volume (P/V) Ratio or Contribution to Sales (C/S) Ratio also known as contribution margin ratio = Contribution÷ Sales

   = Contribution per unit÷ Selling price per unit

   = Change in Contribution÷ Change in Sales
4. Breakeven Point (BEP): Point where there is no profit or no loss.

   at BEP, Contribution = Fixed Cost

   Thus, Break Even Sales (in sales value)=Fixed Cost ÷ P/V ratio
5. Margin of safety = Sales – BEP sales

   = Contribution / P/V ratio - Fixed cost / P/V ratio

= Profit / P/V ratio

6. Profit = (Sales × P/V ratio) – Fixed Cost

   = (Margin of Safety Sales) × P/V ratio
7. BEP - Calculation in different scenario :

(i) Without limiting factor (non- attributable to a single product)

BEP in units = Fixed cost ÷ Average contribution per unit.

(when sales mix in units are given)

BEP in value = Fixed cost ÷ composite P/ V Ratio

(when sales mix in rupee are given)

where Composite P/V Ratio = (Total Contribution÷Total Sales)

$=\sum$[Sales Mix × P/ V Ratio]

(ii) With limiting factor (attributable to a single product)

Find contribution per limiting factor & give rank. Find total contribution from the product ranked as1st. Calculate the amount of fixed cost still to be recovered. Whether it can be recovered by 2nd rank product or not?

(iii) For perishable products apply the same concept in case of opening stock with the identified variable cost.

8. a. BEP in case of process costing is expressed in terms of total raw material input

   b. In capital budgeting, BEP is that sales volume where discounted cash inflow = discounted cash outflow.

   c. Potential BE : On the basis of sales out of current period production only.

   d. Multiple BE : Different BE due to change in sales price, variable costs & fixed costs for different production level.

e. Cash BEP = Cash fixed cost ÷contribution p.u. So do not consider the sunk cost.

f. BEP for decision making purpose: Accept that proposal where BEP is lowest provided the profit cannot be calculated.

2.9 Important Factors in Marginal Costing Decisions

In all recommendations of marginal costing decisions, the following factors are to be considered:

(i) Whether the product or production line in question makes a contribution.

(ii) Where a choice is to be made between two courses of action, the additional fixed overhead, if any, should be taken into account.

(iii) The continuity of demand after expansion or renovation or installation of the sophisticated machine and its impact on the selling price should also be considered. For example, if the selling price goes down when the supply increases the possible drop in profit should be taken into account.

(iv) Cost is not the only criterion for decision making. Non-cost factors like the necessity to retain the experienced employees, etc. should also be considered.

2.10 Pricing Decisions under Special Circumstances

If goods were sold in the normal circumstances under normal business conditions, the price would cover the total cost plus a margin of profit. Selling prices are not always determined by the cost of production. They may be determined by market conditions but in the long run they tend to become equal to the cost of production of marginal firm. Therefore, a business cannot continue to sell below the total cost for a long period. Occasionally, a firm may have to sell below the total cost.

Theproblem of pricing can be summarised under three heads:

(i) Pricing in periods of recession,

(ii) Differential selling prices and

(iii) Acceptance of an offer and submission of a tender.

2.10.1 Pricing in periods of recession : In periods of recession, a firm may sell its articles at a price *less than the total cost but above the marginal cost* for a limited period.

The advantages of this practice are:

(i) The firm can continue to produce and use the services of skilled employees who are well trained and will be difficult to re-employ later if discharged.

(ii) Plant and machinery can be prevented from deterioration through idleness.

(iii) The business would be ready to take advantage of improved business conditions later.

(iv) This avoids the competition of securing the business of the firm.

One thing to remember here is that a situation like this should not lead to a drastic price cutting and the orders accepted should not cover a long period extending over the production facilities of a period when business conditions improve.

It may also be justifiable to sell the product at a price below marginal cost for a limited period provided the following conditions prevail:

(i) Where materials are of perishable nature.

(ii) Where stocks have been accumulated in large quantities and the market prices have fallen. This will save the carrying cost of stocks.

(iii) Is it essential to reduce the prices to such an extent in order to popularise a new product?

(iv) Where such reduction enables the firm to boost the sales of other products having larger profit margin.

2.10.2 Differential selling prices: Use of differential selling price which is above the marginal cost but below the total cost is resorted to in an order to absorb surplus capacity. There are two ways of doing this:

(i) The firm producing a branded article may use the surplus capacity to produce the same article to be sold above marginal cost in a different market.

Dumping of goods in the export market is an example of this type of pricing.

The articles sold in the home market will recover all fixed expenses. Since pricereduction in the home market is injurious to the normal sales, it is not resorted to. Any reduction in the selling prices in the export market will not affect the price prevailing in the home market.

(ii) The firm may produce and sell a branded article, say product A, which covers the entire fixed overheads and use the surplus capacity to produce another product B, which may be sold at a price above its marginal cost. The overall profitability will thus increase. The manufacture of product B, however, should be confined to surplus capacity and it should not have the possibility of becoming a major product at the low price at which it is sold. If it becomes so there will be a reduction in profit as illustrated below:

Example

	Condition 1		***Condition 2***	
	Product A	***Product B***	***Product A***	***Product B***
Capacity	90%	10%	60%	40%
	(₹)	(₹)	(₹)	(₹)
Sales	9,000	2,000	6,000	8,000
Marginal cost	6,000	1,800	4,000	7,200
Gross margin	3,000	200	2,000	800

Fixed expenses	2,000	—	2,000	—
Profit	1,000	200	—	800
Total profit	1,200		800	

Surplus capacity is assumed to be 10% in the above example.

2.11 Acceptance of an offer and submission of a tender

◆ **Acceptance of an offer:** When a firm having surplus capacity receives an offer from a special or export market, a decision as to whether to accept or not to accept the offer can be taken after the analysis of the *incremental cost and incremental revenue*.

Illustration 28

Capacity	*Unit cost (₹)*	*Unit selling price (₹)*
6,000	*80*	*100*
7,000	*75*	*97*
8,000	*74*	*95*
9,000	*72*	
10,000	*71*	

The firm is operating at 8,000 units capacity and has received an order for 2,000 units from an export market at a price of ₹ 70 per unit. Advise the firm as to whether the export order should be accepted or not.

Solution

Apparently, the unit cost at 9,000 and 10,000 units capacity is ₹ 72 and ₹ 71 respectively and since the export order is at ₹ 70 per unit, the order is not profitable. But this is a wrong approach. Let us tabulate the figures again and see the result.

Capacity	Unit cost (₹)	Total cost (₹)	Incremental cost (₹)	Unit price (₹)	Total sales value (₹)	Incremental revenue (₹)
6,000	80	4,80,000		100	6,00,000	
7,000	75	5,25,000	45,000	97	6,79,000	79,000
8,000	74	5,92,000	67,000	95	7,60,000	81,000
9,000	72	6,48,000	56,000			
10,000	71	7,10,000	62,000			

At 8,000 level of output the total sales revenue is ₹ 7,60,000 and the total cost is ₹ 5,92,000 leaving a profit of ₹ 1,68,000. The fact that this level of output leaves a profit means that the fixed expenses have been recovered already. Hence we have to take only the incremental cost for further levels of output. For an additional sales of 2,000 units the incremental cost is ₹ 7,10,000 – ₹ 5,92,000 = ₹ 1,18,000. The cost per unit, therefore, is $\frac{₹\ 1,18,000}{2,000 \text{ units}}$ = ₹ 59 for which the price quoted is ₹ 70 per unit. The offer is, therefore, acceptable.

◆**Submission of Tenders:** For submitting tenders also the incremental cost and incremental revenue approach is useful. Considering the above example, if the firm operates at 8,000 level of output, and quotations are to be given, any price quotation above the unit incremental cost of ₹ 59 would be profitable.

Illustration 29

All Play and Nowork Ltd. are specialists in the manufacture of sports goods. They manufacture croquet mallets but purchase the wooden balls, iron arches and stakes required to complete a croquet set.

Mallets consist of a head and handle. Handles use 1.5 board feet per handle at ₹ 40 per board foot. Spoilage loss is negligible for the manufacture of handles.

Heads frequently split and create considerable scrap. A head requires 0.20 board feet of high quality lumber costing ₹ 70 per board foot. Spoilage normally works out to 20% of the completed heads. 4% of the spoiled heads can be salvaged and sold as scrap at ₹ 10 per spoiled head.

In the department machining and assembling the mallets, 12 men work 8 hours per day for 25 days in a month. Each worker can machine and assemble 15 mallets per uninterrupted 50 minutes time frame. In each 8 hours working day, 15 minutes are allowed for coffee-break, 8 minutes on an average for training and 9 minutes for supervisory instructions. Besides 10% of each day is booked as idle time to cover checking in and checking out changing operations, getting materials and other miscellaneous matters. Workers are paid at a comprehensive rate of ₹ 6 per hour.

The department is geared to produce 40,000 mallets per month and the monthly expenses of the department are as under:

	(₹)
Finishing and painting of the mallets	*50,800*
Lubricating oil for cutting machines	*300*
Depreciation for cutting machine	*700*
Repairs and maintenance	*100*
Power to run the machines	*200*
Plant Manager's salary	*2,700*
Other overheads allocated to the department	*1,20,000*

As the mallets are machined and assembled in lots of 500, prepare a total cost sheet for one lot and advise the management on the selling price to be fixedper mallet in order to ensure a minimum 20% margin on the selling price.

Solution

All Play and Nowork Ltd.
Cost Sheet of one lot of 500 Croquet Mallets

	(₹)	(₹)	(₹)
Direct material:			
Handles (1.5 feet × 500 units × ₹ 40)		30,000	
Heads (1.20 × 500 × 0.20 × ₹ 70)	8,400		
(Refer to working note 1)			
Less: Scrap recovery			
(4% × 100 × ₹ 10)	40	8,360	38,360
Direct labour:			
$\left(\frac{8 \text{ hrs} \times ₹\ 6}{120} \times 500\right)$ *(Refer to working note 2)*			200
Prime cost			<u>38,560</u>
Factory & other overheads:			
Variable			
Finishing & painting			
$\left(\frac{₹\ 50,800}{40,000} \times 500\right)$ *(Refer to working note 3)*			635
Fixed			
$\left(\frac{₹\ 1,24,000}{36,000} \times 500\right)$ *(Refer to working note 4)*			1,722
Total cost			40,917
Price quotation:			
Cost per mallet $\left(\frac{₹\ 40,917}{500 \text{ units}}\right)$			81.834
Add: Profit 25% on cost			20.458
(20% margin on selling price means 25% on cost)			
Selling price			102.29

Working notes:

1. Since 20% of completed heads are spoiled, output of 1 unit requires input of 1 + 0.20 = 1.20 units; so, total heads processed: 1.20 × 500 = 600, of which spoiled heads are 100.

2. Total time in a day: 8 × 60 — 480 minutes

Total time in a day: 8 × 60	480 minutes	
Less : Idle time	48 minutes	
Coffee break	15 minutes	
Instructions	9 minutes	
Training	8 minutes	80 minutes
Productive time per day:		400 minutes

Therefore, mallets to be produced per man per day: $\left(\frac{400}{50} \times 15\right)$ = 120 units.

Since mallets are produced at the rate of 120 mallets per man day, so total monthly production will be: 120 units × 12 men × 25 days = 36,000 mallets.

3. Finishing and painting overheads are assumed to be variable for the production of 40,000 mallets.
4. All the other expenses are fixed and are to be absorbed by 36,000 mallets of monthly production.

Illustration 30

A Company can produce and sell at its maximum capacity 20,000 units of a product. The sales price per unit is ₹100. The present sale is 15,000 units. To produce over 20,000 units and up to another 10,000 units some balancing equipments are to be installed at a cost of ₹10 lakhs and the same will have a life span of 10 years.

The current cost structure is as under:

Direct material	*30% of sale value*
Direct labour	*20% of sale value*
Variable overheads	*₹20 per unit*
Profit	*₹15 per unit*

The present cost is estimated to go up due to price escalation as under:

10% in Direct material from present level of 30%

25% in Direct labour from present level of 20%

₹50,000 in Fixed overheads per year.

There is a concrete proposal from a party to take 10,000 units additionally over the present level of output on a long-term basis at a unit price of ₹90. Apart from the investment of ₹10 lakhs, as shown above, the fixed overheads will increase by ₹ 50,000 due to additional Administrative expenses.

The Company is in a dilemma as to whether to accept the order for 10,000 units or to use the present unused capacity of 5,000 units for which there will be additional selling expenditure of ₹50,000.

Ignore financing charges and give your recommendation.

Solution

Working Note :

Fixed overheads :	(₹)
Present sale value: (A)	15,00,000
(*15,000 units × ₹100*)	
Direct materials	4,50,000
(*30% of sale value*)	
Direct labour	3,00,000
(*20% of sale value*)	
Variable overheads	3,00,000
(*₹20 per unit*)	
Total variable costs: (B)	10,50,000
Contribution: (C) : (A) – (B)	4,50,000
Profit : (D)	2,25,000
(*15,000 units × ₹15*)	
Fixed overheads: (C) – (D)	2,25,000
(*current level*)	
Add: Additional fixed overheads due to price escalation	50,000
Total fixed overheads	2,75,000

Statement of profitability for various alternatives

Alternatives	**I**	**II**	**III**	**IV**
	Rejecting the proposal for the purchase of 10,000 units and continuing with present level of sales only	***Rejecting the proposal for the purchase of 10,000 units from a party and attaining the maximum capacity by incurring additional selling expenditure***	***Accepting the proposal of the party to take 10,000 units @ ₹ 90 per unit by installing a balancing equipment and continuing with pre-sent level of sales***	***Accepting the proposal of party to take 10,000 units @ ₹90 per unit by installing a balancing equipment and attaining sale of maximum available capacity by incurring additional selling expenditure***
Sales (units)	15,000	20,000	25,000	30,000
	(₹)	(₹)	(₹)	(₹)
Sales value: (A)	15,00,000	20,00,000	24,00,000	29,00,000
	(15,000 × ₹ 100)	(20,000 × ₹ 100)	(15,000 × ₹ 100 +	(20,000 × ₹ 100 +
			10,000 × ₹ 90)	(10,000 × ₹ 90)

Variable costs :				
Direct materials (33% of sales value)	4,95,000	6,60,000	8,25,000*	9,90,000*
Direct labour (25% of sale value)	3,75,000	5,00,000	6,25,000*	7,50,000*
Variable overheads (@ ₹ 20 per unit)	3,00,000	4,00,000	5,00,000	6,00,000
Total variable costs : (B)	11,70,000	15,60,000	19,50,000	23,40,000
Fixed costs :				
Fixed overheads *(Refer to working note)*	2,75,000	2,75,000	2,75,000	2,75,000
Additional selling expenditure	—	50,000	—	50,000
Depreciation for balancing equipment	—	—	1,00,000	1,00,000
Additional administrative expenses	—	—	50,000	50,000
Total fixed costs : (C)	2,75,000	3,25,000	4,25,000	4,75,000
Total costs D : [(B)+ (C)]	14,45,000	18,85,000	23,75,000	28,15,000
Profit : (A)–(D)	55,000	1,15,000	25,000	85,000

*** Note :** For computing the material and labour cost under alternatives III & IV the notional sale price of ₹ 100 is taken for additional 10,000 units.

Recommendations: Alternative II is the best as it gives maximum profit.

2.12 Quotation for an Export Order

Illustration31

Somesh of Agra presently operates its plant at 80% of the normal capacity to manufacture a product only to meet the demand of Government of Tamil Nadu under a rate contract.

He supplies the product for ₹4,00,000 and earns a profit margin of 20% on sales realisations. Direct cost per unit is constant.

The indirect costs as per his budget projections are :

Indirect costs	***20,000 units (80% capacity)***	***22,500 units (90% capacity) (₹)***	***25,000 units (100% capacity) (₹)***
Variable	*80,000*	*90,000*	*1,00,000*
Semi-variable	*40,000*	*42,500*	*45,000*
Fixed	*80,000*	*80,000*	*80,000*

He has received an export order for the product equal to 20% of its present operations. Additional packing charges on this order will be ₹1,000.

Arrive at the price to be quoted for the export order to give him a profit margin of 10% on the export price.

Solution

Working Notes :

1. Direct Cost per unit

	₹
Selling price per unit (₹ 4,00,000/20,000 units)	20
Less : Profit margin (20% × ₹ 20/-)	4
Total cost	16
Less: Indirect costs (₹ 2,00,000/20,000 units)	10
Direct cost per unit	6

2. Statement of differential cost for 4,000 units (20% of 20,000 units)

	Present production 20,000 units (₹)	Proposed production 24,000 units (₹)	Differential cost for 4,000 units (₹)
Direct cost @ ₹ 6/- p.u.	1,20,000	1,44,000	24,000
Indirect cost:			
Variable @ ₹4/- p.u.	80,000	96,000	16,000
Semi-variable	40,000	44,000	4,000
Fixed	80,000	81,000	1,000
Total	3,20,000	3,65,000	45,000

Computation for the price to be quoted for the export order of 4,000 units

	(₹)
Differential cost *(Refer to working note 2)*	45,000
Add : Profit (10% of export price or 1/9th of cost)	5,000
Price to be quoted	50,000

Export price per unit : ₹ 12.50

(₹ 50,000/4000 units)

2.13 Make or Buy Decision

Very often management is faced with the problem as to whether a part should be manufactured or it should be purchased from outside market. Under such circumstances two factors are to be considered:

(a) whether surplus capacity is available, and

(b) the marginal cost.

Illustration32

The total cost of a manufactured component is as under:

Prime cost	*₹15*	*Fixed overhead*	*₹ 4*
Variable overhead	*₹7*	*Total cost*	*₹26*

The same part is available in the market at ₹23. Should the firm make it or buy it?

Solution

If surplus capacity is available and will remain idle if the component is bought, the out of pocket expenses will be ₹ 23 per unit,₹ 1 more than the variable (relevant) cost of making component which is ₹ 22 (₹ 15 + ₹ 7). Hence, it is economical to make it. However, if the firm is utilizing or can utilize the capacity in making some other part, which contributes, say ₹ 4 per unit, the effective cost of buying the component will be ₹ 19 (₹ 23 less ₹ 4 contribution from other product). In that case, it would be economical to buy the component at ₹ 23 per unit from outside. The relevant computations for taking decision may be as follows:

	Make (₹)	**Per unit cost Buy and leave capacity idle (₹)**	**Buy and use capacity forother product (₹)**
Cost of making/buying	(22)	(23)	(23)
Contribution from other product	—	—	4
Net relevant cost	(22)	(23)	(19)

Illustration33

Perfect Product Ltd. is currently buying a component from a local supplier at ₹15 each. The supply is tending to be irregular. Two proposals are under consideration:

1. Buy and install a semi-automatic machine for manufacturing this component, which would involve an annual fixed cost of ₹ 9 lakhs and a variable cost of ₹ 6 per manufactured component.

2. Buy and install an automatic machine for manufacturing this component, incurring an annual fixed cost of ₹ 15 lakhs and a variable cost of ₹ 5 per manufactured component.

Determine with necessary computations:

(1) The annual volume required, in each case, to justify a switch over from outside purchase to 'own manufacture'.

(2) The annual volume required, to justify selection of the automatic machine instead of the semi-automatic machine.

(3) If the annual requirement of the coming year is expected to be 5,00,000 Nos. and the volume is expected to increase rapidly thereafter, would you recommend the automatic or semi-automatic machine. Justify your recommendation.

Solution(1)

	Proposal 1 Semi-automatic Machine (₹)	**Proposal 2 Automatic Machine (₹)**
Purchase cost per unit for the component now being bought	15	15
Less : Unit variable cost for 'own manufacture'	6	5
Unit contribution from 'own manufacture'	9	10
Total annual fixed cost to be recouped	9,00,000	15,00,000
Number of units required to fully recover the fixed costs.	1,00,000 Units	1,50,000 Units

These figures show that an annual volume of over 1,00,000 Nos. of the component will justify 'own manufacture' on the semi-automatic machine, instead of purchase from outside.

To justify the installation of the automatic machine, the quantity required is an annual volume of over 1,50,000 Nos.

(2) Incremental annual fixed cost if automatic machine is chosen: ₹ 6,00,000

Saving in unit variable cost by choosing the automatic machine: ₹ 1

Production volume required to recover the additional annual fixed costs through saving in unit variable cost. 6,00,000 Nos.

For annual requirements of over 6,00,000 units of the components, the automatic machine will be more economical as compared to the semi-automatic machine.

(3) If the annual requirement is 5,00,000 units, the semi-automatic machine is to be preferred, as it would involve a lower total cost per unit of the component, as indicated below :

	Semi-automatic (₹)	**Automatic (₹)**
Total variable costs:		
5,00,000 units @ ₹ 6 and ₹ 5 respectively	30,00,000	25,00,000
Total fixed costs	9,00,000	15,00,000

Total costs	39,00,000	40,00,000
Total cost per unit	7.80	8.00

However, the annual requirement is expected to increase rapidly beyond 5,00,000 units; as soon as it is 6,00,000 units the semi-automatic machine will become more expensive as compared to the automatic machine. Then the need for installing the automatic machine will arise which may be within a very short period after commissioning the semi-automatic machine. Replacement of the semi-automatic machine by an automatic machine may then become costly, not only because of the loss that may arise on the semi- automatic machine but also by possibly a higher price of the automatic machine. The management may therefore, install an automatic machine immediately.

Illustration34

Agrocaps Ltd., in manufacturing agricultural machinery, is preparing its annual budget for the coming year. The company has a metal pressing capacity of 20,000 hours, which will be insufficient for manufacture of all requirements of components A, B, C and D.

The company has the following choices:

(i) Buy the components entirely from outside suppliers.

(ii) Buy from outside suppliers and/or use a partial second shift.

The data for the current year are given below:

Standard production cost perunit

Component	***A*** (₹)	***B*** (₹)	***C*** (₹)	***D*** (₹)
Variable cost:				
Direct materials	*37*	*27*	*25*	*44*
Direct wages	*10*	*8*	*22*	*40*
Direct expenses	*10*	*20*	*10*	*60*
Fixed overhead	*5*	*4*	*11*	*20*
Total production cost p.u.	*62*	*59*	*68*	*164*
Requirements in units	*2,000*	*3,500*	*1,500*	*2,800*

Direct expenses relate to the use of the metal presses which cost ₹ 10 per hour, to operate. Fixed overheads are absorbed as a percentage of direct wages.

Supply of all or any part of the total requirement can be obtained at following prices, each delivered to the factory:

Component	*(₹)*
A	*60*
B	*59*
C	*52*
D	*168*

Second shift operations would increase direct wages by 25 percent over the normal shift and fixed overhead by ₹500 for each 1,000 (or part thereof) second shift hours worked.

You are required to present, with calculations:

(a) Which component, and in how much quantities should it be manufactured in the 20,000 hours of press time available?

(b) Whether it would be profitable to make any of the balance of components required on a second shift basis instead of buying them from outside suppliers.

Solution

(a) Working notes:

(i) Press hours required per unit of production

Component	A	B	C	D
Direct expenses per unit	10	20	10	60
No. of press hours per unit,direct expenses per presshour being ₹ 10	1	2	1	6

(ii) Marginal cost of production per unit vs. bought out prices per unit

Component	A (₹)	B (₹)	C (₹)	D (₹)
Marginal (variable) costs				
Direct material	37	27	25	44
Direct wages	10	8	22	40
Direct expenses	10	20	10	60
Marginal cost per unit : (A)	57	55	57	144
Bought out price : (B)	60	59	52	168
Excess of bought out price				
over marginal cost : {(B) – (A)}	3	4	(5)	24
Press hours per unit	1	2	1	6
Excess of bought out price perunit of limiting factor (i.e. press hour)	3	2	(5)	4

The bought-out price for component C is lower by ₹ 5 than the marginal cost of production and so it should be purchased from outside.

In case the remaining components A, B and D are bought, their ranking in terms of loss per unit of limiting factors (press hour) would be D (highest loss per unit), A and B. The capacity available should, therefore, be deployed for making D first and then A and thereafter B.

Components and their quantities to be manufactured in 20,000 hours of press time available(single shift operation)

	(Hours)
Available capacity for metal pressing	20,000
First, produce D hours required (2,800 units × 6 hours)	16,800
Balance hours available	3,200
Second, produce A hours required (2,000 units × 1 hour)	2,000
Balance hours available	1,200
Third, produce B, for the balance hours available (600 units × 2 hours)	1,200
Balance hours available	Nil

So, in 20,000 hours of press time available, all the requirements of components D and A and only 600 units of component B can be manufactured. The balance requirement of component B i.e. 2,900 (3,500 – 600) units will have to be bought out or manufactured in the second shift.

(b) Since the purchase price of Component C (₹ 52) is lower than the marginal cost ofmanufacturing (₹ 57) in even single shift, it will not be profitable to make it, hence it should be purchased from outside.

Now it is to be seen whether 2,900 units of B should be produced in the second shift or bought from outside. The comparative position is given below:

Cost of producing 2,900 units of components B in second shift	(₹)
Variable cost per unit on single shift basis	55.00
*Add.*Increase in direct wages per unit	2.00
Variable cost per unit	57.00
Total variable cost for 2,900 units, (2,900 units × ₹ 57)	1,65,300
Additional fixed cost :	
Hours required for 2,900 units of B (2900 units × 2 hours) = 5,800 hrs.	
Extra fixed cost for 5,800 hours at ₹ 500 for	
every 1,000 hours (or part thereof)	3,000
Total cost for producing 2,900 units of B in second shift : (A)	1,68,300
Bought outside price for 2,900 units of B will be 2,900 units × ₹59:(B)	1,71,100
Disadvantage in buying: (A – B)	(2,800)

Since the cost of manufacturing balance quantity of component B i.e. 2,900 units in second shift is less by ₹ 2,800, it is profitable to make it on a second shift basis instead of buying it from outside suppliers.

Illustration35

A company manufacturing a highly successful line of cosmetics intends to diversify the product line to achieve fuller utilisation of its plant capacity. As a result of considerable research made, the company has been able to develop a new product called "EMO".

EMO is packed in tubes of 50 gram capacity and is sold to the wholesalers in cartons of 24 tubes at ₹240 per carton. Since the company uses its spare capacity for the manufacture of EMO, no additional fixed expenses will be incurred. However the cost accountant has allocated a share of ₹4,50,000 per month as fixed expenses to be absorbed by EMO as a fair share of the company's present fixed costs to the new product for costing purposes.

The company estimates the production and sale of EMO at 3,00,000 tubes per month and on this basis the following cost estimates have been developed:

	₹ per carton
Direct materials	*108*
Direct wages	*72*
Overheads	*54*
Total costs	*234*

After a detailed market survey the company is confident that the production and sales of EMO can be increased to 3,50,000 tubes per month and ultimately to 4,50,000 tubes per month.

The company at present has a capacity for the manufacture of 3,00,000 empty tubes and the cost of the empty tubes if purchased from outside will result in a saving of 20% in material and 10% in direct wages and variable overhead costs of EMO. The price at which the outside firmis willing to supply the empty tubes is ₹1.35 per empty tube. If the company desires to manufacture empty tubes in excess of 3,00,000 tubes, a new machine involving an additional fixed overheads of ₹30,000 per month will have to be installed.

Required:

(i) State by showing your workings whether the company should make or buy the empty tubes at each of the three volumes of production of EMO namely, 3,00,000; 3,50,000 and 4,50,000 tubes.

(ii) At what volume of sales will it be economical for the company to install the additional equipment for the manufacture of empty tubes?

(iii) Evaluate the profitability on the sale of EMO at each of the aforesaid three levels of output based on your decision and showing the cost of empty tubes as a separate element of cost.

Solution

(i) Working notes :

		(₹)
(1)	Overheads for one carton i.e. 24 tubes	54
	Therefore, per tube overheads: (₹ 54/24 tubes)	2.25

Fixed overheads allocated for 3,00,000 tubes: ₹ 4,50,000

Per tube fixed overheads: $\left(\frac{₹\ 4,50,000}{3,00,000\ \text{tubes}}\right)$ = ₹ 1.50

Therefore variable overheads, per tube {₹ 2.25 – ₹ 1.50} = ₹ 0.75

		(₹)
(2)	Direct wages per carton	72
	Therefore, direct wages per tube : (₹ 72/24 tubes)	3
(3)	Direct materials per carton	108
	Therefore, direct materials per tube : (₹ 108/24 tubes)	4.50

(4) *Cost of making one empty tube:*

	Cost per tube of EMO (₹)	% of cost in respect of empty tube	Cost of empty tube (₹)	Cost per tube of EMO without empty tube (₹)
Direct materials	4.50	20	0.90	3.60
Direct wages	3.00	10	0.30	2.70
Variable overheads	0.75	10	0.075	0.675
	8.25		1.275	6.975

Cost of manufacturing / buying of 300,000 empty tubes of EMO

	Emptytube cost (₹)	If emptytubes aremade (₹)	If emptytubes arepurchased (₹)
Direct materials	0.90	2,70,000	—
Direct wages	0.30	90,000	—
Variable overheads	0.075	22,500	—
Purchase price	1.35	—	4,05,000
Total		3,82,500	4,05,000

Since manufacturing capacity is available for the manufacture of 3,00,000 empty tubes at a cost of ₹ 3,82,500 whereas the total cost of purchase of tubes is higher, i.e., ₹ 4,05,000, the company should manufacture the empty tubes for a production volume of 3,00,000 EMO tubes.

Beyond 3,00,000 empty tubes, the company has to install a new machine involving atotaladditional fixed overheads of ₹ 30,000. The cost of making and buying the additional tubes 50,000 and 1,50,000 units of empty tubes will be as under :

	Per tube	Additional empty tubes			
		50,000 tubes		1,50,000 tubes	
		Make	Buy	Make	Buy
	(₹)	(₹)	(₹)	(₹)	(₹)
Direct materials	0.90	45,000		1,35,000	—
Direct wages	0.30	15,000		45,000	—
Variable overheads	0.075	3,750		11,250	—
Additional overheads		30,000		30,000	—
Purchase price	1.35	—	67,500	—	2,02,500
Total		93,750	67,500	2,21,250	2,02,500

The above statement shows that the cost of buying additional empty tubes at both the levels is lower than the cost of their manufacture. Therefore, if the company increases production to 3,50,000 tubes of EMO, 3,00,000 tubes should be made in the factory and additional 50,000 tubes should be purchased at ₹ 67,500.

If the company increases production to 4,50,000 tubes of EMO, 3,00,000 empty tubes should be made in the factory and additional 1,50,000 tubes should be purchased at a cost of ₹ 2,02,500.

(ii) Additional fixed overheads to be incurred on a new machine: ₹ 30,000. Savings per unit if empty tubes are made in the factory instead of buying: ₹ 1.35 – ₹ 1.275 = ₹ .075

Minimum additional quantity of empty tubes to be made to recover the additional fixed costs:

$$\frac{₹\ 30,000}{₹\ 0.075} = 4,00,000 \text{ empty tubes}$$

Thus the company should sell 3,00,000 + 4,00,000 = 7,00,000 tubes of EMO per month to warrant justification for the installation of the new machine for the manufacture of empty tubes.

(iii) Evaluation of the profitability onsale of EMO at the three levels

	Per tube	3,00,000 tubes	3,50,000 tubes	4,50,000 tubes
	(₹)	(₹)	(₹)	(₹)
Sales (₹ 240/24 tubes)	10	30,00,000	35,00,000	45,00,000
Direct materials	3.60	10,80,000	12,60,000	16,20,000
Direct wages	2.70	8,10,000	9,45,000	12,15,000
Variable overheads	0.675	2,02,500	2,36,250	3,03,750
Empty tubes made	1.275	3,82,500	3,82,500	3,82,500
Empty tubes purchased	1.35	—	67,500	2,02,500
Total costs		24,75,000	28,91,250	37,23,750
Profit		5,25,000	6,08,750	7,76,250

Illustration 36

A firmneeds a component in an assembly operation. If it wants to do the manufacturing itself, it would need to buy a machine for ₹ 4 lakhs which would last for 4 years with no salvage value. Manufacturing costs in each of the 4 years would be ₹ 6 lakhs, ₹ 7 lakhs, ₹ 8 lakhs and 10 lakhs respectively. If the firm had to buy the component from a supplier the component would cost ₹ 9 lakhs, ₹ 10 lakhs, ₹ 11 lakhs and ₹ 14 lakhs respectively in each of the 4 years. However, the machine would occupy floor space which could have been used for another machine. This latter machine could be hired at no cost to manufacture an item, the sale of which would produce net cash flows in each of the 4 years of ₹ 2 lakhs; it is impossible to find room for both the machines and there are no other external effects. The cost of capital is 10% and PV factor for each of the 4 years is 0.909, 0.826, 0.751 and 0.683 respectively. Should the firm make the component or buy from outside?

Solution

Evaluation of Make or Buy Proposal

(₹in lakhs)

Year	Present valuefactor at 10%	When the Component ismanufactured		When the Component isbought	
		Cash outflow	Present value	Cash outflow	Present value
0	1.000	4	4.000	—	—
1	0.909	6 + 2	7.272	9	8.181
2	0.826	7 + 2	7.434	10	8.260
3	0.751	8 + 2	7.510	11	8.261
4	0.683	10 + 2	8.196	14	9.562
			34.412		34.264

Saving in buying: ₹ 34.412 lakhs – ₹ 34.264 lakhs = ₹ 0.148 lakh

Thus it is beneficial to buy the component from outside.

Note: The loss of ₹ 2 lakhs cash inflow for each of the 4 years due to inability of the firmto operate another machine when it manufactures the component is to be treated as an opportunity cost.

2.14 Export Vs Local Sale Decision

When the firm is catering to the needs of the local market and surplus capacity is still available, it may think of utilising the same to meet export orders at price lower than that prevailing in the local market.

This decision is made only when the local sale is earning a profit, i.e., where its fixed expenses have already been recovered by the local sales. In such cases, if the export price is more than the marginal cost, it is preferable to enter the export market.

Any reduction in the price prevailing in the local market to fulfill surplus capacity may have adverse effect on the normal local sales. Dumping in the export market at a lower price will not, however, have any such adverse effect on local sales.

Illustration 37

XLtd., having an installed capacity of 1,00,000 units of a product is currently operating at 70% utilization. At current levels of input prices, the FOB unit costs (after taking credit for applicable export incentives) work out as follows:

Capacity Utilisation	*FOB Unit Costs*
Per cent	*(₹)*
70	*97*
80	*92*
90	*87*
100	*82*

The company has receivedthree foreign offers from different sources as under:

Source A 5,000 units at ₹55 per unit FOB

Source B 10,000 units at ₹52 per unit FOB

Source C 10,000 units at ₹51 per unit FOB

Advise the company as to whether any or all export orders should be accepted or not.

Solution

X Limited

Statement showing Differential Costat Different Capacity Utilisation Levels

Installed capacity 1,00,000 Units

Capacityutilisation	Production atdifferent levels of capacityutilisation	FOB unit costs	Total costs	Differential costs	Per unitdifferentialcosts
Percent	Units	(₹)	(₹)	(₹)	(₹)
70	70,000	97	67,90,000	—	—
80	80,000	92	73,60,000	5,70,000	57
90	90,000	87	78,30,000	4,70,000	47
100	1,00,000	82	82,00,000	3,70,000	37

Statement showing Gain or Loss on Accepting the Various Export Orders

Export Order (Source)	Export Order (Unit)	Capacity Utilisation Per cent	Differential Costs Per Unit (₹)	Differential Costs Total (₹)	FOB Priceper Unit (₹)	Sales Revenue from the Export (₹)	Gain/ (Loss) (₹)
A	5,000	75%	5,000 Uts. @ 57	2,85,000	55	2,75,000	(10,000)
B	10,000	85%	5,000 Uts. @ 57 5,000 Uts. @ 47	5,20,000	52	5,20,000	Nil
C	10,000	95%	5,000 Uts. @ 47 5,000 Uts. @ 37	4,20,000	51	5,10,000	90,000
Total	25,000	95%		12,25,000		13,05,000	80,000

It is obvious from the above statement that the company will gain only when all the three export orders are accepted. If the company accepts exports only for one or two of the three sources, it will loose. Therefore, the company should accept all the three exports orders.

2.15 Expand or Contract Decision

Whenever a decision is to be taken as to whether the capacity is to be expanded or not, consideration should be given to the following points:

(a) Additional fixed expenses to be incurred.

(b) Possible decrease in selling price due to increase in production.

(c) Whether the demand is sufficient to absorb the increased production.

Based on these considerations, the cost schedule will be worked out. While deciding about the contraction of business, the saving in fixed expenses and the marginal contribution lost will have to be taken into account. If a branch office is to be closed down, and if the branch is giving a marginal contribution sufficient to cover fixed expenses the contraction may lead to a loss as under:

Example

Branch B:	Sales	₹ 20,000
	P/V ratio	20%
	Marginal contribution	₹ 4,000
	Fixed expenses of the branch	₹ 3,000

The branch is giving an extra contribution of ₹ 1,000. If it is closed, the fixed expenses saving is ₹ 3,000 whereas the contribution lost is ₹ 4,000. Hence it is not advisable to contract the business by closing down the branch.

Illustration38

Nice and Warm Ltd., manufactures and markets hot plates. During the first five years of operation, the company had experienced a gradual increase in sales volume, and the current annual growth in sales of 5% is expected to continue into the foreseeable future. The plant is now producing at its full capacity of one lakh hot plates.

At the monthly Management Advisory Committee meeting, amongst other things, the plan of action for next year was discussed.

Managing Director proposed two alternatives. First, operations could be continued at full capacity and with the existing facilities an output of one lakh hot plates at a selling price of ₹ 100 per unit could be maintained. Secondly, production and sales could be increased by 5% to take advantage of the rate of expansion in demand for the product. But this could increase cost, as to achieve the output, the company will have to resort to weekend and overtime workings. However, a policy of steady growth was preferable to maintaining status quo.

In view of the company's competitors having a substantial share of the market, the Works Director was of the view that it was not enough for the company to maintain merely the present share of the total market. A larger share of the total market should be obtained. For that, the company should increase the production by 10% through a modest expansion of plant capacity. In order to sell the output of 1,10,000 units, the selling price could be reduced to ₹95 per unit.

Thinking on the same lines, the Marketing Director put forth a more radical proposal. The strategy should be to seize the competitive leadership in the market with regard to both price and volume. With this end in view, he suggested that the company should straight away embark on an expensive modernisationprogramme which will initially increase volume by 20%. The entire output of 1,20,000 hot plates could be easily sold at a price of ₹90 per unit.

At this juncture Managing Director expressed concern about the probable behaviour of the company's competitors. They might also expand in order to produce more and sell at lowest prices. Suppose this happened, he wanted also the financial effects of the proposals of the Works Director and the Marketing Director, if in those proposals, the increase in sales were to be only half of that predicted.

As the Cost Accountant of the company you are required to critically evaluate the six alternatives, along with your recommendations and circulate the same to the Directors.

In this connection you have gathered the following details.,

(1) If next year's production was maintained at the current year's level variable costs would remain unchanged at ₹ 30 lakhs.

(2) The weekend and overtime working would increase with the variable and fixed costs. Variable cost would rise to ₹ 55 per unit while fixed costs would increase to ₹30,25,000.

(3) In the proposal of the Works Director, the ratio of variable costs to sales would continue to be 50% and fixed costs would rise to ₹ 32,25,000.

(4) In the proposal of the Marketing Director, as a result of increased production efficiency and some savings from purchase of materials, it is estimated that the ratio of variable cost to sales would decrease to 48% and the fixed costs would increase by ₹ 5,16,000.

Your answer should contain:

(a) A tabular statement of comparative figures pertaining to Total Turnover, Total Contribution, Percentage of Profit to Sales and Break-Even units as regards to each of the six proposals.

(b) Comment on the relative risks involved.

(c) Consideration of the short-term and long-term implications of the Managing Director's proposals.

(d) Comments on the price elasticity of demand for the company's product and your suggestions on the pricing policy and cost structure.

(e) Comments on financial implications of the expansion schemes.

Solution

(a) Tabular Statement of Comparative Figures Pertaining to Total Turnover, Total Contribution, Percentage of Profit to Sales and Break-even Units etc., as regards to each of the Six Proposals.

	Proposals					
	Managing Director's 1st Proposal	Managing Director's 2nd Proposal	Works Director's 1st Proposal	Works Director's 2nd Proposal (1/2 of expected increase)	Marketing Director's 1st Proposal	Marketing Director's 2nd Proposal (1/2 of expected increase)
	(1)	(2)	(3)	(4)	(5)	(6)
Units sold	1,00,000	1,05,000	1,10,000	1,05,000	1,20,000	1,10,000
Unit selling price (₹)	100	100	95	95	90	90
Total turnover (₹ in lakhs)	100.00	105.00	104.50	99.75	108.00	99.00
Unit contribution (₹)	50	45	47.5	47.5	46.80	46.80

Total contribution (₹ in lakhs)	50	47.25	52.25	49.875	56.16	51.48
Fixed cost (₹ in lakhs)	30	30.25	32.25	32.25	35.16	35.16
Profit (₹ in lakhs)	20	17.00	20.00	17.625	21	16.32
Percentage of profit to sales	20%	16.19%	19.14%	17.67%	19.44%	16.48%
Break-even units	60,000	67,222	67,895	67,895	75,128	75,128
Margin of safety in units	40,000	37,778	42,105	37,105	44,872	34,872

(b) At the present full capacity level, it is enough to sell 60,000 units to break even. Other proposals raise the break-even point further. In an uncertain market, if in the proposals of Works Director and the Marketing Director, only half the increase is achieved, the margin of safety will be lower than the present 40,000 units. Profit as a percentage of sales is also lower than existing, in all the proposals. All this is a disquieting feature as the risk involved is greater in all the other proposals.

(c) The company has already reached its full capacity. As a short term measure, the Managing Director's first proposal seems to be all right. From long-term point of view, neither of the proposals can be considered to be satisfactory. Both the proposals of the Managing Director do not provide a lasting solution. Though the second proposal maintains the market share, it results in less profit, both in quantum and percentage. As the capacity has already been reached there is an urgent necessity for the Managing Director to address himself to long range objectives and plans keeping in view the expansion in demand for the company's product.

(d) It seems that both the Works Director and the Marketing Director have very elementary notions on price. They think that if the volume increases in order to sell the increased volume, price has to be lowered. No serious study seems to have been made on the price elasticity of demand for the company's product. On the other hand, we have been told that there is a steady 5% annual growth in demand, which means that the prices need not be reduced, only more market share has to be obtained. For incremental production, differential pricing in certain special markets has to be resorted to; if this is not possible, the increased production can be sold under a different brand name with a different price (A static cost structure, more or less, has been assumed). To beat competition, a better product has to be put in the market and cost reduction offered through value analysis, etc.

(e) The expansion scheme envisaged have to be properly tested for profitability by feasibility study reports, etc. Source of financing the expansion has to be determined.The financial implications of share issue or borrowed funds have to be gone through. Long range objectives have to be defined and plans drawn accordingly to achieve them.

2.16 Product Mix Decision

Many times the management has to take a decision whether to produce one product or another instead. Generally decision is made on the basis of *contribution of each product*. Other things *being the same the product which yields the highest contribution is best one to produce*. But, if there is shortage or limited supply of certain other resources which may act as a key factor like for example, the machine hours, then the contribution is linked with such a key factor for taking a decision. For example, in an undertaking the availability of machine capacity is limited and the machine hours required for one unit of the two products are different. In such cases the contribution is to be linked with the machine hour and the product which yields the highest contribution per machine hour is to be preferred for taking decision.

Illustration39

A firm's operations are at present performed by hand. It has a proposal to install a new machine which can produce at a faster rate. Following information is available. Advise the management about the profitability of mechanisation.

	Hand	***Machine***
Production in units per hour	*1*	*2*
Marginal cost per unit (₹)	*18*	*16*
Additional fixed cost per unit (₹)	—	*3*
Total cost per unit (₹)	*18*	*19*
Selling price per unit (₹)	*24*	*24*

Solution

Let us analyse the figures as under:

	Hand (₹)	**Machine** (₹)
Selling price per unit	24	24
Less: Marginal cost per unit	18	19
(including additional fixed cost)		
Contribution per unit	6	5
Contribution per hour	6	10

If there is a great demand for the products, it is advisable to mechanise because the gross margin per hour is more when machine is used. If however, there is idle capacity and there is an under- absorption of fixed overheads to the extent of ₹ 3 per unit the total cost will be ₹ (19 + 3) = ₹ 22 leaving a contribution of ₹ 2 per unit. The contribution per hour will, therefore, be ₹ 4 which is less than that of obtaining under hand operation. Hence mechanisation will not be advisable under these circumstances.

Illustration40

An engineering company isengaged in producing four products through operations at welding and pressing departments. Products W_1 and W_2 are produced by welders in the welding department whereas products P_1 and P_2 are produced by press-operators in the pressing department. Due to specific skill requirements, the welders and press-operators can only work in their own department.

The following relevant data are available in respect of the products:

	Products			
	W_1	W_2	P_1	P_2
Hours required per unit	4	4	5	2
Selling price per unit (₹)	48	50	77	69
Direct material cost per unit (₹)	18	22	32	44
Direct labour hourly rate (₹)	4	4	4	4
Variable overhead rate per unit (₹)	2	2	3	3

The company incurs ₹50,000 per annum towards fixed costs. The maximum available hours are 20,000 and 16,000 for welding and pressing departments respectively.

The demands keep on fluctuating but the minimum demands which are to be met as per management's decision are 2,000 units of W_l, 2,500 units of W_2, 1,800 units of P_1 and 2,200 unitsof P_2.

The production manager suggests that the welders and press- operators can be trained to perform both welding and pressing jobs so that excess demand of any of the products can be met. This decision is going to increase the burden of fixed costs by ₹5,000 per annum.

Prepare the profitability statement for optimum product-mix and recommend with reasons and appropriate workings whether it is advisable to train the welders and press-operators as suggested by the production manager.

Solution

Optimum Product Mix before Training

Department Maximum Hours Product	**Welding 20,000 hours**		**Pressing 16,000 hours**	
	W_1	**W_2**	**P_1**	**P_2**
Selling price per unit (₹)	48	50	77	69
Less: Variable cost				
Material (₹)	18	22	32	44
Labour (₹)	16	16	20	8

Variable overhead (₹)	2	2	3	3
Total variable cost (₹)	36	40	55	55
Contribution per unit : (A)	12	10	22	14
Labour hours per unit : (B)	4	4	5	2
Contribution per hour (₹) (A)/(B)	3	2.5	4.4	7
Ranking	*I*	*II*	*II*	*I*
Minimum production (units)	2,000	2,500	1,800	2,200
Labour hours needed	8,000	10,000	9,000	4,400
Labour hours used foreach department	8,000 + 10,000 = 18,000		9,000 + 4,400= 13,400	
Balance hours available	20,000 – 18,000 = 2,000		16,000–13,400= 2,600	
Product to be produced	W_I		P_2	
Units to be produced	2,000/4 = 500		2,600/2 = 1,300	
	W_1	W_2	P_1	P_2
Hence, product-mix(units)	2,000 + 500 = 2,500	2,500	1,800	2,200+1,300 = 3,500

Profitability Statement before Training

Product	Product-mix units	Hours		Contribution	
		Per unit	Total	Per hour (₹)	Total
W_1	2,500	4	10,000	3.00	30,000
W_2	2,500	4	10,000	2.50	25,000
			20,000		55,000
P_1	1,800	5	9,000	4.40	39,600
P_2	3,500	2	7,000	7.00	49,000
			16,000		88,600
	Total		36,000		1,43,600
			Less: Fixed costs		50,000
			Net profit		93,600

Optimum Product-Mix after Training

After training, the capacity will be taken as a whole at 36,000 labour hours because of interchangeability of available labour force. The ranking will be done on the basis of contribution per hour among all the four products, since a workman is trained to work in any of the departments.

Production	W_1	W_2	P_1	P_2
Maximum hours	36,000 hours as a whole			
Hours needed for minimum production	8,000	10,000	9,000	4,400
	(Total hours = 31,400 hours)			
Balance hours	(36,000 – 31,400 hours = 4,600 hours)			
Product priority (Ranking)	III	IV	II	I
Product in balance hoursP_2 (I rank):				4,600 ÷ 2 = 2,300 units
Product-mix (units)	2,000	2,500	1,800	2,200 + 2,300= 4,500

Profitability Statement after Training

Product	Product-mix units	Hours		Contribution	
		Per unit	Total	Per hour	Amount
				(₹)	(₹)
W_1	2,000	4	8,000	3.00	24,000
W_2	2,500	4	10,000	2.50	25,000
P_1	1,800	5	9,000	4.40	39,600
P_2	4,500	2	9,000	7.00	63,000
			36,000		1,51,600
				Less: Fixed costs	55,000
				Net profit	96,600

Recommendation:

Since the net profit after training will be more by ₹ 3,000 (₹ 96,600 – ₹ 93,600), it is advisable to train the welders and press operators as suggested by the production manager.

Illustration41

Veejay Ltd. makes and sell two products, 'Vee' and 'Jay'. The budgeted selling price of 'Vee' is ₹1,800 and that of 'Jay' is ₹2,160. Variable costs associated with producing and selling the 'Vee' are ₹900 and with 'Jay'₹1,800. Annual fixed production and selling costs of Veejay Ltd. are ₹88,000.

The company has two production/sales option. The 'Vee' and 'Jay' can be sold either in the ratio of two 'Vees' to three 'Jays' or in the ratio of one 'Vee' to two 'Jays'.

What will be the optimal mix and why ?

Solution

Selection of best optimal mix

	Products	
	'Vee'	**'Jay'**
	(₹)	(₹)
Budgeted selling price p.u.	1,800	2,160
Less : Variable cost p.u.	900	1,800
Contribution p.u.	900	360

Option-I

(I) *Production/Sales option : (2 units of 'Vee' and 3 units of 'Jays')*

Total contribution under 1st option

= (2 units × ₹ 900 + 3 units × ₹ 360)

= ₹ 1,800 + ₹ 1,080 = ₹ 2,880

$$\textit{Break-even point} = \frac{\text{Annual fixed production \& selling costs}}{\text{Total contribution under 1st option}}$$

$$= \frac{₹\ 88{,}000}{₹\ 2{,}880} = 30.56 \text{ (sets of 5 units)}$$

	Products		**Total**
	'Vee'	**'Jay'**	
Break-even point (units)	30.56 × 2 units = 61.12 units = 61 (units approx.)	30.56 × 3 units = 91.68 units = 92 (units approx.)	
Break–even sales (₹)	= 1,09,800 (61 units × ₹ 1,800)	= 1,98,720 (92 units × ₹ 2,160)	3,08520

Option-II

(II) *Production/Sales option : (1 unit of 'Vee' and 2 units of 'Jays')*

Total contribution under IIndoption

= (1 unit × ₹ 900 + 2 units × ₹ 360)

= ₹ 900 + ₹ 720 = ₹ 1,620

$$\text{Break–even point} = \frac{\text{Annual fixed production \& selling costs}}{\text{Total contribution under 1st option}}$$

$= \dfrac{₹\ 88,000}{₹\ 2,880} = 54.32$ (sets of 3 units)

	Products		Total
	'Vee'	'Jay'	
Break–even points (units)	54.32 × 1 unit = 54 (units approx.)	54.32 × 2 units = 109 (units approx.)	
Break–even sales (₹)	= 97,200 *(54 units × ₹1,800)*	= 2,35,440 *(109 units × ₹2,160)*	3,32,640

Note:

The given amount of annual fixed production and selling cost is such that it fails to determine the exact figure of break–even point under two given sales options. The approximations made in the above solutions under option- I, at break–even level over recovery of₹ 20; whereas under option- II of the solution there is an under recovery of fixed cost to the extent of ₹ 160.

Decision &reasoning:

Option I is preferred over option II, as it results in a lower level of sales to reach break–even (because of higher average contribution per unit sold). The average contribution per unit (under option I) is ₹ 576 (₹ 2,880/5 units) and (under option II) it is ₹ 540 (₹ 1,620/3 units). Option I contains a higher percentage (40% as against 33 1/3%) of more profitable products.

Illustration42

ZED Ltd. manufactures two products P and Q and sells them at ₹215 and ₹320 per unit respectively. The variable costs per unit are as under.

	Product-P (₹)	**Product-Q (₹)**
Raw materials :		
Material-X	*22.00*	*28.00*
Material-Y	*8.00*	*32.00*
Direct wages (₹6 per labour hour) :		
Department-A	*36.00*	*54.00*
Department-B	*18.00*	*36.00*
Department-C	*54.00*	—
Department-D	—	*72.00*
Variable overheads	*23.00*	*14.30*

The company procures raw materials against import licence. The company operates at single shift a day of 8 hours for 300 days in a year. The numbers of workmen engaged are 30, 16, 18 and 24 in departments A, B, C and D respectively. Neither the workers are subject to transfer from one department to another nor any new recruitment is possible at present. Fixed costs are ₹12,000 per month.

You are required to find out the following:

(a) The product-mix to yield maximum profit

(b) The most profitable product if only one product is to be manufactured. Whether the answer will differ if licence to import raw materials is released only for ₹ 1,80,000.

Solution

Working Notes :

1. Computation of total labour hours available

Departments (a)	No. of workmen (b)	Days (c)	Hrs./day (d)	Total hours (e) = (b) × (c) × (d)
A	30	300	8	72,000
B	16	300	8	38,400
C	18	300	8	43,200
D	24	300	8	57,600

2. Computation of hours required per unit of each product

Departments	Product P			Product Q		
	Wages (₹)	Wages/hr. (₹)	Hrs.	Wages (₹)	Wage/hr. (₹)	Hrs
	(a)	*(b)*	*(c) = (a)/(b)*	*(d)*	*(e)*	*(f) = (d)/(e)*
A	36	6	6	54	6	9
B	18	6	3	36	6	6
C	54	6	9	–	–	–
D	–	–	–	72	6	12
	Total hours per unit :		18			27

3. Statement showing maximum output permissible

Departments	Hours available	Product P		Product Q	
		Hrs. required/ Unit	Maximum output in Units	Hrs. required/ Unit	Maximum output in Units
	(a)	(b)	(c) = (a)/(b)	(d)	(e) = (a)/(d)
A	72,000	6	12,000	9	8,000
B	38,400	3	12,800	6	6,400
C	43,200	9	4,800*	–	–
D	57,600	–		12	4,800*
	Total hours p.u.	18		27	

* This shows that either 4,800 units of product P or Q can be obtained by utilising the available hours in the four departments.

4. Product wiseContribution per hour

		Product P (₹)	Product Q (₹)
Selling price p.u.	(A)	215	320
Total raw material cost p.u.			
(₹ 22 + ₹ 8)		30	
(₹ 28 + ₹ 32)			60
Total wages per unit			
(₹ 36 + ₹ 18 + ₹ 54)		108	
₹ 54 + ₹ 36 + ₹ 72)			162
Variable overheads p.u.		23	14.30
Total variable cost p.u.	(B)	161	236.30
Contribution p.u. [(A) – (B)]		54	83.70
Labour hours p.u.		18	27
Contribution per labour hour		3	3.10
		(₹ 54/18 hrs.)	(₹ 83.70/27 hrs.)

(a) Though the contribution per labour hour of Product Q is better but there is a constraint viz., the numbers of workers in each department can neither be interchanged nor newly recruited, hence due to this, following alternatives would arise which may help in deciding about the product mix to yield maximum profit.

Alternative I : Producing 4,800 units of Product Q and utilising the remaining available hours of labour for making units of Product P.

Alternative II : Producing 4,800 units of Product P and utilising the remaining available hours of labour for making units of Product Q.

Statement of Product mix under alternative I

Departments	Available hours	Hours required for 4,800 units of Q	Remaining hours	Hrs./Unit of Product P	Units of Product
	(a)	(b)	(c) = (a)–(b)	(d)	(e) = (c)/(d)
A	72,000	43,200	28,800	6	4,800
B	38,400	28,800	9,600	3	3,200
C	43,200	–	43,200	9	4,800
D	57,600	57,600	Nil	–	–

The above table shows that out of the available hours under alternative I; 4,800 units of Product Q and 3,200 units of Product P can be made.

Statement of Product mix under alternative II

Departments	Available hours	Hours required for 4,800 units of P	Remaining hours	Hrs./Unit of Product Q	Units of Product
	(a)	(b)	(c) = (a)–(b)	(d)	(e) = (c)/(d)
A	72,000	28,800	43,200	9	4,800
B	38,400	14,400	24,000	6	4,000
C	43,200	43,200	–	–	–
D	57,600	–	57,600	12	4,800

The above table shows that out of the available hours under alternative II; 4,800 units of Product P and 4,000 units of Product Q can be made.

Profit Statement under above alternatives

Products	First alternative			Second alternative		
	Units	Contribution p.u. (₹)	Amount (₹)	Units	Contribution p.u. (₹)	Amount (₹)
P	3,200	54.00	1,72,800	4,800	54.00	2,59,200
Q	4,800	83.70	4,01,760	4,000	83.70	3,34,800
Total Contribution:			5,74,560			5,94,000
Less : Fixed cost p.a.			1,44,000			1,44,000
Profit			4,30,560			4,50,000

Second alternative is the most profitable product mix.

(b) Statement of most profitable product if only one product is to be manufactured

Products	P	Q
Contribution per unit (₹) : A	54.00	83.70
Maximum possible output (in units) : (B)	4,800	4,800
Total Contribution : (A) × (B)	2,59,200	4,01,760

Product Q is to be preferred

Statement of most profitable product if only one product is to be manufactured and licence to import the raw material is only for materials worth ₹ 1,80,000

Products	P	Q
Raw material required p.u. (₹)	30	60
Permissible output in units out of importedmaterial of ₹ 1,80,000	6,000	3,000
Maximum output possible in the available hours	4,800	4,800
Output possible keeping in view the availability of imported materialand labour hours (Units)	4,800	3,000
Contribution per unit (₹)	54	83.70
Total Contribution (₹)	2,59,200	2,51,100
	(4,800 units × ₹ 54)	(3,000 units × ₹ 83.70)

Product P is to be preferred (i.e. answer differs) because of import licence restriction, which is only available for purchasing material worth only ₹ 1,80,000.

Illustration 43

The relevant data of X Ltd. for its three products A, B and C are as under:

	A	*B*	*C*
Direct material (₹/Unit)	*260*	*300*	*250*
Direct labour (₹/Unit)	*130*	*270*	*260*
Variable overheads (₹/Unit)	*110*	*230*	*180*
Selling price (₹/Unit)	*860*	*1,040*	*930*
Machine Hours required (Per Unit)	*12*	*6*	*3*

The estimated fixed overheads at four different levels of 3,600; 6,000; 8,400; and 10,800 machine hours are ₹ 1,00,000; ₹ 1,50,000; ₹ 2,20,000 and ₹ 3,00,000 respectively. The maximum demand of A, B and C in a cost period are 500; 300 and 1,800 units respectively.

You are required to find out (i) the most profitable product-mix at each level and (ii) the level of activity where the profit would be maximum.

Solution

Working Note :

Ranking of three products A, B and C

Products	A	B	C
Selling price (p.u.) (₹)	860	1,040	930

Less : Variable cost p.u. (₹)	500	800	690
Contribution p.u. (R.s)	360	240	240
Machine hrs. required p.u.	12	6	3
Contribution per machine hour (₹)	30 *(₹360/12. hrs)*	40 *(₹240/6 hrs.)*	80 *(₹240/3 hrs.)*
Ranking	III	II	I
Maximum demand in units	500	300*	1,800

(i) **Statement of the most profitable product mixat each level of machine hours**

(*Refer to working note*)

Product (mix)	**Machine hours**			
	3,600	**6,000**	**8,400**	**10,800**
C	1,200units *(3,600 hrs/ 3 hrsp.u.)*	1,800units *(5,400 hrs/ 3 hrsp.u.)*	1,800units	1,800units
B	–	100units *(600 hrs/6 hrsp.u.)*	300units *(1800 hrs/6 hrsp.u.)*	300units
A	–	–	100units *(1,200 hrs/ 12 hrsp.u.)*	300units *(3,600 hrs/ 12 hrsp.u.)*

(ii) **Statement of level of activity wherethe profit would be maximum**

[*Refer to the answer of part (i)*]

	Products								
Level of activity (Machine hours)	**C units**	**Contribution* (₹)**	**B units**	**Contribution* (₹)**	**A units**	**Contribution* (₹)**	**Total Contribution (₹)**	**Fixed Cost (₹)**	**Net Profit (₹)**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7) = (2) + (4) + (6)**	**(8)**	**(7)–(8)**
3,600	1,200	2,88,000 *(1200 × ₹ 240)*	–	–	–	–	2,88,000	1,00,000	1,88,000
6,000	1,800	4,32,000 *(1,800 × ₹ 240)*	100	24,000 *(100 × ₹ 240)*	–	–	4,56,000	1,50,000	3,06,000
8,400	1,800	4,32,000	300	72,000	100	36,000	5,40,000	2,20,000	3,20,000

10,800	1,800	4,32,000	300	*(300 × ₹ 240)* 72,000	300	*(100 × ₹ 360)* 1,08,000 *(300 × ₹ 360)*	6,12,000	3,00,000	3,12,000

Recommandation :

* At 8,400 machine hour level of capacity the company would earn maximum profit i.e. ₹ 3,20,000

* Refer to working note.

2.17 Price-Mix Decision

When a firm can produce two or more products from the same production facilities and the demand of each product is affected by the change in their prices, the management may have to choose price mix which will give the maximum profit, particularly when the production capacity is limited.

In such a situation, the firm should compute all the possible combinations and select a price-mix which yields the maximum profitability.

Illustration44

Sellaway Ltd. manufactures and markets 2 products A and B, the demand in the market of which fluctuates with the prices quoted. As a result of the deliberations of its recent Sales Conference the following data were agreed upon as a working basis:

Product	***A***			***B***		
Selling price per unit (₹)	*32*	*30*	*28*	*22*	*20*	*18*
Expected demand per month (Nos.)	*900*	*1,000*	*1,500*	*1,600*	*2,000*	*3,000*

8 labour hours are required to produce product A and 4 labour hours to produce product B and the maximum capacity of the factory is restricted to 20,000 labour hours per month.

The cost structure per unit of production is as under:

Product	***A (₹)***	***B (₹)***
Direct material	*4*	*3*
Direct labour	*6*	*5*
Variable overheads	*10*	*6*
	20	*14*

Fixed overheads are ₹32,400 per quarter.

You are required to compute the possible combinations and arrive at a proper price mix for maximum profitability.

Solution.

Workings :

Product	**A**			**B**		
Selling price per unit (₹)	32	30	28	22	20	18
Expected demand per month (Nos.)	900	1,000	1,500	1,600	2,000	3,000
Total labour hours required	7,200	8,000	12,000	6,400	8,000	12,000
Variable cost per unit (₹)	20	20	20	14	14	14
Contribution per unit (₹)	12	10	8	8	6	4
Contribution per hour (₹)	1.5	1.25	1	2	1.5	1
Total contribution (₹)	10,800	10,000	12,000	12,800	12,000	12,000

Possible combinations

Products		***Contribution ₹***	***Labour Hrs. Reqd.***
A	***B***		
32	22	23,600	13,600
32	20	22,800	15,200
32	18	22,800	19,200
30	22	22,800	14,400
30	20	22,000	16,000
30	18	22,000	20,000
28	22	24,800	18,400
28	20	24,000	20,000
28	18	24,000	24,000

Recommendation:

The above computations show that the maximum contribution of ₹ 24,800 is possible at 18,400 labour hours. Therefore, profitable *price mix* is A ₹ 28 and B ₹ 22.

Summary

- For the purpose of decision making and control, costs are distinguished on the basis of their relevance to the different type of decisions.
- ***Relevant Costs.*:** These are the estimated future costs that are different under alternative courses of action for a specific problem and hence are appropriate to a specific management decision.
- **Conditions to be satisfied for relevant costs :-**

- ✓ **Occur in the Future**-every decision deals with selecting a course of action based on its expected future results.
- ✓ **Differ among the alternative courses of action-** costs and revenues that do not differ will not matter and will have no bearing on the decision being made.

- ***Differential cost:*** Difference in total cost that will arise from the selection of one alternative instead of another.
- ***Opportunity Cost:*** Value of the next best alternative.
- ***Sunk Cost:*** Cost which do not change under given circumstance and do not play any role in decision making process.
- ***Out of Pocket Costs :*** Portion of costs which involves payments to outsiders i.e., it gives rise to cash expenditure as opposed to other costs such as depreciation, which do not involve any cash expenditure.
- The investment decisions involve current outlays in return for a stream of benefits in future year.
- **Shut Down or Continue Decision**

  A firm to temporarily close down the factory due to trade recession with a view to reopening it in the future. In such cases, the decision should be based on the marginal cost analysis.

  $$\text{Shut down point} = \frac{\text{Total fixed cost} - \text{Shut down costs}}{\text{Contribution per unit}}$$

- The cut off rate is defined as the minimum rate of return expected from the investment.
- Cost-Volume-Profit analysis explores the relationship existing amongst costs, volume and profit.
- **Pricing Decisions under Special Circumstances**

  If goods were sold in the normal circumstances under normal business conditions, the price would cover the total cost plus a margin of profit.

  The problem of pricing can be summarised under three heads:

  - ✓ Pricing in periods of recession,
  - ✓ Differential selling prices, and
  - ✓ Acceptance of an offer and submission of a tender.

  Pricing in periods of recession: Sell its articles at a price *less than the total cost but above the marginal cost* for a limited period.

  Differential selling prices: Which is above, the marginal cost but below the total cost is resorted to in order to absorb surplus capacity.

- **Make or Buy Decision**

  Very often management is faced with the problem as to whether a part should be manufactured or it should be purchased from outside market. Under such circumstances two factors are to be considered:

  ✓ Whether surplus capacity is available, and

  ✓ The marginal cost.

- **Export vs Local Sale Decision**

  When the firm is catering to the needs of the local market and surplus capacity is still available, it may think of utilising the same to meet export orders at price lower than that prevailing in the local market.

- **Price-Mix Decision**

  When a firm can produce two or more products from the same production facilities and the demand of each product is affected by the change in their prices, the management may have to choose price mix which will give the maximum profit, particularly when the production capacity is limited.

3

Pricing Decisions

LEARNING OBJECTIVES

After studying this chapter you will be able to understand :

- The importance of pricing decision in achieving the organizational goal and long term survival.
- Theory of price, economic theory of pricing and pricing in different market structures.
- Pricing policy to achieve the objectives and overall organisational goal.
- Different principles governing the pricing of a product or services.
- How to price a new product with different characteristics.
- Pricing strategies to be adopted to market the products successfully.
- Appreciate the different strategies of pricing followed by the company and their objectives.
- The importance of products which generate substantial revenue with the help of Pareto Analysis.

3.1 Introduction

A pricing decision is one of the most crucial & difficult decision that a firm has to make. Such a decision affects the long term survival of any profit oriented enterprise.

Accounting information is often an important input to pricing decisions. Most firms need to make decision about setting or accepting selling prices for their products or services. In some firms selling price is derived directly from cost information by estimating future product's cost & adding a suitable profit margin. In others an established market price is accepted.

Generally, pricing decisions are influenced by the pricing policy followed by an organisation. Pricing policies are made taking overall objectives of an organisation into account. Thus, before fixing price of a product, objectives of the organisation must be understood first to achieve the organisation's goal. Objective of an organisation may be either to maximise the profit or maximise the sales or maximise the output or optimal utilisation of resources etc.

In this chapter we will learn how pricing is done in different market structures, pricing principles, pricing policies and pricing strategies and Pareto analysis.

3.2 Theory of Price

The basic approach in most of the micro-economic theory (theory of the individual firm and its relation to other firms) defines the term optimum price as that price which yields the maximum profits (excess of total revenues over total costs). Thus the basic assumption of the pricing theory is that the firm's main objective is to maximise its profits. It also assumes that the firm takes into consideration the position of demand and cost functions and that the firm produces one product.

If a firm sells unlimited number of units, the total revenue line will be a straight line arrived at by TR= *mx.*

Where,

TR = Total revenue line

m = quantity of units sold

x = price per unit.

In most of the market situations, however, additional units can be sold by reducing the price. This means that although the total sales revenue will increase as more and more units are sold, the increase in total revenue will decline gradually as sales increases. Consider the following example:

Example- 1:

A firm's pricing of a product is as under:

20 units @ ₹4.00 per unit.
21 units @ ₹3.90 per unit.
22 units @ ₹3.80 per unit.

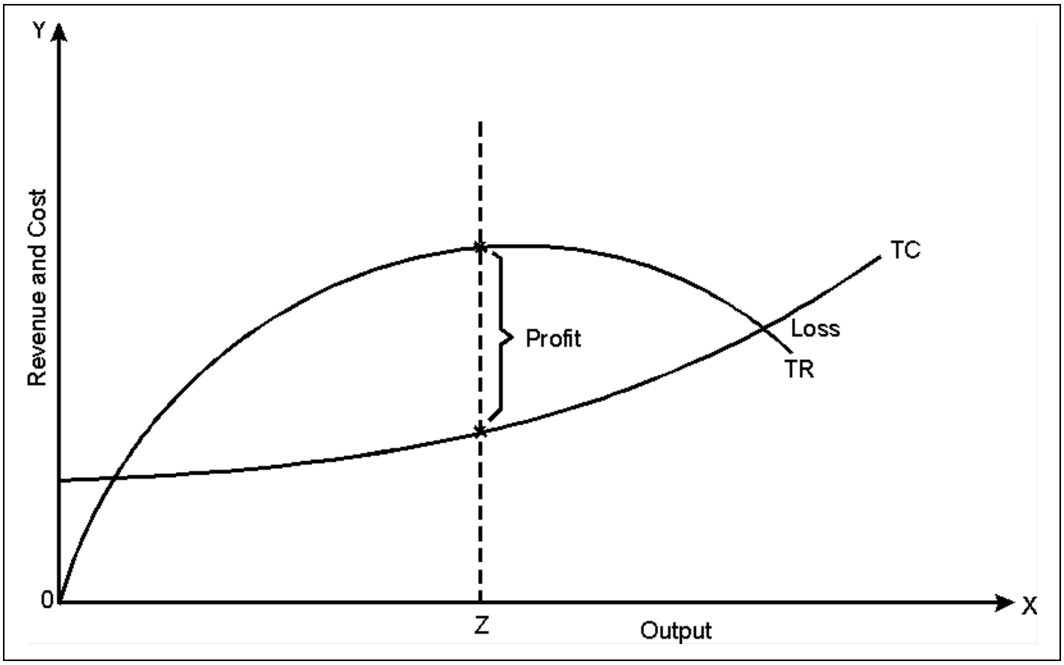
The sales figures can be summarised as under:

Quantity	Price (₹)	Total sales revenue (₹)	Addition to total revenue (₹)
20	4.00	80.00	—
21	3.90	81.90	1.90
22	3.80	83.60	1.70

The reduction in the price of each additional unit reflects a gradual reduction in the steepness of the total revenue curve as shown in the diagram given on next page. The total cost curve will however, register an increase in the steepness because as the volume increases, the cost also increases because of the difficulty of expanding output with a given productive resources.

The slope of the total revenue and total cost curves due to the addition of one unit will be equal to the increase in total revenue. This is the point where there will be no profit increase due to increase of one unit of output. In the figure [Fig. No. (1)], the situation has been depicted at point Z, where the gap between the total cost line and total revenue is the maximum, thus Z is the point of optimum volume. Any attempt to increase the volume beyond this point will reduce the profit because the incremental cost will be more than the incremental revenue.

These relations are expressed in terms of marginal revenue and marginal cost. Marginal revenue is the increase in total revenue that results from the sale of one additional unit. In the example given above, the marginal revenue of increasing one unit from 20 units to 21 units is ₹1.90. Marginal cost is the increase in total cost that results from the production of one additional unit.



3.2.1 Pricing Model: Pricing model is a mathematical model which uses economic theory of pricing.

(i) As per economic theory of pricing profit is maximum at a level of output where marginal revenue (MR) is equal to marginal cost (MC) i.e.

Marginal Revenue (MR) = Marginal Cost (MC)

This model determines the level of production upto which production can be continued.

(ii) The Basic price equation, which is used to determine the price where profit is maximum. The equation is written as

P = a – bQ

Where, P = Price

b = Slope of the demand curve, calculated as $\left[b = \frac{(\text{Change in price})}{(\text{Change in quantity})} \right]$

Q = Quantity demanded

a = Price at which demand is zero.

(iii) The marginal revenue equation is written as

Marginal Revenue (MR) = P = a – 2bQ

Example- 2:

Aditya Heavy Engineering Ltd. (AHEL) produces its only product A_7. To manufacture a unit of A_7 a variable cost of ₹2,20,000 is incurred. Market research has indicated that at a selling price of ₹ 5,10,000 no order will be received, but the demand for A_7 will be increased by two units with every ₹5,000 reduction in the unit selling price below ₹5,10,000.

Determine the unit selling price for A_7 that will maximize the profit of AHEL.

Answer:

We assume that

Selling price per unit of A_7 is 'P', and Quantity demanded is 'Q'

The Marginal Cost of a unit of A_7 is ₹2,20,000

Price equation for 'A_7'

P	=	a – bQ
P	=	₹5,10,000 – (5,000 / 2) × Q
Revenue (R)	=	Q x [5,10,000 – 2,500 × Q]
	=	5,10,000 Q – 2,500 Q^2
Marginal Revenue (MR)	=	a – 2bQ
	=	₹5,10,000 – 2 × (5,000 / 2) × Q
	=	₹5,10,000 – 5,000 Q
Marginal Cost (MC)	=	₹ 2,20,000

Profit is Maximum where Marginal Revenue (MR) equals to Marginal Cost (MC)

₹5,10,000 – 5,000 Q	=	₹2,20,000
Q	=	58 units

By putting the value of 'Q' in *Price equation*, value of 'P' is obtained.

P	=	₹5,10,000 – (5,000/ 2) × Q
	=	₹5,10,000 – 2,500 × 58 units
	=	₹3,65,000

At selling price of ₹3,65,000 AHEL's profit will be maximum.

3.2.2 Pricing under different market structures: The determination of optimal price can be considered under the following market structures:

Types of market and price determination

Perfect competition | **Monopoly** | **Imperfect competition**

- **Monopolistic**
- **Oligopoly**

(a) Perfect competition: Under perfect competitive market, there are **large numbers of sellers** selling a **homogeneous product** using identical production process and all of them **have perfect information** about the market and price. Perfect market allows **free entry and exit** of firms into and out of the industry.

Under this type of market, firm has no pricing policy of its own as the **sellers are price takers** (i.e. it has to accept the price determined by the market) and sell as much as they are capable of selling at the prevailing market price. Since each firm produces and sells a homogeneous product, it cannot increase its price beyond the market price. If it does so then it has to lose all of its market demand to the competitors.

There is no control over market price which will equate the quantities available with the quantities which the buyers are willing to buy. The firm has to take a decision in favour of the quantity to sell. The firm can continue to produce so long as its marginal cost is less than or equal to its selling price, upto the point at which the marginal cost is equal to price, increase in output will add to revenue and thereafter the increase will add to cost. It can be seen in following example:

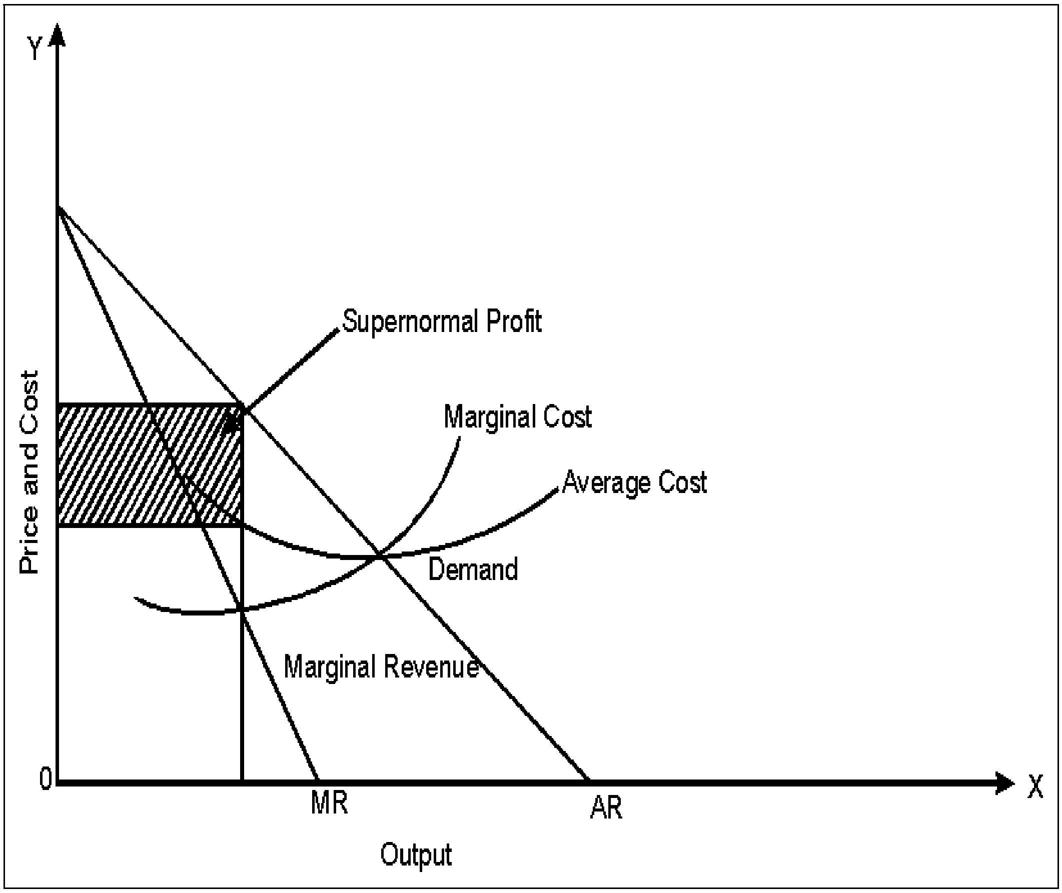
Example- 3:

Aditya LLP produces a product- X, the market for the product X is competitive and the prevailing market price for a unit of product-X is ₹40. The following table presents the marginal cost and profit for the product-X:

Units	Total revenue (₹)	Total cost (₹)	Marginal cost (₹)	Profit (₹)
0	0	20	-	(20)
1	40	30	10	10
2	80	50	20	30
3	120	85	35	35
4	**160**	**125**	**40**	**35**
5	200	170	45	30
6	240	217	47	23

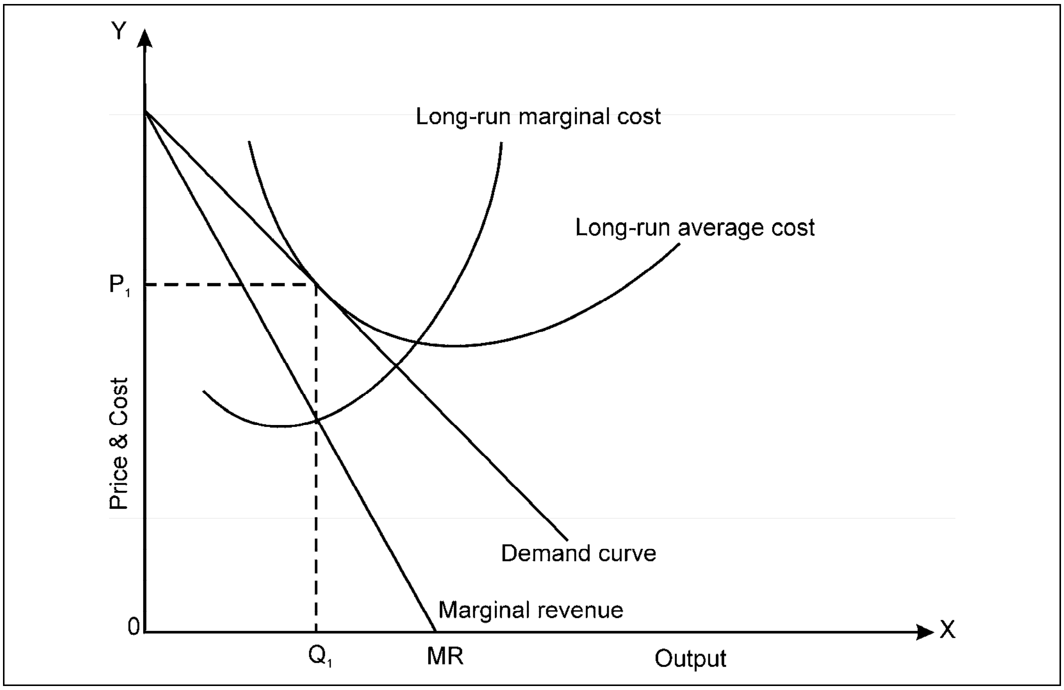
The marginal cost for producing 4^{th} unit is equal to the price per unit. Thus, Aditya LLP can maximize its profit at 4 unit level.

(b) Monopoly: Monopoly is a market condition where there is only **one supplier** or producer of a **homogeneous product** for which there is **no close substitute** but has **many buyers**. Under the monopoly **a firm is a price setter** i.e. it can fix any price but here also the pricing is done taking elasticity of demand for the product into consideration. That means though the seller/ producer can fix any price but it will go for the price where demand for the product and consequent profit will be maximum.



(c) Monopolistic competition: The monopolistically competitive market is one in which there are **large number** of **firms producing similar but not identical products**. Since there is limit to the growth of competitors the excess profits earned by monopolistic situation attracts new competition. This will have a long-run effect on the excess profits which will tend to diminish because of the price competition with close substitutes. The company will, however, have to compare marginal cost and marginal revenue in maximising its profits.

Under monopolistic condition, consumers may buy more at a lower price than at higher price. The profit can be maximised by equating marginal revenue with marginal cost. This cost be seen from the following diagram.



The point at which MR and MC curves meet will determine the price level. So the price will be P_1 and output to be manufactured will be Q_1. The firm may both fix output Q_1 and leave the price to be fixed at P_1 or *vice versa.* There is, however, no protection for the existing firms from the entry of substitute firms in the market.

(d) Oligopoly: A market structure where there are **few firms** producing or selling **homogenous or identical product**. In this type of market structure the firms are aware of the mutual interdependence of investment, production process, advertising and sales plan of its rival firm. Hence, any change in any variable by a firm is likely to have an equal reaction on the part of other competing firms. It is therefore, clear that the oligopolistic firm, while determining the price for its product, consider not only the demand for the product but also the reactions of the other firms in the industry to any action or decision it may take.

If a firm does not follow or adapt its pricing policy in consonance with its competitor, the shift in the sales will be sensitive. That means demand will shift towards the lower price. Thus each firm will study the potential reaction before increasing or decreasing the selling price. The firms in oligopolistic market maintain the price of the product either by close analysis of each other's behavior or by means of cooperation and collusion.

3.3 Pricing Policy

The pricing policy plays an important role in a business because the long run survival of a business depends upon the firm's ability to increase its sales and derive the maximum profit from the existing and new capital investment. Although cost is an important aspect of pricing, consumer demand and competitive environment are frequently far more significant in pricing decisions. These are also known as determinants of pricing or market powers. Thus costs alone do not determine price. Cost is only one of the many complex factors which determine prices. There must however, be some margin in prices over total cost if capital is to be unimpaired and production maximised by the utilisation of internal surplus.

The pricing policy and the relative price structure should:

(a) Provide an incentive to producer for adopting improved technology and maximising production;

(b) Encourage optimum utilisation of resources;

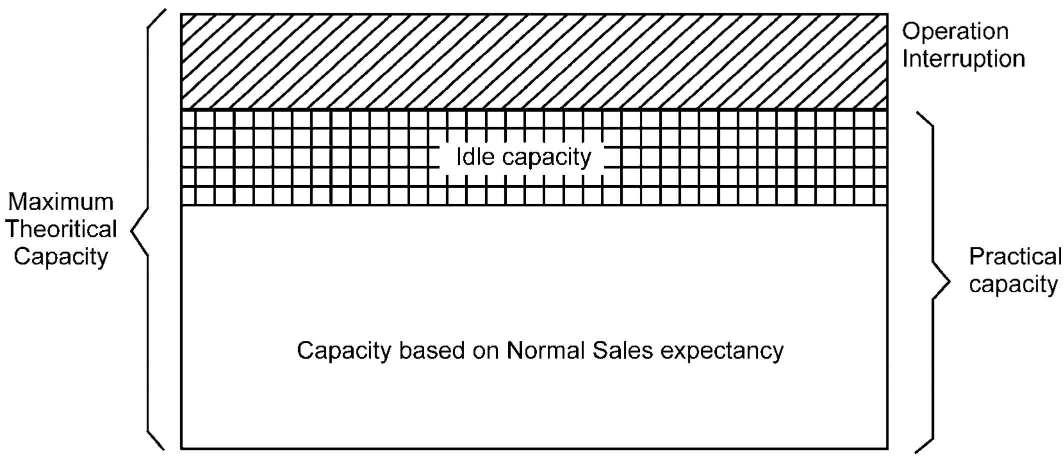
(c) Work towards better balance between demand and supply;

(d) Promote exports; and

(e) Avoid adverse effects on the rest of the economy.

An individual manufacturer may take his cost of production into account and arrive at a price at which the products are to be sold in the concerned region. A manufacturer having several factories all over the country may determine the weighted average cost of each of the factories and include the same in his computations so as to arrive at a uniform price for the country as a whole, e.g., if prices are to be fixed by a statutory authority, like the Tariff Commissions etc. Weighted average price is also taken into account if large numbers of factories are owned by one manufacturer.

The price may also be fixed after taking into account the cost of a representative unit which may fall within the range of lowest cost unit and the highest cost unit. Alternatively, the factories may be classified into (i) small size factories, (ii) medium size factories, and (iii) large size factories. The cost of medium size factories can be taken into account if this group forms the greater part of the industry. Where, however, the units in an industry are very large as in the case of textile industry for example, some representative sample has to be taken. The sample should be of economic size and also be of representative of the conditions of the different regions. The demand of the product and the cost of the marginal unit may have to be taken into account in fixing the price so that the marginal units are not driven out of the market.



In order to frame a price policy, one of the elements that should receive consideration is the determination of normal capacity. Normal capacity is the utilisation of plant that is necessary to meet the average commercial demand over a period of time, long enough to level out peaks which come with seasonal and cyclical variations. The following chart illustrates the major relationships involved. Price determination should normally be based on the level of production and capacity utilisation likely to be achieved. Any assumption of low utilisation may result in over estimating the cost. Conversely, a high utilisation assumption may result in under estimating the cost. It is, therefore, desirable that the level of production and capacity utilisation which are likely to apply, say in the next three years should be arrived at with utmost care on realistic basis keeping in view not only the past performance but also the future demand. A uniform system of costing should also be devised and introduced in each industry for the fixation of price.

3.4 Principles of Product Pricing

Creating value for the customers is one of the important objectives of a firm. A firm makes all the efforts to create value and to achieve this it formulate its marketing strategy in that direction. Understanding customers' wants and needs is foundation for building this value. To create value, a firm makes the following marketing strategies:

(i) First it develops a product that satisfy the wants and needs of the customers,

(ii) After identification and development, it designs a promotion program to convey the value of the products to the customers.

(iii) It chooses the right distribution channel through which its product will reach to the customers

(iv) At last it has to design a pricing strategy that creates incentive to purchaser to buy the product and to seller to sell the product.

Pricing of a product plays the role in marketing strategy to tap into the value created and to generate revenues. So the biggest concern on the part of any organisation is to getting the right price. To arrive at a right price, the following important points to be kept in the mind.

3.4.1Value-pricing approach: Before discussing this approach, few terms used here is to be understood first.

(i) Objective value or True economic value (TEV): This is a measure of benefits that a product is intended to deliver to the consumers relative to the other products without giving any regard whether the consumer can recognize these benefits or not.

True economic value for a consumer is calculated taking two differentials into consideration:

TEV = Cost of the next best alternative + Value of performance differential

Cost of the next best alternative is the cost of a comparable product offered by some other company. Value of performance differential is the value of additional features provided by the seller of a product.

Example- 4:

Mr. Aditya went for a holiday vacation with his family consists of his wife and a three years old son, to Munnar in Kerala through Deccan Tour Ltd. a tour & travel agency which took ₹ 15,000 for the tour. The feature of the holiday package was as follows:

Number of days:	3 days and 2 nights
Stay:	At 3 star hotel
Food & beverages:	Breakfast and Dinner
Sightseeing:	By Indica car with A/c

There is an another travel agency Munnar Crocks Pvt. Ltd. which provides the same tour package with almost the same features with an additional airport pickup and drop facility at ₹ 14,500 only. Airport pickup and drop facility costs ₹1,600.

In the above example, if we have to calculate True economic value (TEV) of the package offered by Munnar Crocks Pvt. Ltd. then it will be

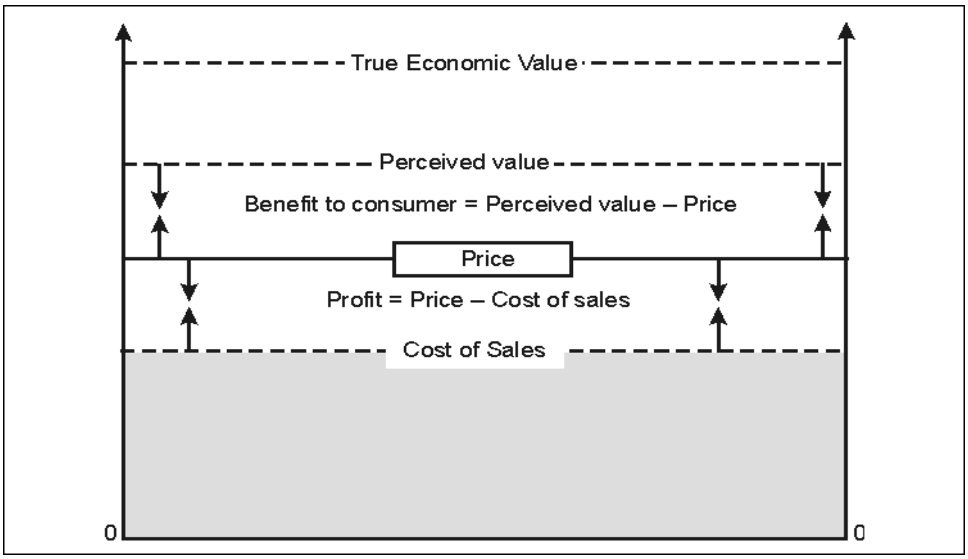
Cost of the next best alternative + Value of performance differential

In this example, the package cost taken by Deccan Tours Ltd. is the cost of the best alternative. Munnar Crocks Pvt. Ltd. has a product differential that is airport pickup and drop facility. Putting these into the formula we can get the TEV of the package.

TEV of the package = ₹15,000 + ₹1,600 = ₹ 16,600

(ii) Perceived value: This is the value that consumer understands the product deliver to it. It is the price of a product that a consumer is willing to spend to have that product.

At the time of fixing price it is to be kept in the mind that any price which set below the perceived value but above the cost of goods sold give incentives to both buyers and the seller. This can be understood with the help the diagram given below.



3.4.2 Price sensitivity: It measures the customer's behaviour to the change in price of a product. The customer behaviour towards the changes in the price is influenced by the following factors:

(i) The magnitude of price: Price sensitivity tends to be far greater in high cost than lowest cost product categories. For example a 10% change in the price of diamond is more sensitive than the price of toothpaste.

(ii) Cost bearer: Price sensitivity for a product is relatively lower where the user is different from the buyer of the product. For example, in a charitable health centre the beneficiary (i.e. patients) are different from the administrator, hence, users are less price sensitive towards the cost of service.

(iii) Competitors reaction on pricing decision: When price of any product changes without reciprocate change on side of competitors, results in large price sensitivity. In a perfect competitive market lower price attracts more demands. But in case of oligopoly market condition, price sensitivity is neutralised by simultaneous reaction from the competitors in the same direction. For example, when a soft drink company lowers its price, a reciprocal downward change is also seen from the other competitive soft drink company. This reaction neutralise the price sensitivity.

3.4.3 Price customisation: Pricing of a product is some time customised keeping taste, preference and perceived value of a consumer into consideration. Price customisation is done in various ways like:

(i) Based on product line: Based on the requirement of the consumer products can be customized and accordingly the prices. For example, some may like to have a smartphone with 16 GB over 32 GB. In this case pricing for the product can be based on memory specification.

(ii) Based on customers' past behaviour: A customer with good payment record may be given more discounts then the others.

(iii) Based on demographics: Different pricing may be adopted based on age or social status. For example railway fare concession for senior citizen and concessional price tickets for military personnel.

(iv) Based on time differential: Pricing for a product or service is also done on the basis of time differential i.e. different price for different time period. For example, discounted price for data usage provided by a broadband service provider if subscription paid for six months at a time.

Apart from above pricing principles, other macro economic and legal factors should also be given due importance while chalking out a pricing strategies.

3.5 New Product Pricing

The pricing of new product poses a bigger problem because of the uncertainty involved in the estimation of their demand. In order to overcome this difficulty experimental sales are conducted in different markets using different prices to see which price is suitable. A company may, for example, choose three different markets and by using the same amount of sales promotional activities, ascertain what the right price is. In such circumstances, it may even prove that the highest price yielding the largest unit contributory margin need not necessarily maximise the profits. A lower price may well go to maximise the profits. But at the same time if a product is priced very low to attract more demand, it may be difficult in the future to raise the price as it may not be acceptable to the consumers. So pricing of a new product is very critical issue which should be decided after a thorough market study and consumer behavior analysis.

A new product is analysed into three categories for the purpose of pricing.

(i) Revolutionary Product: A product is said to be revolutionary when it is new for the market and has the potential to create its own value. This type of product has revolutionary impact on the market and consumer behaviour. It replaces the existing method or technology and the approach to doing a work is quite different and unique. These products enjoy the benefit of product differentials and have the potential of being market leader.

Revolutionary product may enjoy the premium price as a reward for its innovation and taking first initiative.

(ii) Evolutionary Product: A product introduces upgraded version with few additional characteristics of the product is known as evolutionary product.

The evolutionary products may be priced taking cost-benefit, competitor and demand for the product into account.

(iii) Me-too Product: A product is said to be me-too product when its emergence is a result of the success of a revolutionary product. These type of products are very similar (in ordinary language imitation) to revolutionary and/ or evolutionary products' of other firms. The firm while producing me-too products, generally follows the similar production process and technology that is used by the other firms. These are known as market followers.

The me-too products are price takers as the price is determined by the market mainly by the competitive forces.

Illustration 1

Novice Ltd. is about to introduce a new product with the following estimates:

Price per unit (in ₹)	*Demand (in thousand units)*
30.00	*400*
31.50	*380*
33.00	*360*
34.50	*340*
36.00	*315*
37.50	*280*
39.00	*240*

Costs:

Direct material	*₹ 12 per unit*
Direct labour	*₹ 3 per unit*

Variable overhead	*₹ 3 per unit*
Selling expenses	*10% on sales*
Fixed production overheads	*₹ 14,40,000*
Administration expenses	*₹ 10,80,000*

Judging from the estimates, determine the tentative price of the new product to earn maximum profit.

Solution

Novice Ltd.
Statement for determining tentative price of the new product, from estimates, to earn maximum profit

Price per unit (₹)	Demand (in lakhs of unit)	Sales revenue (in ₹ lakhs)	Variable costs (in ₹ lakhs) (₹ 18* p.u. + 10% of selling price)	Contribution (in ₹ lakhs)
(a)	(b)	(c) = (a) × (b)	(d)	(e) = (c) – (d)
30.00	4.00	120.00	84.00	36.00
31.50	3.80	119.70	80.37	39.33
33.00	3.60	118.80	76.68	42.12
34.50	3.40	117.30	72.93	44.37
36.00	3.15	113.40	68.04	45.36
37.50	2.80	105.00	60.90	44.10
39.00	2.40	93.60	52.56	41.04

[*DM- ₹12 + DL- ₹3 + VO- ₹3 = ₹18]

The tentative price of the new product should be ₹ 36 per unit. At this price the contribution of Novice Ltd. is maximum and maximum profit of the concern comes to ₹ 20,16,000 *(Refer to working note).*

Working note:

Maximum profit

= Maximum contribution – (Fixed production overheads + Administration expenses)

= ₹ 45,36,000 – (₹ 14,40,000 + ₹ 10,80,000) = ₹ 20,16,000

3.6 Pricing of Finished Product

3.6.1 Cost plus pricing: In many businesses the common method of price determining is to estimate the cost of product & fix a margin of profit. The term 'cost' here means full cost at current output and wages level since these are regarded as most relevant in price determination. In arriving at cost of production, it is necessary to determine the size of the unit whose products are to be priced.

If a firm wants to survive and stay in business, it has to maintain its fixed capital intact so that its fixed assets may be replaced at the end of their useful working life out of the funds generated from profits retained in the business. In a period of relatively stable price levels, depreciation based on historical cost of fixed assets would perhaps be adequate for achieving this object. In periods when the price level is continuously changing, the firm may not be left with adequate funds generated out of accumulated depreciation at the end of the life of the plant to replace the plant at a higher price. Hence depreciation should be properly included as a part of cost so as to leave sufficient profits for asset replacement.

Advantages:

1. ***Fair method***: It is a fair method of price fixation. The business executives are convinced that the price fixed will cover the cost.
2. ***Assured profit***: If price is greater than cost, the risk is covered. This is true when normal expected capacity basis of cost estimation is used.
3. ***Reduced risks and uncertainties***: A decision maker has to take decisions in the face of many uncertainties. He may accept a pricing formula that seems reasonable for reducing uncertainty.
4. ***Considers market factors***: This sort of pricing does not mean that market forces are ignored. The mark up added to the cost to make a price reflect the well established customs of trade, which guide the price fixer towards a competitive price.

Disadvantages:

1. ***Ignores demand***: It ignores demand. It f ails to take into account the buyers' needs and willingness to pay which govern the sales volume obtainable at each series of prices.
2. ***Ignores competition***: It fails to reflect competition adequately.
3. ***Arbitrary cost allocation*****:** It takes for granted that the costs have been estimated with accuracy which is not often true particularly in multi-product firms because the common costs are allocated arbitrarily.
4. ***Ignores opportunity cost*****:** For many decisions incremental costs rather than full costs play a vital role in pricing. This aspect is ignored.
5. ***Price-volume relationships***: Since the fixed overheads are apportioned on the basis of volume of production, the cost will be more if a sales volume is less and cost will be less if sales volume is more. The increase or decrease in sales volume again is dependent on price. Thus it is a vicious circle–cost plus mark up is price based on sales volume and sales volume is based on price.

Illustration 2

Rational Ltd. produces 3,00,000 kgs. of S and 6,00,000 kgs. of Y from an input of 9,00,000 kgs. of raw material- Z simultaneously The selling price of S is ₹ 8 per kg and that of Y is ₹ 6 per kg.

Processing costs amount to ₹ 54 lakhs per month as under:

Raw material- Z- 9,00,000 kgs. at ₹ 3 per kg	*₹ 27,00,000*
Variable processing costs	*₹ 18,00,000*
Fixed processing costs	*₹ 9,00,000*
Total	*₹ 54,00,000*

There is an offer to purchase 60,000 kgs of Y additionally at a price of ₹4 per kg. The existing market for Y will not be affected by accepting the offer. But the price of S is likely to be decreased uniformly on all sales.

Find the minimum reduced average price for S to sustain the increased sales.

Solution

Since, S and Y are produced simultaneously in the ratio of 1:2 from an input of material- Z, therefore when additional 60,000 kgs of Y will be produced then 30,000 kgs. of S will also be produced.

The input of material- Z required for these additional 60,000 kgs of Y and 30,000 kgs. of S will be 90,000 kgs. Hence, the cost of processing 90,000 kgs. of material- Z will be as follows :

	(₹)
Cost of Raw material- Z (90,000 kgs × ₹ 3)	2,70,000
Variable processing cost (90,000 kgs × ₹ 2)	1,80,000
Total cost of processing	4,50,000
Less: Sales revenue from 60,000 kgs. of Y (60,000 kgs × ₹ 4)	2,40,000
Balance cost to be recovered	2,10,000
Current sales revenue from the sale of 3,00,000 kgs. of S (3,00,000 kgs.× ₹ 8)	24,00,000
Total sales revenue to be earned from the sale of S (existing- 3,00,000 kgs. + additional- 30,000 kgs.)	26,10,000
Hence minimum price per kg of S to recover ₹26,10,000 from the sale of 3,30,000 kgs. of S $\left(\frac{₹\,26,10,000}{3,30,000\text{ kgs.}}\right)$	7.91

3.6.2 Rate of Return Pricing: Determination of return on capital employed is one of the most crucial aspects of price fixation process. In this process instead of arbitrarily adding a percentage on cost for profit, the firm determines an average mark up on cost necessary to produce a desired rate of return on its investment. Under this method three issues arise:

(a) The basis on which the capital employed is computed.

(b) Which items should be covered in the return on capital?

(c) What rate of return can be regarded as fair?

The rate of return to be earned by the firm or industry must depend on the risk involved. The desirability of earning adequate profits for the purpose of ploughing back into business should be kept in mind.

It would be correct to assume that allowing the industry to earn adequate return on the capital employed would attract additional capital and increase the number of factories and production of all commodities which must ultimately lead to competition and reduction in costs and prices.

Illustration 3

Electromatic Excellers Ltd. specialises in the manufacture of novel transistors. They have recently developed a technology to design a new radio transistor capable of being used as an emergency lamp also. They are quite confident of selling all of the 8,000 units that they would be making in a year. The capital equipment that would be required will cost ₹ 25 lakhs. It will have an economic life of 4 years and no significant terminal salvage value.

During each of the first four years promotional expenses are planned as under:

Year	*1*	*2*	*3*	*4*
Advertisement (₹)	*1,00,000*	*75,000*	*60,000*	*30,000*
Other expenses (₹)	*50,000*	*75,000*	*90,000*	*1,20,000*

Variable costs of producing and selling the unit would be ₹ 250 per unit.

Additional fixed operating costs incurred because of this new product are budgeted at ₹ 75,000 per year.

The company's profit goals call for a discounted rate of return of 15% after taxes on investments on new products. The income tax rate on an average works out to 40%. You can assume that the straight line method of depreciation will be used for tax and reporting.

Work out an initial selling price per unit of the product that may be fixed for obtaining the desired rate of return on investment.

Present value of annuity of ₹ 1 received or paid in a steady stream throughout 4 years in the future at 15% is 2.855.

Solution: Determination of initial selling price

Let the selling price be ₹ X; Sales value: ₹ 8,000 X

Annual cash costs are:

	(₹)
Variable cost : 8,000 units × ₹ 250	20,00,000
Advertisement and other expenses	1,50,000
Additional fixed costs	75,000
Total cash cost	22,25,000

Depreciation per annum = $\left(\frac{₹\ 25,00,000}{4}\right)$ = ₹ 6,25,000

Profit for taxation: 8,000 × ₹ X – (₹ 22,25,000 + ₹ 6,25,000) = ₹ 8,000 X – ₹ 28,50,000

Tax at 40% on profit: 40% of {₹ 8,000 X – ₹ 28,50,000} = ₹ 3,200 X – ₹ 11,40,000

Total annual cash outflow: ₹22,25,000 + (₹ 3,200 X – ₹ 11,40,000) = ₹ 3,200 X + ₹ 10,85,000

Net annual cash inflow: ₹ 8,000 X – (₹ 3,200 X + ₹ 10,85,000) = ₹ 4,800 X – ₹ 10,85,000

Now, present value of initial cash outflow = Present value of cash inflow

or, ₹ 25,00,000 = (₹ 4,800 X – ₹ 10,85,000) × 2.855

or, ₹ 13,704 X = ₹ 30,97,675 + ₹ 25,00,000

or, X = ₹ 408.47

Hence selling price should be ₹ 408.47 per unit.

Alternative Solution

	(₹)
Total variable costs per year (8,000 × ₹ 250)	20,00,000
Promotional costs per year	1,50,000
Fixed operating costs per year	75,000
	22,25,000
Less: Income tax 40% (tax shield)	(8,90,000)
	13,35,000
Less: Tax saving on depreciation $\left(\frac{₹\ 25,00,000}{4} \times 40\%\right)$	(2,50,000)
Net annual cash outflow	10,85,000
PV factor for 4 years @15%: is 2.855	
Therefore, present value of annual cash outflow (₹10,85,000 × 2.855):	30,97,675
Initial investment	25,00,000
Present value of total outlay	55,97,675
Divide the present value of total outlay by PV factor to get required	
Annual revenue after tax (₹55,97,675 ÷ 2.855)	19,60,657
Required annual revenue before tax $\left(₹\ 19,60,657 \times \frac{100}{60}\right)$:	32,67,761
Unit selling price $\left(\frac{₹\ 32,67,761}{8,000 \text{ units}}\right)$:	408.47

3.6.3 Variable costs pricing: We have seen a number of decisions based on variable or marginal costing principle in the last chapter. Pricing based on total costs is subjected to two limitations. They are:

(a) The allocation of inter-departmental overheads is based on an arbitrary basis; and

(b) The allocation overheads will require estimation of normal output which often cannot be done precisely.

In order to avoid these complications, variable costs which are considered as relevant costs are used for pricing, by adding a mark up to include fixed costs allocation also.

Illustration 4

A small scale manufacturer produces an article at the operated capacity of 10,000 units while the normal capacity of his plant is 14,000 units. Working at a profit margin of 20% on sales realisation, he has formulated his budget as under:

Units	***10,000 (₹)***	***14,000 (₹)***
Sales realisation	*2,00,000*	*2,80,000*
Variable overheads	*50,000*	*70,000*
Semi-variable overheads	*20,000*	*22,000*
Fixed overheads	*40,000*	*40, 000*

He gets an order for a quantity equivalent to 20% of the operated capacity and even on this additional production profit margin is desired at the same percentage on sales realisation as for production to operated capacity.

Assuming prime cost is constant per unit of production, what should be the minimum price to realise this objective?

Solution

Working Notes:

(i) Computation of prime cost:

	(₹)	(₹)
Cost of sales (The profit margin is 20% on sales, therefore, cost of sales shall be 80% of ₹ 2,00,000)		1,60,000
Less: Variable overheads	50,000	
Semi-variable overheads	20,000	
Fixed overheads	40,000	(1,10,000)
Prime cost		50,000

(ii) Semi-variable overheads:

$$\text{Variable overhead per unit} = \frac{₹22{,}000 - ₹20{,}000}{14{,}000\,\text{units} - 10{,}000\,\text{units}} = ₹0.50$$

Fixed Overhead = ₹20,000 – (10,000 units × ₹0.50) = ₹15,000

Computation of differential cost of production of 2,000 additional units for determining minimum price

	Amount (₹)
Prime cost $\left(\frac{₹50{,}000}{10{,}000\,\text{units}} \times 2{,}000\,\text{units}\right)$	10,000
Variable overheads	10,000

$\left(\frac{₹50{,}000}{10{,}000\,\text{units}} \times 2{,}000\,\text{units}\right)$	
Semi-variable overheads (Variable part only) (₹0.50 × 2,000 units)	1,000
Differential cost	21,000

The minimum price to be quoted for the additional order

	Amount (₹)
Differential cost	21,000
Add: Profit margin (20% on sales realization i.e. 25% on cost) (25% of ₹ 21,000)	5,250
Total sale value	26,250
Price per unit $\left(\frac{₹26{,}250}{2{,}000\,\text{units}}\right)$	13.125

3.6.4 Competitive pricing : When a company sets its price mainly on the consideration of what its competitors are charging, its pricing policy under such a situation is called competitive pricing or competition-oriented pricing. It is not necessary under competitive pricing to charge the same price as charged by the concern's competitors. But under such a pricing the concern may keep its prices lower or higher than its competitors by a certain percentage. *Its own costs or demand may* change, but the concern maintains its price because its competitors maintain their prices. Conversely, the concern will change its price when its competitors change their price, even if its own costs or demand have not altered. Different types of competitive pricing in vogue are as follows:

(i) Going rate pricing

(ii) Sealed bid pricing

(i) *Going rate pricing*: It is a competitive pricing method under which a firm tries to keep its price at the average level charged by the industry. The use of such a practice of pricing is especially useful where it is difficult to measure costs. Adoption of going rate pricing will not only yield fair return but would be least disruptive for industry's harmony.

Going rate pricing primarily characterises pricing practice in homogeneous product markets, The concern selling a homogeneous product in a highly competitive market has actually very little choice about the setting of its price. There is apt to be a market determined price for the product, which is not established by any single firm or clique of firms but through the collective interaction of buyers and sellers. The concern which is going to charge more than the going rate would attract virtually no customers. The concern should not charge less because it can dispose of its entire output at the going rate. Thus, under highly competitive conditions in a homogeneous product market (such as food, raw materials and textiles) the concern really has no pricing decision to make. The major challenge before such a concern is good cost control.

Since promotion and personnel selling are not in the picture, the major marketing costs arise in physical distribution.

In pure oligopoly, where a few large concerns dominate the industry, the concern also tends to charge the same price as is being charged by its competitors. Since there are only a few concerns, each firm is quite aware of others' prices, and so are the buyers.

This does not mean that the going price in an oligopoly market will be in practice indefinitely. It cannot, since industry costs and demand change over time.

(ii) ***Sealed bid-pricing:*** Competitive pricing also dominates in those situations where firms compete on the basis of bids, such as original equipment manufacturer and defense contract work. The bid is the firms offer price, and it is a prime example of pricing based on expectations of how competitors will price rather than on a rigid relation based on the concern's own costs or demand. The objective of the firm in the bidding situation is to get the contract, and this means that it hopes to set its price lower than that set by any of the other bidding firms. But however the firm does not ordinarily set its price below a certain level. Even when it is anxious to get a contract in order to keep the plant busy, it cannot quote price below marginal cost. On the other hand, if it raises its price above marginal cost, it increases its potential profit but reduces its chance of getting the contract.

3.6.5 Incremental pricing: Incremental pricing is used because it involves comparison of the impact of decisions on revenues and cost. If a pricing decision results in a greater increase in revenue than in costs, it is favourable. Such a decision is not merely confined to comparison of revenues and costs. It also permits that consideration being given to other objectives of the business. Thus profitability can be set as the matter of primary consideration and then the decision can be adjusted to bring it in consonance with the other decision of the business.

The following points will be useful to show how this technique gives consideration to all repercussions of a decision.

1. ***Relevant cost analysis*****:** This technique considers changes in costs rather than in average cost. Overhead allocations are irrelevant.

2. ***Product-Line relationship analysis:*** This technique necessitates consideration being given to possible complementary relations in demand. Sale of one product may lead to the sale of a complementary product. This overall effect on profitability has to be evaluated.

3. ***Opportunity cost analysis:*** The opportunity costs should be covered by the incremental revenue. A price which results in an incremental revenue which in turn merely covers the incremental costs is not sufficient. If the opportunity foregone is greater than incremental revenue, the decision is not sound.

4. ***Time factor analysis:*** The decision should take into account the short run and long run effects. A high price may increase its immediate profits but may lead to loss of revenue in the long run owing to competitors snatching the business.

5. ***CVP analysis:*** In fixing prices consideration should be given to price volume relationship. The responsiveness of the market to the price should be such that the volume is increased so that fuller utilisation of plant is achieved.

6. ***Risk analysis:*** Consideration should also be given to the evaluation of uncertainty. The decision taken should be able to maximise the expected value.

Illustration 5

Prompt Printers Ltd., uses a scheme of pricing based on cost plus. All the overheads are charged, based on direct labour and based on the total cost arrived at, the selling price is fixed.

The following figures are obtained from the Annual Budget for 2015 prepared by the company:

	(₹)
Sales	*10,00,000*
Direct material	*1,80,000*
Direct labour	*3,20,000*
Factory superintendent's salary	*30,000*
Commission paid on sales (5%)	*50,000*
Foreman's salaries	*60,000*
Insurance	*10,000*
Advertisement	*20,000*
Depreciation on assets	*30,000*
Administration expenses	*90,000*
Variable factory costs :	
Repairs and maintenance	*60,000*
Tools consumed	*40,000*
Miscellaneous supplies	*10,000*

The company has submitted a tender quoting ₹ 10,000 on a large order with a cost of ₹ 1,800 direct materials and ₹ 3,200 direct labour. The customer strikes the business at ₹ 8,900 on a 'take it or leave it' basis. If the company accepts the order, the total sales for 2015 would be ₹ 10,08,900. The company is reluctant to accept the order as it would be against its policy of accepting an order below cost.

Write a note to the Managing Director, recommending the acceptance of the order, substantiating your recommendation fully with supporting figures to explain that the price offered would not be below cost and a sizeable profit also would be made. Also comment on the pricing policy of the company.

Solution:

To : Managing Director

From : Management Accountant Date: ....

Subject: Additional order

The additional order for which the company has submitted a tender quoting ₹ 10,000/- and for which the customer has offered to strike the business at ₹ 8,900/- on a 'take it or leave it' basis and as the company is reluctant to accept the order as it would be against its policy of accepting an order below cost, the following is submitted for consideration of the Managing Director with the recommendation that the acceptance of the order will be profitable to the company as is substantiated by the following figures:

At present the company determines the sales value as follows:

	(₹)	(₹)
Direct material		1,80,000
Direct labour		3,20,000
Variable overheads:		
-Repairs and maintenance	60,000	
-Tools consumed	40,000	
-Miscellaneous supplies	10,000	
-Commission paid on sales	50,000	1,60,000
Fixed overhead		
-Factory superintendent's salary	30,000	
-Foreman's salaries	60,000	
-Insurance	10,000	
-Advertisement	20,000	
-Depreciation on assets	30,000	
-Administration expenses	90,000	2,40,000
Total cost		9,00,000
Profit (balancing figure)		1,00,000
Sales		10,00,000

While applying overhead rate, the company does not distinguish between variable and fixed overheads. Overhead, as can be seen, is charged at 125% of direct labour i.e. $\frac{(₹1,60,000+₹2,40,000)}{₹3,20,000}$ and profit at 1/9 of total cost i.e. $\frac{₹1,00,000}{₹9,00,000}$. On the same basis, the quotation price has been submitted as follows:

	(₹)
Direct material	1,800
Direct labour	3,200
Overhead (125% of direct labour i.e. 125% of ₹3,200)	4,000
Total cost	9,000
Profit 1/9 of total cost	1,000
Tender price	10,000

But the above quotation against the tender is not giving true picture. This is an additional activity and the total sales after the acceptance of this order would be ₹ 10,08,900. To meet this order, only the variable overheads will be incurred as fixed overheads are absorbed by normal production. Therefore, the revised figures are:

	(₹)	(₹)
Price offered		8,900
Direct material	1,800	
Direct labour	3,200	
Variable overhead excluding sales commission*	1,100	
Sales commission (5% of ₹8,900)	445	6,545
Profit		2,355

*Variable overhead excluding sales commission is ₹1,10,000 (Total variable cost- ₹1,60,000 – Sales commission- ₹50,000) when the prime cost is ₹5,00,000 (DM- ₹1,80,000 + DL- ₹3,20,000), therefore, when prime cost for the additional order is ₹ 5,000 (DM- ₹1,800 + DL- ₹3,200), the variable cost excluding sales commission shall be ₹1,100.

It can be seen from the above that ₹ 8,900 price offered by the customer is well above the incremental cost of the additional order and the profit is above 1/3 of cost, much more than 1/9 of cost. Hence the company should accept the order. In making this recommendation, it has been assumed that the existing sales will remain unaffected by the acceptance of this order.

Comments on the pricing policy of the company

When overheads are capable of being distinguished as variable and fixed, it is wrong to club them together and charge overheads indiscriminately as a percentage of direct labour. Cost plus, as a basis of pricing, is all right for normal activity; but for incremental activity, the relevant cost is only the marginal cost and the profit on marginal activity, is normally greater than that of the original activity. Taking the whole cost and basing the price on cost plus will be misleading. It will result in loosing valuable profit opportunity for earning an additional profit.

3.7 Pricing Strategies

Pricing strategy is defined as a broad plan of action by which an organisation intends to reach its goal. Some illustrative strategies are:-

- Expanding product lines that enjoy substantial brand equity
- Offer quantity discounts to achieve increase in sales volume.

Since the right amount of volume has to be selected to optimise profit, sufficient promotional activities are necessary. In some cases it may even take a long time for the producer to establish. There are various types of pricing strategies which firm can adopt. Few of them are as follows:

3.7.1 Market-Entry strategies: While preparing to enter the market with a new product, management must decide whether to adopt a skimming or penetration pricing strategy.

(a) Skimming pricing*:* It is a policy of high prices during the early period of a product's existence. This can be synchronised with high promotional expenditure and in the later years *the* prices can be gradually reduced. The reasons for following such a policy are:

(i) ***Inelastic demand:*** The demand is likely to be inelastic in the earlier stages till the product is established in the market.

(ii) ***Sales boost:*** The charge of high price in the initial periods serves to skim the cream of the market that is relatively insensitive to price. The gradual reduction in price in the later year will tend to increase the sales.

(iii) ***Assured profit:*** This method is preferred in the beginning because in the initial periods when the demand for the product is not known the price covers the initial cost of production.

(iv) ***Cost revenue matching:*** High initial capital outlays, needed for manufacture, results in high cost of production. Added to this, the manufacturer has to incur huge promotional activities resulting in increased costs. High initial prices will be able to finance the cost of production particularly when uncertainties block the usual sources of capital.

(b) Penetration pricing: This policy is in favour of using a low price as the principal instrument for penetrating mass markets early. It is opposite to skimming price. The low price policy is introduced for the sake of long-term survival and profitability and hence it has to receive careful consideration before implementation

Penetrating pricing, means a pricing suitable for penetrating mass market as quickly as possible through lower price offers. This method is also used for pricing a new product. In order to popularise a new product penetrating pricing policy is used initially.

The company may not earn profit by resorting to this policy during the initial stage. Later on, the price may be increased as and when the demand picks up. The use of this policy by the existing concerns will discourage the new concerns to enter the market. This pricing policy is also known as **"stay-out-pricing".**

The three circumstances in which penetrating pricing policy can be adopted are as under:

(i) ***Elastic demand:*** When demand of the product is elastic to price. In other words, the demand of the product increases when price is low.

(ii) ***Mass production:*** When there are substantial savings on large scale production. Here increase in demand is sustained by the adoption of low pricing policy.

(iii) ***Frighten competition:*** When there is threat of competition. The prices fixed at a low level act as an entry barrier to the prospective competitors.

3.7.2 Price discounts and differentials:

Distributors' discounts: It means price deductions that systematically make the net price vary according to buyer's position in the chain of distribution. These discounts are given to various distributors in the trade channel e.g., wholesalers, dealers and retailers. As these discounts creates differential prices for different customers on the basis of marketing functions performed by them, so these are also called as functional discounts.

Various forms of Distributors discounts: Distributors discounts can be classified under the following three categories:

(i) ***Different net prices for different distributor levels:*** Net prices are commonly used as the device for quoting differential prices to distributors. A list of such prices is given to authorised dealers by manufacturers to facilitate their task of billing.

(ii) A uniform list price modified by a structure of discounts, each rate so determined is applied to a different level of distributor: This method is commonly used as it is easy to deal with its use in diverse trade channels. By merely varying the discounts it facilitates cyclical and seasonal adjustments in prices also. Its use helps in keeping actual prices a secret not only among distributors but also among competitors and customers. This method gives to the manufacturers a greater control over the realised margin of different categories of distributors.

(iii) ***A single discount combined with differing supplementary discounts to different levels of distributors:*** Supplementary discount gives clear cut picture about the trade channel structure or the suggested resale prices. These discounts reflect distributors cost at different stages and competition between different kinds of distributors. These discounts are often very elaborate and are in use due to tradition in the industry.

- ***Pre-requisites for determining Distributors' Discounts:*** The economic function of distributors discount is to induce different categories of distributors to perform nicely, their respective marketing functions. As such, to build up a discount structure on sound economic lines, it is essential to know about :

  (i) The services to be performed by the distributors at different levels.

  (ii) Knowledge about distributors' operating costs.

  (iii) Discount structure adopted by competitors.

  (iv) Effect of discounts on distributor's population.

  (v) Costs of selling to different channels.

  (vi) Availability of opportunities for market segmentation.

- **Quantity discounts:** Quantity discounts are price reductions related to the quantities purchased. It may take several forms. It may be related to the size of the order which is being measured in terms of physical units of a particular commodity. This is practicable where the commodities are homogeneous or identical in nature, or where they may be measured in terms of truck-loads.

However, this method is not applicable in the case of heterogeneous commodities as it is difficult to add them in terms of physical units or truck loads e.g. textile and drug industry. Quantity discounts are useful in the marketing of materials and supplies but are rarely used for marketing equipment and components.

Objectives of quantity discounts: The main objective of quantity discounts is to reduce the number of small orders and thus avoid the high cost of servicing them.

Advantages: Quantity discount system enables the dealer to avail such discounts by buying larger lots. The economic buying by a dealer may enable him to charge lower prices from his customers thereby benefiting them. Finally, lower prices to customers may increase their demand for the commodities which in turn may enable the dealer to purchase larger quantities, reaping still greater discounts, and the manufacturer to reap economies of large-scale production. In some cases, discounts become a matter of trade custom.

Disadvantages: Dealers may find it cheaper to purchase their requirement of commodities from wholesalers, availing themselves of these quantity discounts than from the manufacturers directly. This is because the wholesalers may pass on some of their discount to the dealers. Such a practice may affect the image of the manufacturer in the minds of the dealers.

Cash Discounts: Cash discounts are price reductions based on promptness of payment. It is a convenient device to identify and overcome bad credit risks. In those trades where credit risk is high, the percentage of cash discount given is also high. If a buyer decides to purchase goods on credit, he has to pay a higher price by foregoing the cash discount.

Time differentials: Charging different prices on the basis of time is another kind of price discrimination. Under time differentials the objective of the seller is to take advantage of the fact that buyer's demand elasticity vary over time. Time differentials can be classified under the following heads.

(i) ***Clock-time differentials:*** The price differentials are known as clock-time differentials when different prices are charged for the same service or commodity at different times within a 24 hour period. Common examples of these are the differences between the day and night tariff of a phone call; difference between the rates charged in morning and regular shows in cinema houses.

(*ii)* ***Calendar-time differentials:*** Here price differences are based on a period longer than 24 hours. For example; seasonal price rate variations in the case of winter clothing or hotel accommodation at a hill station and a tourist resort. The main objective here is to exploit the time preferences of the buyers.

(*iii)* ***Geographical price differentials*****:** It refers to price differentials based on buyers location. The objective here is to exploit the differences in transport-cost, due to the varying distances between the locations of the plants and customers.

(iv) ***Consumer category price differentials*****:** Price discriminations are frequently practiced according to consumer categories in the case of public utilities, e.g., electricity, transportation, etc. Electricity companies charge different rates for residential consumers and industrial consumers. The rates may also differ to domestic power, light and fan.

3.7.3 Price Discrimination: Price discrimination means charging different prices and it takes various forms according to whether the basis is customer, product, place or time. These are illustrated as under:

(a) ***Price discrimination on the basis of customer:*** In this case, the same product is charged at different prices to different customers. It is, however, potentially disruptive of customer relations.

(b) ***Price discrimination based on product version:*** In this case, a slightly different product is charged at a different price regardless of its cost-price relationship. If, for example, a table with wooden top can be sold at ₹ 400, a table with sunmica top costing ₹ 175 extra is sold at ₹ 575. The higher premium in the latter case does not necessarily reflect the higher production cost.

(c) ***Price discrimination based on place:*** An example of this method is the seats in cinema theatre where the front seats are charged at lower rates than the back seats.

(d) ***Price discrimination based on time:*** An example of this method is the practice of giving off-season concession in sale of fans or refrigerators just after the summer season.

Price discrimination is possible if the following conditions are satisfied:

(a) the maker must be capable of being segmented for price discrimination;

(b) the customers should not be able to resell the product of the segment paying higher price; and

(c) the chance of competitors' underselling in the segment of higher prices should not be possible.

Example- 5:

A company chooses three prices to be charged in three different markets and it can be seen from the figures given below that the intermediate price maximises profit.

Particulars	*Price (₹)*	*Price (₹)*	*Price (₹)*
Selling price per unit	*1.20*	*1.50*	*1.80*
Estimated sales (units)	*8,000*	*6,000*	*3,000*
Sales revenue : (A)	*9,600*	*9,000*	*5,400*
Costs:			
Manufacturing costs	*5,200*	*3,900*	*1,950*
Selling expenses	*1,000*	*500*	*450*
Fixed expenses	*1,500*	*1,500*	*1,200*
Total : (B)	*7,700*	*5,900*	*3,600*
Profit : {(A) – (B)}	*1,900*	*3,100*	*1,800*

3.7.4 Geographic Pricing Strategies: In pricing, a seller must consider the costs of shipping goods to the buyer. These costs grow in importance as freight becomes a larger part of total variable costs. Pricing policies may be established whereby the buyer pays all the freight expense, the seller bears the entire cost, or the seller and buyer share this expense. The strategy chosen can influence the geographic limits of a firm's market, locations of its production facilities, sources of its raw materials, and its competitive strength in various geographic markets.

***Point-of-Production Pricing**:* In a widely used geographic pricing strategy, the seller quotes the selling price at the point of production and the buyer selects the mode of transportation and pays all freight costs. This method of pricing is referred as ex- factory/ works pricing.

Uniform Delivered Pricing: Under uniform delivered pricing, the same delivered price is quoted to all buyers regardless of their locations. Uniform delivered pricing is typically used where freight costs are a small part of the seller's total cost. This strategy is also used by many retailers who believe "free" delivery is an additional service that strengthens their market position.

Zone-Delivered Pricing: Zone-delivered pricing divides a seller's market into a limited number of broad geographic zones and then sets a uniform delivered price for each zone.

Freight-Absorption Pricing: Under freight-absorption pricing, a manufacturer will quote to the customer a delivered price equal to its factory price *plus* the freight costs that would be charged by a competitive seller located near that customer.

3.8 Pareto Analysis

Pareto Analysis is a rule that recommends focus on the most important aspects of the decision making in order to simplify the process of decision making. It is based on the 80: 20 rule that was a phenomenon first observed by Vilfredo Pareto, a nineteenth century Italian economist. He noticed that 80% of the wealth of Milan was owned by 20% of its citizens. This phenomenon, or some kind of approximation of it say, (70: 30 etc.) can be observed in many different business situations. The management can use it in a number of different circumstances to direct management attention to the key control mechanism or planning aspects. It helps to clearly establish top priorities and to identify both profitable and unprofitable targets.

3.8.1 Usefulness of Pareto Analysis: It provides the mechanism to control and direct effort by fact, not by emotions. It helps to clearly establish top priorities and to identify both profitable and unprofitable targets.

Pareto analysis is useful to:

- Prioritize problems, goals, and objectives to Identify root causes.
- Select and define key quality improvement programs.
- Select key customer relations and service programs.
- Select key employee relations improvement programs.
- Select and define key performance improvement programs.
- Maximize research and product development time.
- Verify operating procedures and manufacturing processes.
- Product or services sales and distribution.
- Allocate physical, financial and human resources.

3.8.2 Application of Pareto Analysis: Pareto analysis may be applicable in the presentation of Performance Indicators data through selection of representative process characteristics that truly determine or directly or indirectly influence or conform the desired quality or performance result or outcome. The Pareto Analysis is generally applicable to the following business situations:

(i) Pricing of a product:

- In the case of a firm dealing with multi products, it would not be possible for it to analyse cost-profit- price -volume relationships for all of them. In practice, in case of such firm approximately 20% of products may account for about 80% of total sales revenue. Pareto Analysis is used for analysing the firm estimated sales revenues from various products and it might indicate that approximately 80% of its total sales revenue is earned from about 20% of its products.
- Such analysis helps the top management to delegate the pricing decision for approximately 80% of its products to the lower levels of management, thus freeing themselves to concentrate on the pricing decisions for products approximately 20% which are essential for the company's survival.
- Thus, a firm can adopt more sophisticated pricing methods for small proportion of products that jointly accounts for approximately 80% of total sales revenue. For the remaining 80% of the products which account for 20% of total sales revenue the firm may use cost based pricing method.

(ii) Customer Profitability analysis:

- Instead of analysing products, customers can be analysed for their relative profitability to the organisation.
- Again it is often found that approximately 20% of customers generate 80% of the profit. There will always be some customers who are less profitable than others, just as some products are less profitable than others.
- Such an analysis is useful tool for evaluation of the portfolio of customer profile and decision making such as whether to continue serving a same customer group, what is the extent of promotion expenses to be incurred.

(iii) ABC analysis- Stock Control: Another application of Pareto analysis is in stock control where it may be found that only a few of the goods in stock make up most of the value. In practice approximately 20% of the total quantity of stock may account for about 80% of its value. The outcome of such analysis is that by concentrating on small proportion of stock items that jointly accounts for 80% of the total value, a firm may well be able to control most of monetary investment in stocks.

(iv) Application in Activity Based Costing: In Activity Based Costing it is often said that 20% of an organisation cost drivers are responsible for 80% of the total cost. By analysing, monitoring and controlling those cost drivers that cause most cost, a better control and understanding of overheads will be obtained.

(v) Quality Control:

- Pareto analysis seeks to discover from an analysis of defect report or customer complaints which "vital few" causes are responsible for most of the reported problems.

- Often, 80% of reported problems can usually be traced to 20% of the various underlying causes. By concentrating once efforts on rectifying the vital 20%, one can have the greatest immediate impact on product quality.
- The Pareto Analysis indicates how frequently each type of failure (defect) occurs. The purpose of the analysis is to direct management attention to the area where the best returns can be achieved by solving most of quality problems, perhaps just with a single action.

Example- 6:

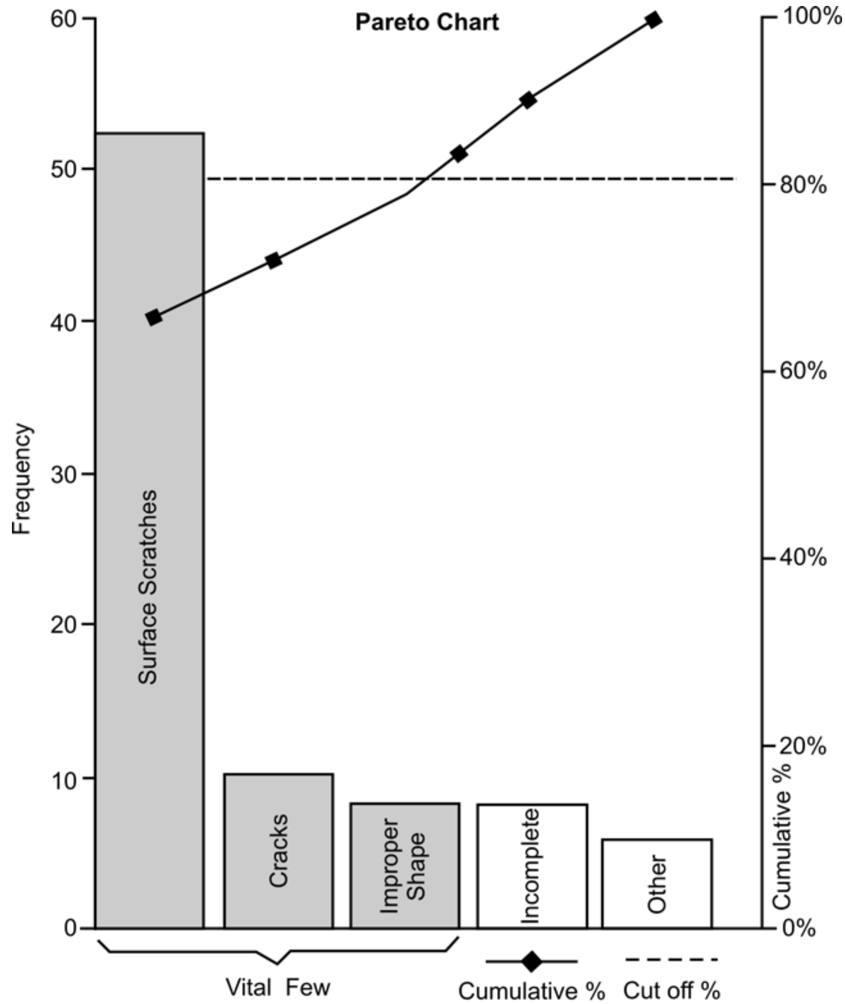
A Toy company performs a Pareto analysis, given a set of 'defect types' and frequencies of their occurrence. The sample data consists of information about 84 defective items. The items have been classified by their 'defect types' as follows:

Defect Types	*No. of items*
Cracks (due to mishandling of raw material)	*10*
Improper shapes	*8*
Incomplete	*8*
Surface scratches	*53*
Other (due to bad quality raw material)	*5*

Frequency table indicating the frequency of occurrence of defects in decreasing order of their occurrence will be as follows:

Defect type	*No. of items*	*(%)*	*Cumulative %*
Surface scratches	*53*	*63.0952*	*63.0952*
Cracks	*10*	*11.9048*	*75.0000*
Improper shape	*8*	*9.5238*	*84.5238*
Incomplete	*8*	*9.5238*	*94.0476*
Other	*5*	*5.9524*	*100.00*

The Pareto chart is then constructed for defect type.



The purpose of Pareto analysis in this example, is to direct attention to the area where best returns can be achieved by solving most of the quality problems, perhaps just with a single action. In this case, use of good quality raw material say plastic may solve 63% of problem and if raw material is handled properly at least 75% the problems may be overcome.

Summary

- Generally pricing decisions are influenced by the pricing policy followed by an organisation. Pricing policies are made taking overall objectives of an organisation into account.
- Under perfect competition, firm has no pricing policy of its own as the sellers are price takers and sells as much as they are capable of selling at the prevailing market price.
- Under the monopoly market, a firm is a price setter i.e. it can fix any price but pricing is done taking elasticity of demand for the product into consideration.

- Under monopolistic condition, consumers may buy more at a lower price than at higher price. The profit can be maximised by equating marginal revenue with marginal cost.
- The oligopolistic firm, while determining the price for its product, takes into consideration not only the demand for the product but also the reactions of the other firms in the same industry to any action or decision it may take.
- Pricing of a product plays the role in marketing strategy to tap into the value created and to generate revenues. So the biggest concern on the part of any organisation is to getting the right price.
- Pricing of a new product is generally a difficult task because of the uncertainty involved in the estimation of their demand.
- Objective value or True economic value (TEV): This is a measure of benefits that a product is intended to deliver to the consumers relative to the other products without giving any regard whether the consumer can recognize these benefits.
- Perceived value: The value that consumer understands the product deliver to it.
- Revolutionary product: When it is new for the market and has the potential to create its own value.
- Evolutionary Product: An existing product with some added new features and updations.
- Me-too products: A product which is very similar to any other existing product.
- There are various methods available for pricing of a finished product viz. cost- plus pricing, rate of return pricing, variable cost pricing, competitive pricing, incremental pricing etc.
- Pricing strategy is defined as a broad plan of action by which an organisation intends to reach its goals.
- Skimming pricing: It is a strategy of keeping high prices during the early period of product introduction to the market to take the advantage of being market leader and to cover the high product development costs.
- Penetration pricing: This is a pricing strategy adopted to introduce a product into the market at a reasonably low price. This pricing is adopted to get entry into the market and to keep pricing barriers to other firms from entering into the market.
- Pareto Analysis is a rule that recommends focus on the most important aspects of the decision making in order to simplify the process of decision making.
- The purpose of Pareto analysis is to identify the area which is very crucial for the organisation's survival.
- As per Pareto's theory there are only 20% of total products/ operations/ inventory which has significant importance, requires direct attention of the top level management.

4

Budget & Budgetary Control

LEARNING OBJECTIVES

After studying this chapter, you should be able to :

- Explain terms Budget and Budgetary Control
- Learn more about using, limiting factors for preparation of Budgets
- Understand concept of Functional Budget, Fixed and Flexible Budget
- Understand concept of Zero Base Budgeting and Performance Base Budgeting
- Explain recent trend such as Budget Variance
- Calculate Budget Ratios

4.1 Introduction

In this chapter you shall further increase your understanding of the above mentioned and learn more about concepts related to identifying and using limiting factors for preparation of budgets, the relationship between strategic, operational and budgetary planning, the actual making of functional budgets from given data and other important budgetary tools like zero based budgeting and performance budgeting.

4.2 Strategic Planning, Budgetary Planning and Operational Planning

Planning is perhaps one of the most important tools in the hands of management to decide upon future course of action. It will be useful at this stage to distinguish in broad terms between three different types of planning:

Strategic Planning: Strategic planning is concerned with preparing long-term action plans to attain the organization's objectives by considering the changes at horizon.

Budgetary Planning: Budgetary planning is mainly concerned with preparing the short to medium term plan of the organisation. It will be carried out within the framework of the strategic plan as already set. An organization's annual budget is considered as an intermediary step towards achieving the strategic plan.

Operational Planning: It concerns with the short-term or day-to-day planning process. It plans the utilisation of resources and will be carried out within the framework of the budget. Each step in the operational planning process is an interim step towards achieving the budget.

4.3 The Preparation of Budgets

The process of preparing and using budgets will differ from organisation to organisation. However, there are a number of key requirements in the design of a budgetary planning and control process.

Co-ordination: Budgets provide a means of co-ordination of the business as a whole. In the process of establishing budgets, the various factors like production capacity, sales possibilities, and procurement of material, labour, etc. are balanced and co-ordinates so that all the activities proceed according to the objective. For this purpose a budget committee is formed which includes all the departmental heads together to solve a common problem.

The need for co-ordination in the planning process is immense. The interrelationship between the functional budgets (e.g. sales production, purchasing) means that one budget cannot be completed without reference to several to several others.

Participative budgeting: Participative budgeting is known as 'bottom-up budgeting'. It contrasts with imposed or top-down budgets where the ultimate budget holder does not have the opportunity to participating in the budgeting process.

CIMA defines participative budgeting as: *"A budgeting system in which all budget committee members are given the opportunity to apply their own budgets in practice"*

The advantages of participative budgeting are as follows:

- **Improved quality of forecasts to use as the basis for the budget:** Managers who are doing a job on a day-to-day basis are likely to have a better idea of what is achievable, what is likely to happen in the forthcoming period, local trading conditions, etc.
- **Improved motivation:** Budget holders are more likely to want to work to achieve a budget that they have been involved in setting themselves, rather than one that has been imposed on them from above.
- **Better results:** being the executor of the budget the applicant can control the costs better than any other manager.

The Budget Manual: Effective budgetary planning relies on the provision of adequate information to the individuals involved in the planning process.

Many of these information needs are contained in the budget manual. A budget manual is a collection of documents that contains key information for those involved in the planning process. Typical contents could include the following:

- An introductory explanation of the budgetary planning and control process, including a statement of the budgetary objective and desired results.
- A form of organisation chart to show that is responsible for the preparation of each functional budget and the way in which the budgets are interrelated.
- A timetable for the preparation of each budget. This will prevent the formation of a 'bottleneck' with the late preparation of one budget holding up the preparation of all others.

- Copies of all forms to be completed by those responsible for preparing budgets, with explanations concerning their completion.
- A list of the organization's account codes, with full explanations of how to use them.
- Information concerning key assumptions to be made by managers in their budgets, for example the rate of inflation, key exchange rates, etc.

Identification of the principal budget factor: The principal budget factor is the factor that limits the activities of functional budgets of the organisation. The early identification of this factor is important in the budgetary planning process because it indicates which budget should be prepared first.

In general sales volume is the principal budget factor. So sales budget must be prepared first, based on the available sales forecasts. All other budgets should then be linked to this.

Alternatively, machine capacity may be limited for the forthcoming period and therefore machine capacity is the principal budget factor. In this case the production budget must be prepared first and all other budgets follow it.

Failure to identify the principal budget factor at an early stage could lead to delays later on when managers realize that the targets they have been working with are not feasible.

In case of one limiting factor, we shall need to apply the concept of Marginal costing. In this we initially allot the limiting resource on the basis of highest contribution per limiting factor.

How to identify the principle budget factor:

- <u>In case of single product organization: Steps to follow:</u>
    1. Identify the capacity of the production departments. Generally normal capacity is consider for budget / estimation. Capacity of a department is defined as facility available for work & generally expressed in terms of labour hour, machine hour or unit.

        There are 4 different expression of capacity.

        - Maximum Capacity: Maximum No. of Days in a period x No. of Workers x Hrs/Days.
        - Practical Capacity: Maximum Capacity – Sunday & Statutory holidays & Normal Maintenance & Idle time.
        - Normal Capacity: It is the average of the last 3-year of normal performance if there is any abnormal is any abnormal data don't consider in the computing the average.
        - Actual Capacity: It can be determined only at the end of the period. So it has no importance for preparation of budget.
    2. Maximum production in a dept. = Normal Capacity ÷ Time per unit
    3. Select the minimum production volume among the above results. The dept. producing that result is known as bottleneck among the production department.
    4. Identify the sale or demand of the product.

5. Now by comparing the above 2 steps we can identify the principle budget factor.

- <u>In case of multi product organization:</u>
  1. Sale / Demand is the Principle Budget Factor
  2. Capacity is in short supply or Limiting Factor i.e. capacity requirement according to demand is more than its supply
     - Only one limiting factor
     - More than one limiting factor- the technique of linear programming is applied.

Illustration 1

In a year, 15 workers are working in a dept. on a single shift basis. Statutory holidays in that year are 18. Normal maintenance requires 250 hrs./ p.m. The capacity utilization during last 5 years:

Year	*Labour Hour*
2007	*30,000*
2008	*38,000*
2009	*31,000*
2010	*30,900*
2011	*26,000*

Calculate capacity of the organisation.

Solution

Maximum Capacity = 365 Days × 15 Workers × 8 Hrs per Day = 43,800 Labour Hours

Practical Capacity = {365- (52+18) days} × 15 workers × 8 Hrs. per Day – 250 Hrs p.m. ×12

= 32,400 Labour Hours

The capacity utilization during last 5 years:

Years	Labour Hours	
2007	30,000	
2008	38,000	(to high)
2009	31,000	
2010	30,900	
2011	26,000	(to low)

$\therefore$ Normal Capacity of 2012 = (30,000 + 31,000 + 30,900) ÷ 3 = 30,633 Labour Hours

While preparing the budget we consider the normal capacity as budgeted production level

$\therefore$ 100% of budgeted capacity always implies the normal capacity.

Illustration 2

There are 3 departments with different normal capacity & time required p.u. is given:

	Machine	Assembly	Finishing
(a) Capacity	12,000 Machine Hours	8,000 Labour Hours	9,000 Labour Hours
(b) Time required/unit	4 Machine Hours	5 Labour Hours	3 Labour Hours
(c) Maximum production in a department (a÷ b)	3,000	1,600	3,000

∴ Feasible production =1,600 units

∴ Assembly department is considered as bottleneck to the above production line.

Illustration 3

Solo products Ltd. manufactures and sells a single product and has estimated sales revenue of ₹ 126 lakhs this year based on a 20 per cent profit on selling price.

Each unit of the product requires 3 lbs of material P and 1½ lbs of material Q for manufacture as well as a processing time of 7 hours in the Machine shop and 2½ hours in the Assembly Section. Overheads are absorbed at a blanket rate of 33.3333% of Direct Labour.

The factory works 5 days of 8 hours a week in a normal 52 weeks a year. On an average statutory holidays, leave and absenteeism and idle time amount to 96 hours, 80 hours and 64 hours respectively, in a year. The past performance (in Hours) of factory in last 3 yrs is as follows-

	Machine Shop	Assembly shop
In 2003	11,00,000	3,45,000
In 2004	10,30,000	3,20,000
In 2005	10,80,000	3,40,000

The other details are as under:

		₹
Purchase price	Material P Material Q	6 per lb 4 per lb
Comprehensive Labour Rate	Machine Shop Assembly	4 per hour 3.20 per hour
No. of Employees	Machine Shop Assembly	600 180

	Finished Goods	Material P	Material Q
Opening Stock	20,000 units	54,000 lbs	33,000 lbs
Closing Stock (Estimated)	?	30,000 lbs	66,000 lbs

You are required to calculate the closing stock of finished goods.

Solution

Computation of Sale Volume

		(₹) p.u.	(₹) p.u.
Material	P: 3 lb @ ₹6	18	
	Q: 1 ½ lb @ ₹4	6	24
Labour - Machine	7 hrs. @ ₹ 4	28	
- Assembly	2.5 hrs. @ ₹ 3.2	8	36
Overhead	33.33% of Direct Labour		12
Cost of Production			72
Add: Mark up (25%)			18
Selling Price (a)			90
Sales Value (b)			₹126 lakhs
No. of units sold (b)/(a)			1,40,000

Computation of Principal Budgeted Factor

(1) Sales/Demand (units) 1,40,000

(2) Feasible Production = Normal Capacity ÷ Time p.u.

Machine Dept: 10,70,000* ÷ 7 1, 52,857

Assembly Dept.: 3,35,000# ÷ 2.5 1, 34,000

Feasible Production during this period 1,34,000 units as Assembly Department is the Bottleneck.

Computation of Expected Closing stock of Finished Goods

Opening Stock (units)	20,000
Add: Estimated Production (units)	1,34,000
	1,54,000
Less: Demand (units)	1,40,000
Closing Stock (units)	14,000

* (11,00,000+10,30,000+10,80,000)/3

\# (3,45,000 + 3,20,000 + 3,40,000)/3

Illustration 4

P. H. Ltd. has specialised in the manufacture of three kinds of sub-assemblies required by the manufacturers of certain equipments. The current pattern of sales of sub-assemblies is in the ratio (in units) of 1: 2: 4 for sub-assemblies P, Q and R respectively.

The sub-assemblies consist of the following components:

Sub-assembly	*Selling price (₹)*	*Requirement of Components*			
		Frame	*Part X*	*Part Y*	*Part Z*
P	*430*	*1*	*10*	*2*	*8*
Q	*500*	*1*	*2*	*14*	*10*
R	*600*	*1*	*6*	*10*	*2*
Purchase Price (₹)		*40*	*16*	*10*	*6*

The direct labour hours required for the manufacture of each of the sub-assemblies are:

Sub-assembly	*Skilled Hours*	*Un-Skilled Hours*
P	*4*	*4*
Q	*3*	*4*
R	*3*	*6*
Wage rate per hour (₹)	*6*	*5*

The labourers work for 8 hours a day for 25 days a month. Variable overheads per sub-assembly are P ₹ 10, Q ₹ 8 R ₹ 7.

Fixed overheads budget per month is an under:

	₹
Production	*15,80,000*
Selling & Distribution	*7,28,000*
Administration	*6,76,000*

All fixed overheads are incurred evenly throughout the year. The target of profit for the current year is ₹ 120 lakhs before tax. The company has to plan to reduce the closing stock of sub-assemblies and components by 10 % as compared to the opening stock.

Find the Sales in quantities

Solution

Computation of Variable Cost per Unit of 3 Products

Items	P	Q	R
Frame @ ₹ 40	40	40	40
Part- X	160	32	96
	(10×16)	(2×16)	(6×16)
Y	20	140	100
	(2×10)	(14×10)	(10×10)
Z	48	60	12
	(8×6)	(10×6)	(2×6)
Wages –Skilled @ ₹ 6	24	18	18
–Unskilled @ ₹ 5	20	20	30
Overhead	10	8	7
Variable cost	322	318	303

Computation of Average Contribution

Product	Selling Price	Variable Cost	Contribution	Mix	Total Contribution
P	430	322	108	1	108
Q	500	318	182	2	364
R	600	303	297	4	1,188
				7	1,660

Average Contribution = ₹ 237.14

Required Profit (p.m.)	₹ 10, 00,000
Required Fixed Cost	₹ 29, 84,000
Required Contribution	₹ 39, 84,000

Required Sales = ₹39,84,000/₹237.14 = 16,800 units

Sales P 16,800 units ×1/7 = 2,400
Q 16,800 units ×2/7 = 4,800
R 16,800 units ×4/7 = 9,600

Illustration 5

The sales, cost, selling price and processing time of three different herbal drinks produced by a company for the year just concluded are given below:

Product	***Strong***	***Normal***	***Mild***
Annual sales (no. of packs 250 gm)	*6,000*	*5,000*	*1,000*
Selling price (₹/ pack)	*50*	*40*	*30*
Unit cost (₹/ pack)	*42*	*36*	*21*
Processing time/ per pack (hrs)	*1.5*	*1*	*2*

The total processing hours available to the company is fully utilised for this sale. Fixed manufacturing overheads are fully absorbed in unit cost at rate of 200% of variable cost. For the coming year the demand for the three products has been estimated as under:

Strong – 6,000 packs   Normal – 6,000 packs   Mild – 2,000 packs

Considering that the selling prices are fixed and the processing time can be switched from one product line to another, calculate the best production programme for next operating year indicating the increase in net profit that will result.

Solution

Statement Showing Profit and Total Processing Time

Particulars		Products			Total
		Strong	Normal	Mild	
(a)	Production & Sales (Packs)	6,000	5,000	1,000	
	Sales (₹ /Pack)	50	40	30	
	Less: Variable Cost (₹ /Pack) [1/3 of total cost]	14	12	7	
(b)	Contribution (₹ /Pack)	36	28	23	
(c)	Total Contribution(₹) [(a) x (b)]	2,16,000	1,40,000	23,000	3,79,000
(d)	Fixed Cost [total cost × 2/3] (₹ /Pack)	28	24	14	
(e)	Total Fixed Cost [(a)×(d)]	1,68,000	1,20,000	14,000	3,02,000
	Profit [(c)-(e)]				77,000
(f)	Processing Time per pack (Hours)	1.5	1.0	2.0	
(g)	Total Processing Time (a) × (f)	9,000	5,000	2,000	16,000 hr.

Capacity Requirement or Process Time Required *in next year*

Product	Demand (Packs)	Hrs/Pack	Total Hours
Strong	6,000	1.5	9,000
Normal	6,000	1.0	6,000
Mild	2,000	2.0	4,000
			19,000
Less: Average Process Time			16,000
Shortage			3,000

Statement of Rank

Product	Contribution/Pack	Hrs/Pack	Contribution/Hr.	Rank
Strong	36	1.5	24	II
Normal	28	1.0	28	I
Mild	23	2.0	11.5	III

Statement of Profit

	Hrs.	Units	Contribution	Total
Process time available	16,000			
Less: R-1 -Normal	6,000	6,000	28	1,68,000
	10,000			
Less: R-II -Strong	9,000	6,000	36	2,16,000
	1,000			

Less: R-III –Mild	1,000	500 (1,000 hrs.÷2/hr.)	23	11,500
				3,95,500
Less: Fixed Cost				3,02,000
Profit *in next year*				93,500
Less: Current year's Profit				77,000
Increase in Profit				16,500

4.4 The Interrelationship of Budgets

The critical importance of the principal budget factor stems from the factor that all budgets are interrelated. For example, if sales are the principal budget factor this is the first budget to be prepared. This will then provide the basis for the preparation of several others budgets, including the selling expenses budget and the production budget.

However, the production budget cannot be prepared directly from the sales budget without a consideration of stockholding policy. For example, management may plan to increase finished goods stock in anticipation of a sales drive. Production quantities would then have to be higher than the budgeted sales level. Similarly, if a decision is taken to reduce the level of material stocks held, it would not be necessary to purchase all of the materials required for production.

4.5 Using Spreadsheets in Budget Preparation

It is clear from just this simple example that exchange in one budget can have a knock-on effect on several others budgets. For this reason spreadsheets are particularly useful in budget preparation. Budgetary planning is an iterative process. Once the first set of budgets has been prepared they will be considered by senior managers. They may require amendments to be made or they may wish to see the effect of changes in key decision variables.

A well-designed spreadsheet model can take account of all of the budget interrelationships. This means that it will not be an onerous task to alter decision variables and produce revised budgets for management's consideration.

4.6 Zero Base Budgeting

ZBB is defined as 'a method of budgeting which requires each cost element to be specifically justified, as though the activities to which the budget relates were being undertaken for the first time. Without approval, the budget allowance is zero'.

Zero – base budgeting is so called because it requires each budget to be prepared and justified from zero, instead of simple using last year's budget as a base. Incremental level of expenditure on each activity is evaluated according to the resulting incremental benefits. Available resources are then allocated where they can be used most effectively.

CIMA has defined it *"as a method of budgeting whereby all activities are revaluated each time a budget is set".*

Zero based budgeting is a decision oriented approach .In Zero Based budgeting no reference is made to previous level expenditure. Zero based budgeting is completely indifferent to whether total budget is increasing or decreasing.

Characteristics of Zero-base budgeting:

- Manager of a decision unit has to completely justify why there should be at all any budget allotment for his decision unit. This justification is to be made a fresh without making reference to previous level of spending in his department.
- Activities are identified in decision packages.
- Decision packages are ranked in order of priority.
- Packages are evaluated by systematic analysis.
- Under this approach there exist a frank relationship between superior and subordinates. Management agrees to fund for a specified service and manager decision of the decision unit clearly accepts to deliver the service.
- Decision packages are linked with corporate objectives, which are clearly laid down.
- Available resources are directed towards alternatives in order of priority to ensure optimum results.

Traditional Budgeting vs Zero- based budgeting: Following are the points of difference between traditional budgeting and zero based budgeting:

- Traditional budgeting is accounting oriented. Main stress happens to be on previous level of expenditure. Zero-based budgeting makes a decision oriented approach. It is very rational in nature and requires all programmes, old and new, to compete for scarce resources.
- In traditional budgeting, first reference is made to past level of spending and then demand for inflation and new programmes. In zero based budgeting a decision unit is broken into understandable decision packages, which are ranked according to importance to enable to top management to focus attention to only on decision packages, which enjoy priority to others.
- In tradition budgeting, some managers deliberately inflate their budget request so that after the cuts they still get what they want. In zero-base budgeting, a rationale analysis of budget proposals is attempted. The managers, who unnecessarily try to inflate the budget request, are likely to be caught and exposed. Management accords its approval only to a carefully devised result-oriented package.
- Traditional budgeting is not as clear and as responsive as zero base budgeting is.
- In traditional budgeting. Its for top management to decide why a particular amount should be spent on a particular decision unit. In Zero-base budgeting, this responsibility is shifted from top management to the manager of decision unit.

- Traditional budgeting makes a routine approach. Zero-base budgeting makes a very straightforward approach and immediately spotlights the decision packages enjoying priority over others.

Process of Zero-base Budgeting: The process of zero-base budgeting involves the following steps:

- Determination of a set of objectives is one of pre-requisites and essential step in the direction of ZBB technique.
- Deciding about the extent to which the technique of ZBB is to be applied whether in all areas of organizations' activities or only in a few selected areas on trial basis.
- Identify those areas where decisions are required to be taken.
- Developing decision packages and ranking them in order of performance.
- Preparation of budget that is translating decision packages into practicable units/items and allocating financial/resources.
- In real terms the Zero base budgeting is simply an extension of the cost, benefit, analysis method to the area of corporate planning and budgeting. It, however, provides a number of advantages to the organizational efficiency and effectiveness.

Advantages of Zero-base budgeting: The advantages of zero-base budgeting are as follows:

- It provides a systematic approach for the evaluation of different activities and rank them in order of preference for the allocation of scarce resources.
- It ensures that the various functions undertaken by the organization are critical for the achievement of its objectives and are being performed in the best possible way.
- It provides an opportunity to the management to allocate resources for various activities only after having a thorough cost-benefit-analysis. The chances of arbitrary cuts and enhancement are thus avoided.
- The areas of wasteful expenditure can be easily identified and eliminated.
- Departmental budgets are closely linked with corporation objectives.
- The technique can also be used for the introduction and implementation of the system of 'management by objective.' Thus, it cannot only be used for fulfillment of the objectives of traditional budgeting but it can also be used for a variety of other purposes.

Zero base budgeting s superior to traditional budgeting: Zero base budgeting's superior to traditional budgeting in the following manner:

- It provides a systematic approach for evaluation of different activities.
- It ensures that the function undertaken is critical for the achievement of the objectives.
- It provides an opportunity for management to allocate resources to various activities after a thorough – cost benefit analysis.

- It helps in the identification of wasteful expenditure and then their elimination. If facilitates the close linkage of departmental budgets with corporate objectives
- It helps in the introduction of a system of Management by Objectives.

Disadvantage of ZBB:

- The work involves in the creation of decision-making and their subsequent ranking has to be made on the basis of new data. This process is very tedious to management.
- The activities selected for the purpose of ZBB are on the basis of the traditional functional departments. So the consideration scheme may not be implemented properly.

4.7 Performance Budgeting (PB)

Performance Budgeting provide a meaningful relationship between estimated inputs and expected outputs as an integral part of the budgeting system. 'A performance budget is one which presents the purposes and objectives for which funds are required, the costs of the programmes proposed for achieving those objectives, and quantities data measuring the accomplishments and work performed under each programme. Thus PB is a technique of presenting budgets for costs and revenues in terms of functions. Programmes and activities are correlating the physical and financial aspect of the individual items comprising the budget.

Traditional budgeting vs. Performance budgeting

- The traditional budgeting (TB) gives more emphasis on the financial aspect than the physical aspects or performance. PB aims at establishing a relationship between the inputs and the outputs.
- Traditional budgets are generally prepared with the main basis towards the objects or items of expenditure i.e. it highlights the items of expenditure, namely, salaries, stores and materials, rates rents and taxes and so on. In the PB latter the emphasis is more on the functions of the organisation, the programmes to discharge these function and the activities which will be involved in undertaking these programmes.

Steps in Performance Budgeting: According to the Administrative Reforms Commission (ARC) the following steps are the basic ones in PB:

- Establishing a meaningful functional programme and activity classification of government operations ;
- Bring the system of accounting and financial management in accord with this classification
- Evolving suitable norms, yardsticks, work units of performance and units costs, wherever possible under each programme and activity for their reporting and evaluation.

The Report of the ARC use the following terms in an integrated sequence:

Function → Programme → Activity → Project

The team 'function' is used in the sense of 'objective'. For achieving objectives 'programmes' will have to be evolved. In respect of time horizon, it is essentially a replacement of traditional annual fiscal budgeting by a more output-oriented, but still an annual, exercise.

For an enterprise that wants to adopt PB, it is thus imperative that:

- The objectives of the enterprise are spelt out in concrete terms.
- The objectives are then translated into specific functions, programmes, activities and tasks for different levels of management within the realities of fiscal; constraints ;
- Realistic and acceptable norms, yardsticks or standards and performance indicators should be evolved and expressed in quantifiable physical units.
- A style of management based upon decentralised responsibility structure should be adopted, and
- An accounting and reporting system should be developed to facilities monitoring, analysis and review of actual performance in relation to budgets.

Performance Reporting at various levels of management:

Report: A major part of the management account's job consists of preparing reports to provide information for purposes of control and planning:

The important consideration in drawing up of reports and determining their scope are the following:

Significance	:	Are the facts in the reports reliable? Does it either called for action or demonstrate the effect of action? It is material enough.
Timeliness	:	How late can the information be and still be of use? What is the earliest moment at which it could be used if it were available? How frequently is it required?
Accuracy	:	How small should be an inaccuracy which does not alter the significance of the information?
Appropriateness	:	Is the recipient the right person to take any action that is needed? Is there any other information which is required to support the information to anyone else jointly interested?
Discrimination	:	Will anything be lost by omitting the item? Will any of the items gain from the omission? Is the responsibility for suppressing the item acceptable?
Presentation	:	Is the report clear and unbiased? Is the form of it is suitable to the subject? Is the form of it suitable to the recipient?

The following are certain types of reports which are to be prepared and submitted to management regularly at predetermined time interval:

1. Top Management: (Including Board of Directors and financial managers)
   (i) Balance Sheet

(ii) Profit & Loss Statement

(iii) Position of stocks

(iv) Disposition of funds or working capital;

(v) Capital expenditure & forward commitments together with progress of projects in hands;

(vi) Cash-flow statements;

(vii) Sales, production, and other appropriate statistics.

2. Sales Management:

(i) Actual sales compared with budgeted sales to measure performance by:

- Products,
- Territories
- Individual salesmen, and
- Customers.

(ii) Standard profit and loss by product:

- For fixing selling prices, and
- To Concentrate on sales of most profitable products.

(iii) Selling expenses in relation to budget and sales value analyzed by:

- Products,
- Territories
- Individual salesmen, and
- Customers.

(iv) Bad debts and accounts which are slow and difficult in collection.

(v) Status reports on new or doubtful customers.

3. Production Management:

(i) To Buyer: Price variations on purchases analysed by commodities.

(ii) To Foreman:

- Operational efficiency for individual operators duly summarized as departmental average;
- Labour utilization report and causes of lost time and controllable time;
- Indirect shop expenses against the standard allowed; and
- Scrap report.

(iii) To Works Managers:

- Departmental operating statement;

- General works operating statements (Expenses relating to all works expenses not directly allocable or controllable by departments);
- Plant utilization report;
- Department Scrap report; and
- Material usage report.

4. Special Reports:

These reports may be prepared at the request of general management or at the initiative of the management accountant. The necessity for them may, in some cases, arise on account of the need for more detailed information on matters of interest first revealed; by the routine, reports. These reports may range over a very wide area. Some of the matters in respect of which such reports may be required can be:

(i) Taxation legislation and its effect on profits.

(ii) Estimates of the earning capacity of a new project.

(iii) Break-even analysis

(iv) Replacement of capital equipment.

(v) Special pricing analysis

(vi) Make or buy certain components

(vii) Statement of surplus available for payment of bonus under the labour appellate tribunal formula.

4.8 Budget Ratio

These ratios provide information about the performance level, i.e., the extent of deviation of actual performance from the budgeted performance and whether the actual performance is favourable or unfavorable. If the ratio is 100% or more, the performance is considered as favourable and if the ratio is less than 100% the performance is considered as unfavourable.

The following ratios are usually used by the management to measure development from budget.

Capacity Usage Ratio: This relationship between the budgeted number of working hours and the maximum possible number of working hours in a budget period.

Standard Capacity Employed Ratio: This ratio indicates the extent to which facilities were actually utilized during the budget period.

Level of Activity Ratio: This may be defined as the number of standard hours equivalent to work produced expressed as a percentage of the budget of standard hours.

Efficiency Ratio: This ratio may be defined as standard hours equivalent of work produced expressed as a percentage of the actual hours spent in producing the work.

Calendar Ratio: This ratio may be defined as the relationship between the number of working days in a period and the number of working das in the relative budget period.

Budget Ratios:

Efficiency Ratio	=	(Standard Hours ÷ Actual Hours) × 100
Activity Ratio	=	(Standard Hours ÷ Budgeted Hours) × 100
Calendar Ratio	=	(Available Working Days ÷ Budgeted Working Days) × 100
Standard Capacity Usage Ratio	=	(Budgeted Hours ÷ Max. Possible Hours in the Budgeted Period) × 100
Actual Capacity Usage Ratio	=	(Actual Hours Worked ÷ Maximum Possible Working Hours in a Period) × 100
Actual Usage of Budgeted Capacity Ratio	=	(Actual working hours ÷ Budgeted hours) × 100

Illustration 6

Following data is available for T.T.D and Co:

Standard working hours	*8 hours per day of 5 days per week*
Maximum capacity	*50 employees*
Actual working	*40 employees*
Actual hours expected to be worked per four week	*6,400 hours*
Std. hours expected to be earned per four weeks	*8,000 hours*
Actual hours worked in the four week period	*6,000 hours*
Standard hours earned in the four week period	*7,000 hours.*

The related period is of 4 weeks. In this period there was a one special day holiday due to national event. Calculate the following ratios :

(1) Efficiency Ratio, (2) Activity Ratio, (3) Calendar Ratio, (4) Standard Capacity Usage Ratio, (5) Actual Capacity Usage Ratio. (6) Actual Usage of Budgeted Capacity Ratio.

Solution

Maximum Capacity *in a budget period*

= 50 Employees × 8 Hrs. × 5 Days × 4 Weeks

= 8,000 Hrs.

Budgeted Hours

40 Employees × 8 Hrs. × 5 Days × 4 Weeks

= 6,400 Hrs.

Actual Hrs. = 6,000 Hrs. (from the sum)

Standard Hrs. for Actual Output = 7,000 Hrs.

Budget No. of Days = 20 Days (4 Weeks x 5 Days)

Actual No. of Days = 20 – 1 = 19 Days

1. Efficiency Ratio = $\frac{\text{Standard Hrs}}{\text{Actual Hrs}}$ ×100 = {(7,000÷6,000)×100}= 116.67%

2. Activity Ratio = $\frac{\text{Standard Hrs}}{\text{Budgeted Hrs}}$ ×100 = {(7,000÷6,400) × 100} = 109.375%

3. Calendar Ratio = (Available Working Days ÷ Budgeted Working Days) × 100

   = {(19÷20) × 100} = 95%

4. Standard Capacity Usage Ratio = (Budgeted Hours ÷ Max. Possible Hours in the Budgeted Period) × 100

   = {(6,400÷8,000) × 100} = 80%

5. Actual Capacity Usage Ratio =

   (Actual Hours Worked ÷ Maximum Possible Working Hours in a period) × 100

   = {(6,000÷8,000) × 100} = 75%

6. Actual Usage of Budgeted Capacity Ratio

   = (Actual Working Hours ÷ Budgeted Hours) × 100

   = {(6,000÷6,400) × 100} = 93.75%

4.9 Miscellaneous Illustrations

Cash Budget

Illustration 7

Prepare cash budget for July - December from the following information:

(i) The estimated sales, expenses etc. are as follows :

(₹ in lacs)

	June	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
Sales	*34*	*40*	*40*	*50*	*50*	*60*	*65*
Purchase	*24*	*16*	*17*	*20*	*20*	*25*	*28*
Wages and Salaries	*12*	*14*	*14*	*18*	*18*	*20*	*22*
Miscellaneous	*5*	*6*	*6*	*6*	*7*	*7*	*7*
Interest Received	*2*	—	—	*2*	—	—	*2*
Sales of Shares	—	—	*20*	—	—	—	—

(ii) 20% of the sales are on cash with 3% cash discount and the balance on credit.

(iii) 1% of the credit sales are returned by the customers. 2% of the net receivable constituted bad debt losses. 50% of the good accounts receivable are collected in the month following the sales with 1% cash discount, 30% of the good accounts receivable are collected in the 2^{nd} month following the sales and the rest in the 3^{rd} month following sales.

(iv) The time lag in the payment of misc. expenses and purchases is one month. Wages and salaries are paid fortnightly with a time lag of 15 days.

(v) The company keeps a minimum cash balance of ₹ 25.00 lakhs. Cash in excess of ₹ 27 lakhs is invested in 9% Govt. securities in the multiple of ₹ 1 lakh. Interest is receivable on monthly basis. Shortfalls in the minimum cash balance are made good by borrowings from banks in multiple of ₹ 2 lakhs & also repaid by same amount. The rate of interest is 12% p.a. (compound interest)

(vi) The opening cash balance is ₹ 26 lacs.

(vii) Sales in the month April & May was ₹ 44 & 40 lacs respectively.

Solution

Working Note 1:

Lets Sales	100	
Less: Cash Sales	20	20×97% = 19.4% of Sale (monthly)
Debtors	80	
Less: Return 1%	0.08	
Net Receivable	79.2	
Less: Bad Debt (2%)	1.584	
Good Debtors	77.62	

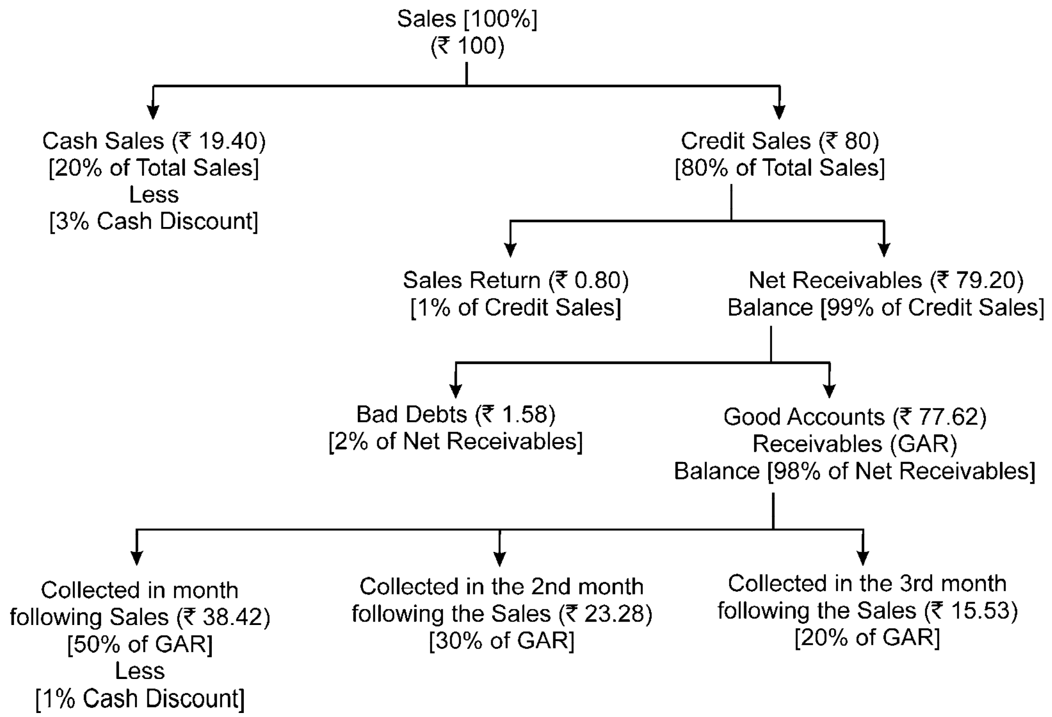
Good Debtors 77.62×50%×99%= 38.42% of Previous Month Sales.

Months Following Sales:

2^{nd} Month *following sales* = 77.62×30% = 23.28% of 2^{nd} Previous Month Sale (PMS)

3^{rd} Month *following sales* = 77.62×20% = 15.53% of 3^{rd} Previous Month Sale (PMS)

Collection Pattern



Cash Budget

	July	Aug.	Sept.	Oct.	Nov.	Dec.
Opening Balance	26.00	26.96	27.10	27.87	27.43	27.27
Collection from Sales:						
Cash Sales 19.4% of Sales	7.76	7.76	9.70	9.70	11.64	12.61
Collection: 38.42% of PMS	13.06	15.37	15.37	19.21	19.21	23.05
23.28% of 2nd PMS	9.31	7.92	9.31	9.31	11.64	11.64
15.53% of 3rd PMS	6.83	6.21	5.28	6.21	6.21	7.76
Interest-Business	—	—	2.00	—	—	2.00
Govt. Bond	—	—	0.11	0.13	0.14	0.16
Sales of Shares	—	20.00	—	—	—	—
(a) Total	62.96	84.22	68.87	72.43	76.27	84.49
Payments p.m. [1 m lag]	24.00	16.00	17.00	20.00	20.00	25.00
- Misc Expenses [1 m lag]	5.00	6.00	6.00	6.00	7.00	7.00
-Wages & Salary [1/2 m lag]	7.00	7.00	9.00	9.00	10.00	11.00
[½ of previous month]	6.00	7.00	7.00	9.00	9.00	10.00
(b) Total	42.00	36.00	39.00	44.00	46.00	53.00

(c) Gross Balance (b)–(a)	20.96	48.22	29.87	28.43	30.27	31.49
(d) Borrowing	6.00	—	—	—	—	—
(e) Refund of Borrowing with Interest	—	6.12	—	—	—	—
(f) Inv. in Govt. Bond	—	15.00	2.00	1.00	3.00	4.00
(g) Net Cash Balance	26.96	27.10	27.87	27.43	27.27	27.49

Illustration 8

The 1st January cash balance of the Jay Company is ₹ 5,000. Sales for the first four months of the year are expected to be as follows: January, ₹ 65,000; February, ₹ 54,000; March, ₹ 66,000; and April, ₹ 63,000.

On January1, uncollected amounts for November and December of the previous year, are ₹ 13,500 and ₹ 39,150, respectively. Collections from customers follow this pattern; 55% in the month of sale, 30% in the month following the sale, 13% in the second month following the sale, and 2% un-collectible.

Materials purchases for December were ₹ 10,000. Forecast purchases for the coming year are January, ₹ 12,500; February, ₹ 16,500; March, ₹ 13,000; and April, ₹ 14,000. Purchases are usually paid by the 10th of the month following the month of purchase. Other cash expenditures of ₹ 41,000 are forecast for each month.

Calculate:

(i) Expected cash collections during February

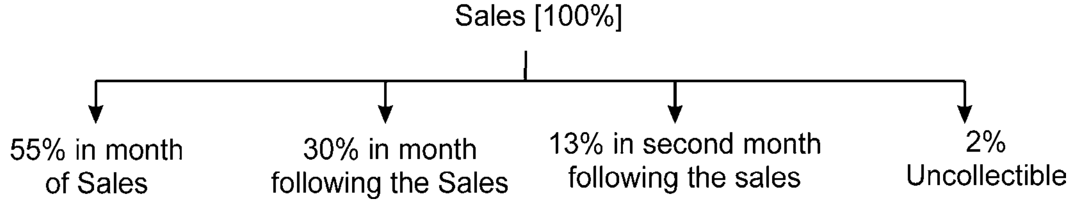
(ii) Expected cash balance, February1

(iii) Expected cash balance, February 28.

Solution

Working note

Collection Pattern



Sales	100%	
Less: Cash Sales	55%	Collection in the month of sale
Recivable at the end of the month of Sale	45%	
Less: Collection	30%	Collection in the month following the sale

Receivable at the end of the month following the Sale	15%	
Less: Collection	13%	Collection in the second month following the sale
Bad Debt	2%	

Cash Budget

Particulars	Jan	Feb
Opening Cash Balance	5,000	27,550
Collection:		
-Cash Sales @ 55%	35,750	29,700
-Collection Previous Month @ 30% of sales	26,100*	19,500
-Collection of 2nd Previous Month @ 13% of sales	11,700@	11,310#
Total (a)	78,550	88,060
Payments:		
-Purchase of Material (1 months lag)	10,000	12,500
-Other Expenses	41,000	41,000
Total (b)	51,000	53,500
Net Cash Balance (a)-(b)	27,550	34,560

Working Note:

* $\left[\frac{39{,}150}{45\%} \times 30\%\right]$

@ $\left[\frac{13{,}500}{15\%} \times 13\%\right]$

\# $\left[\frac{39{,}150}{45\%} \times 13\%\right]$

Fixed and Flexible Budgets

Illustration 9

The budgeted level of activity of a production department of a manufacturing company is 5,000 hours in a period. But a technical study assumes overhead behaviour mentioned below:-

	₹ ('00) Per hr.	Total in ₹ ('000)
Indirect Wages, variable cost	*0.40*	–
Rent and Tax, fixed cost	–	*320*
Consumable supplies, variable	*0.24*	–
Repairs : up to 2,000 hours	–	*100*
additional each extra 500 hrs up to 4,000 hrs.	–	*35*
additional 4,001 to 5,000 hrs	–	*60*

additional, above 5,000 hrs	–	*70*
Supervision up to 2,500 hrs	–	*400*
additional each extra 600 hrs up to 4,900 hrs	–	*100*
additional, above 4,900 hrs	–	*150*
Power variable up to 3,600 hrs	*0.25*	–
for hrs above 3,600 additional cost	*0.20*	–
Depreciation up to 5,000 hrs	–	*650*
above 5,000 hrs	–	*820*
Clearing up to 4,000 hrs	–	*60*
above 4,000 hrs	–	*80*
Lighting 2,100 to 3,500 hrs	–	*120*
3,501 hrs to 5,000 hrs	–	*150*
above 5,000 hrs	–	*175*

(a) Prepare fixed budget and a flexible budget at 70%, 85% and 110% of budgeted level of activity in one statement.

(b) Calculate a departmental hourly rate of overhead absorption.

Solution

Particulars	**Flexible budget**			**Fixed budget**
Capacity	70%	85%	110%	100%
Hours	3,500	4,250	5,500	5,000
	₹ '000	₹ '000	₹ '000	₹ '000
Indirect Wages @ ₹ 40/hr.	140	170	220	200
Rates & Taxes	320	320	320	320
Consumable Supplies @ ₹ 24/hr.	84	102	132	120
Repair	205	300	370	300
	(100+35×3)	(100+35×4+60)	(100+35×4+60+70)	(100+35×4+60)
Supervision	600	700	950	950
	(400+100×2)	(400+100×3)	(400+100×4+150)	(400+100×4+150)
Power	87.5	103	128	118
Depreciation	650	650	820	650
Clearing	60	80	80	80
Lighting	120	150	175	150
Total Cost	2,266.5	2,575	3,195	2,888
Rate/hour	0.647	0.605	0.580	0.577

Illustration 10

From the information given below prepare a flexible budget of M/s Piston Bearings Ltd. for a production capacity of 15,000, 20,000, 25,000 and 30,000 tonnes.

(a) The production capacity of the plant is 30,000 tonnes.

(b) The sales for the year just concluded have been 25,000 tonnes at a unit realization of ₹ 400 per tonne ex-works. This rate is likely to be maintained in the coming year as well.

(c) The sales manager feels that with a little more effort on the part of the sales staff, he can achieve a sales programme of 30,000 tonnes.

(d) Raw material consumption is twice the quantum of finished products and the price of raw material is ₹ 40 per tonne.

(e) The other major material used is furnace oil which is available at ₹ 300 per tonne and the consumption ratio of oil to the finished products is 30%.

(f) Power is bought outside from the State Electricity Board and a per present tariffs, the cost of power would be as under:

Kwh Purchased per annum (in lakhs)	Rent per unit (applicable to entire purchase-in paise)
25 to 30	15
31 to 35	14
36 to 40	13
41 to 45	12
over 45	10

Power requirements of the plant are normally 200 kwh per tonne of product at a production level of 20,000 tonnes and are estimated to come down to 175 kwh per tonne at a production level of 25,000 tonnes per annum and 150 kwh per tonne at 30,000 tonnes per annum. Similarly, the consumption is expected to be 220 kwh per tonne at a production level lower than 20,000 tonne p.a.

(g) Labour is employed on a daily rate basis of ₹ 10 per day on an employment of 300 days p.a. There are at present 350 men employed and though lower production would result in some 20% of them being rendered surplus, because of an agreement with the labour union, there cannot be any retrenchment.

(h) Consumption of stores during the last four years had been as under:

Year	Production level	Stores consumed
2011	25,000 tonnes	₹ 5.20 lakhs
2010	20,000 tonnes	3.84 lakhs
2009	22,500 tonnes	3.95 lakhs
2008	25,000 tonnes	4.00 lakhs

Prices over the base year 2008 have been increasing at the rate of 10% p.a. in the current year, the increases is expected to be maintained at the same rate over the prices of 2011.

(i) Selling and distribution overheads are expected to be maintained at ₹ 15 per tonne.

(j) Administrative expenses of the organization in 2008 were ₹ 7.50 lakhs and have been increasing at the rate of 5% p.a. over the immediately preceding year's level. No additional staff is expected to be employed for achieving addition production.

Your working should form part of the answer.

Solution

M/s Piston Bearings Ltd.
Flexible Budget For 2012

Production (tonnes)	15,000 (₹)	20,000 (₹)	25,000 (₹)	30,000 (₹)
Raw Materials	12,00,000	16,00,000	20,00,000	24,00,000
Furnace Oil (see note 1)	13,50,000	18,00,000	22,50,000	27,00,000
Power (see note 2)	4,62,000	5,20,000	5,25,000	5,40,000
Labour	10,50,000	10,50,000	10,50,000	10,50,000
Stores (see note 3)	3,43,200	4,57,600	5,72,000	6,86,400
Factory Cost	44,05,000	54,27,600	63,97,000	73,76,400
Administrative Overhead (see note 4)	9,11,630	9,11,630	9,11,630	9,11,630
Selling & Distribution Overheads	2,25,000	3,00,000	3,75,000	4,50,000
Cost of Sales	55,41,630	66,39,230	76,83,630	87,38,030
Net profit	458,370	13,60,770	23,16,370	32,61,970
Sales	60,00,000	80,00,000	1,00,00,000	1,20,00,000

Working Notes:

1. Furnace oil is 30% of the finished product. For example, for the production of 15,000 tonnes; 4,500 tonnes, of furnace oil will be re required. The cost is ₹ 300 per tonne.

2. Power requirements are:

(i)	Capacity (in tonnes)	15,000	20,000	25,000	30,000
(ii)	Total Requirements (per tonne)	220	200	175	150
(iii)	Rate [per kwh (paise)]	14	13	12	12
(iv)	Total Power Cost	₹ 4,62,000	5,20,000	5,25,000	5,40,000

3. Consumption of Stores:

Cost (per tonne) in 2011 = $\frac{₹\ 5,20,000}{25,000}$ = ₹ 20.8 (per tonne)

Price has increased by 10% over 2011

Price for 2012 is ₹ 20.8+ ₹2.08 = ₹ 22.88 (per tonne)

Cost of Stores *at various levels of capacity*:

Levels of Capacity (tonnes)	15,000	20,000	25,000	30,000
Cost [per tonne (₹)]	22.88	22.88	22.88	22.88
Total Cost (₹)	3,43,200	4,57,600	5,72,000	6,86,400

4. Administration Expenses for 2008 — 7,50,000
Increase in 2009 at *5% over preceding year* — 37,500
Expenses for 2009 — 7,87,500
Increase in 2010 at 5% — 39,375
Expenses for 2010 — 8,26,875
Increase in 2011 at 5% — 41,344
Expenses for 2011 — 8,68,219
Increase in 2012 at 5% — 43,411
Estimated Expenses for 2012 — 9,11,630

Functional Budgets and Budgeted Financials

Illustration 11

The direct labour requirements of three of the products manufactured in a factory, each involving more than one labour operation, are estimated as follows:

Direct Labour hours per unit (in minutes)

	Product 1	***Product 2***	***Product 3***
Operation 1	*18*	*42*	*30*
Operation 2	—	*12*	*24*
Operation 3	*9*	*6*	—

The factory works 8 hours per day, 6 days in a week. The budget quarter is taken as 13 weeks and during a quarter lost hours due to leave & holiday is estimated to be 124 hours.

The budgeted hourly rates for the workers manning the operations 1,2 and 3 are ₹ 2.00, ₹ 2.50 and ₹ 3.00 respectively.

The budgeted sales of the products during the quarter are:

Product	*1*	*9,000 units*
	2	*15,000 units*
	3	*12,000 units*

There is a carry over of 5,000 units of product 2 and 4,000 units of product 3 and it is proposed to build up a stock at the end of the budget quarter as follows:

Product	*1*	*1,000 units*
	3	*2,000 units*

Prepare a man-power budget for the quarter showing for each operation, (i) direct labour hours, (ii) direct labour cost and (iii) the number of workers.

Solution

Man Power Budget ***for the quarter***

Operation 1		**Product**		
		1	**2**	**3**
(a)	Labour (minutes/unit)	18	42	30
(b)	Production (units) [Working Note-1]	10,000	10,000	10,000
(c)	Direct Labour (minutes) [(a)×(b)]	1,80,000	4,20,000	3,00,000
(d)	Direct Labour Hours [(c)/(60 minutes)]	3,000	7,000	5,000
(e)	Rate (per hour)	2	2	2
(f)	Labour Cost [(d)×(e)]	6,000	14,000	10,000
(g)	Capacity/ Worker (hrs) [Working Note- 2]	500	500	500
(h)	No. of workers [(d)/(g)]	6	14	10

Operation 2		**Product**		
		1	**2**	**3**
(a)	Labour (minutes/unit)	—	12	24
(b)	Production (units) [Working Note-1]	10,000	10,000	10,000
(c)	Direct Labour (minutes) [(a)×(b)]	—	1,20,000	2,40,000
(d)	Direct Labour Hours [(c)/(60 minutes)]	—	2,000	4,000
(e)	Rate (per hour)	2.50	2.50	2.50
(f)	Labour Cost [(d)×(e)]	—	5,000	10,000
(g)	Capacity/ Worker (hrs) [Working Note- 2]	500	500	500
(h)	No. of workers [(d)/(g)]	—	4	8

Operation 3		**Product**		
		1	**2**	**3**
(a)	Labour (minutes/unit)	9	6	—
(b)	Production (units) [Working Note-1]	10,000	10,000	10,000
(c)	Direct Labour (minutes) [(a)×(b)]	90,000	60,000	—
(d)	Direct Labour Hours [(c)/(60 minutes)]	1,500	1,000	—
(e)	Rate (per hour)	3.00	3.00	3.00
(f)	Labour Cost [(d)×(e)]	4,500	3,000	—
(g)	Capacity/ Worker (hrs) [Working Note- 2]	500	500	500
(h)	No. of workers [(d)/(g)]	3	2	—

Working Note 1:

Product	1	2	3
Sales	9,000	15,000	12,000
Add: Closing Stock	1,000	—	2,000
Less: Opening Stock	—	5,000	4,000
	10,000	10,000	10,000

Working Note 2:

Practical capacity per worker [(8 hrs.×6 days ×13 weeks) –124 hours = 500 hrs.]

Illustration 12

Good Producers Ltd. is specialist in the manufacturing of Industrial Products. They manufacture and market three types of products under the name PNM, PNO, PNP. Company produces three products from three basic raw materials in three departments. Company follows budgetary control system. From the following data, they require your assistance to prepare the following for the month of Aug 2012.

(a) Production Budget

(b) Material Usage Budget

(c) Material Purchase Budget

(d) Budgeted Profit/(Loss) statement for each product

Budgeted data for Aug 2012

Product	*Sales (In ₹)*	*Stock at Aug 1, 2012 (In units)*	*Department*	*Production Overhead (₹)*	*Direct Labour Hours*	*Direct Material*	*Stock at Aug 1, 2012 (In units)*
PNM	*46,80,000*	*4,500*	*DMM*	*4,39,875*	*87,975*	*MPD*	*36,750*
PNO	*16,32,000*	*3,000*	*DMO*	*3,76,350*	*1,25,450*	*MND*	*30,750*
PNP	*22,96,000*	*3,750*	*DMP*	*5,15,250*	*85,875*	*MOD*	*26,250*

The company makes its stock to finished goods on a total cost basis. According to company's anticipation, raw material stock and finished product stock as at 31st Aug 2012 will be reduce by 20% and 10% respectively.

Fixed production overhead is absorbed on a direct labour hour basis.

It is expected that there will be no work-in-progress at the beginning or end of the month.

Administration cost is absorbed by products at a rate of 40% of production cost and selling and distribution cost is absorbed by products at a rate of 20% production cost.

Profit is budgeted as a 25% of total cost.

Standard cost data per unit of product:

Direct Material		*Product*		
	Price per unit	*PNM*	*PNO*	*PNP*
	(₹)	*Units*	*Units*	*Units*
MPD	*3*	*8*	*-*	*18*
MND	*6*	*-*	*15*	*14*
MOD	*2*	*8*	*8*	*-*

Direct Wages:	*Rate per Hour (₹)*	*Hrs*	*Hrs*	*Hrs*
DMM	*4*	*6*	*3*	*3*
DMO	*3*	*9*	*3*	*4*
DMP	*2*	*3*	*6*	*9*

	Product		
	PNM	*PNO*	*PNP*
Other Variable Cost per unit (₹)	*28*	*41*	*19*

Solution

(a) Production Budget for Aug 2012

	PNM (units)	PNO (units)	PNP (units)
Sales*	11,700	3,400	4,100
Less: Opening Stock	4,500	3,000	3,750
	7,200	400	350
Add: Closing stock 10% reduction	4,050	2,700	3,375
	11,250	3,100	3,725

***** (sales in ₹ /price per unit)

(b) Material Usage Budget for Aug 2012

Product	Units of Product	MPD Qty. per unit of Product	Quantity Required	MND Qty. per unit of Product	Quantity Required	MOD Qty. per unit of Product	Quantity Required
PNM	11,250	8	90,000	-	-	8	90,000
PNO	3,100	-	-	15	46,500	8	24,800
PNP	3,725	18	67,050	14	52,150	-	-
			1,57,050		98,650		1,14,800

(c) Material Purchase Budget for Aug 2012

	MPD		MND		MOD	
	Units	₹ (3/-)	Units	₹ (6/-)	Units	₹ (2/-)
Usage	1,57,050	4,71,150	98,650	5,91,900	1,14,800	2,29,600
Less: O/stock	36,750	1,10,250	30,750	1,84,500	26,250	52,500
	1,20,300	3,60,900	67,900	4,07,400	88,550	1,77,100
(Add:C/stock)	29,400	88,200	24,600	1,47,600	21,000	42,000
	1,49,700	4,49,100	92,500	5,55,000	1,09,550	2,19,100

(d) Budgeted Profit/(Loss) Statement for *each product and in total* for Aug 2012

	PNM	PNO	PNP
Sales	₹ 46,80,000	₹ 16,32,000	₹ 22,96,000
Less: Cost	37,44,000	13,05,600	18,36,800
	9,36,000	3,26,400	4,59,200

Working Notes:

Statement showing Selling Price for each product:

	PNM		PNO		PNP	
		(₹)		(₹)		(₹)
Material:						
MPD	(₹3x8)	24		-	(₹3x18)	54
MND		-	(₹6x15)	90	(₹6x14)	84
MOD	(₹2x8)	16	(₹2x8)	16		-
		40		106		138
Labour:						
DMM	(₹4x6)	24	(₹4x3)	12	(₹4x3)	12
DMO	(₹3x9)	27	(₹3x3)	9	(₹3x4)	12
DMP	(₹2x3)	6	(₹2x6)	12	(₹2x9)	18
		57		33		42
Variable Overhead:		28		41		19
Fixed Cost:						
DMM	(₹5x6)	30	(₹5x3)	15	(₹5x3)	15
DMO	(₹3x9)	27	(₹3x3)	9	(₹3x4)	12
DMP	(₹6x3)	18	(₹6x6)	36	(₹6x9)	54
		75		60		81
Total Production Cost:		200		240		280
Adm.(Based on 40% of		80		96		112

production cost)						
Selling and Dist. Cost		40		48		56
(Based on 20% of production cost)						
Total Cost:		320		384		448
Profit (25%)		80		96		112
Selling Price per unit		**400**		**480**		**560**

Fixed Overhead Rate:

DMM = ₹ 4,39,875 / 87,975 hours= ₹ 5 per hour

DMO = ₹3,76,350 / 1,25,450 hours= ₹ 3 per hour

DMP = ₹ 5,15,250 / 85,875 hours = ₹ 6 per hour

Cost of Products:

PNM = 11,700 units x ₹ 320 = ₹ 37,44,000

PNO = 3,400 units x ₹ 384 = ₹ 13,05,600

PNP = 4,100 units x ₹ 448 = ₹ 18,36,800

Illustration 13

The following is the Sales Budget of a company engaged in manufacturing and marketing certain consumer products (in 3 lines), their markets being Eastern and Western Zones.

(₹ in lakhs)

Product	*Eastern*		*Western*		*Total*	
	Units	*₹*	*Units*	*₹*	*Units*	*₹*
A	*30,000*	*15.00*	*16,000*	*8.00*	*46,000*	*23.00*
B	*10,000*	*6.00*	*15,000*	*9.00*	*25,000*	*15.00*
C	*4,000*	*3.20*	*6,000*	*4.80*	*10,000*	*8.00*

No changes are expected in the inventory levels. The following are the unit standard cost details for the 3 products :

	A ₹	*B* ₹	*C* ₹
Direct Materials			
Material X @ ₹ 6 per kg.	*12*	*24*	*18*
Material Y @ ₹ 4 per kg	*8*	*—*	*4*
Direct Labour:			
₹ 4 per hour	*12*	*16*	*20*
Factory Overheads:			
Variable @ Re. 1.00 per Std. Hr.	*3*	*4*	*5*
Fixed @ ₹ 2.00 per Std. Hr.	*6*	*8*	*10*

Variable Overheads comprise Indirect Material, Indirect Labour and Indirect Expenses in the ratio of 50 : 25 : 25.

Fixed Factory overheads stated above are based on the following Product Mix:–

Product A	*20,000 units*
Product B	*15,000 units*
Product C	*10,000 units*

The mix of fixed factory overheads consists of Indirect material, Indirect Labour and Indirect Expenses in the ratio of 30 : 30 : 40.

Price of Material X is expected to increase by ₹ 0.20 per kg. in the budget period. There will be 2% inefficiency (i.e. 2% wastage allowance) in case of Direct Materials. A 3% increase in productivity of direct labour is expected. No other variances in direct costs are expected. Therese variances and any other variances in indirect items have to be built into the Budgets.

The selling and Distribution cost budget for the two zones are as follows:

	Eastern	***Western***
Zonal Manager's Control		
Commission	*10% on Std. Gross Profit*	*10% Std. Gross Profit*
	₹	*₹*
Travelling	*40,000*	*35,000*
Advertising	*15,000*	*12,000*
Office Expenses	*9,000*	*9,000*
Salaries	*20,000*	*20,000*
Perquisites	*2,000*	*2,000*
Depreciation	*5,000*	*4,000*
Insurance	*1,000*	*1,000*

The head office selling and distribution expenses are: Advertising and Sales Promotion ₹ 70,000; Salaries ₹ 42,000; Stationary Postage etc. ₹ 5,000; Depreciation ₹ 5,000; insurance ₹ 1,000. Head Office Administrative expenses are ₹ 2,00,000 and this should be met out of gross profit. The average rate of Tax is 30%.

You are required to prepare the Budgeted Income Statement for the Company.

Solution

Budgeted Income Statement for the period …

	Note	(₹)	(₹)
Sale:		23,00,000	
Product A			
Product B		15,00,000	
Product C		8,00,000	46,00,000
Less: Cost of Goods Sold:			
Direct Materials: X	1	14,03,928	
Y	1	4,16,160	18,20,088
Direct Labour:	2		11,17,440
VariableOverhead: Indirect Material	3	1,44,400	
: Indirect Labour	3	72,000	
: Indirect Expenses	3	72,000	2,88,000
Fixed Overhead : Indirect Material	4	1,02,000	
: Indirect Labour	4	1,02,000	
: Indirect Expenses	4	1,36,000	3,40,000
Gross Profit------			10,34,472
Less : Head Office Expenses			
Administration:		2,00,000	
Selling & Distribution:		1,23,000	3,23,000
Less: Selling Distribution and Other Fixed Expenses of the Two Zones :			
Commission (Note 5)	5	84,400	
Travelling Expenses		75,000	
Advertising		27,000	
Office Expenses		18,000	
Salaries & Perks		44,000	
Depreciation		9,000	
Insurance		2,000	2,59,400
Net Profit -------			4,52,072
Less Tax @ 30%			1,35,622
Profit After Tax-----			3,16,450

Working Notes :

(1) Calculation of Direct Material Cost:

		Material X Kg.		*Material Y Kg.*
Product A	46,000 × 2	92,000	46,000 × 2	92,000
Product B	25,000 × 4	1,00,000		—
Product C	10,000 × 3	30,000	10,000 × 1	10,000
Total Material Required *as per standard*		2,22,000		1,02,000
Add: 2% Wastage Allowance *(assumed to be based on output)*		4,440		2,040
Total Material Requirement..(a)		2,26,440		1,04,040
Material Cost Price per Kg….(b)		₹6.20		₹4
Total Material Cost …(a) x (b)		₹14,03,928		₹4,16,160

(2) Calculation of Direct Labour Cost:

		Hrs.
Product A	46,000 × 3 hrs.	1,38,000
Product B	25,000 × 4 hrs.	1,00,000
Product C	10,000 × 5 hrs.	50,000
		2,88,000
Less :Saving of 3% Due to Efficiency		8,640
Hours (to be paid for)		2,79,360
Total Direct Labour Cost (2,79,360 @ ₹ 4/- per hour)		₹11,17,440

(3) Calculation of Variable Factory Overheads:

	(₹)
Product A 46,000 units @ ₹ 3 per unit	1,38,000
Product B 25,000 units @ ₹ 4 per unit	1,00,000
Product C 10,000 units @ ₹ 5 per unit	50,000
	2,88,000
Indirect Material Cost:	1,44,000
Indirect Labour Cost	72,000
Indirect Expenses	72,000
(Ratio of 50 : 25 : 25 respectively)	

(4) Calculation of Fixed Factory Overheads:

	(₹)
Product A 20,000 units @ ₹ 6 per unit	1,20,000
Product B 15,000 units @ ₹ 8 per unit-	1,20,000
Product C 10,000 units @ ₹ 10 per unit	1,00,000
	3,40,000
Indirect Material Cost	1,02,000
Indirect Labour Cost	1,02,000
Indirect Expenses	1,36,000
(Ratio of 30 : 30 : 40 respectively)	

(5) Calculation of Commission of the 2 Zones:

Standard Gross Profit (per unit): Selling Price – Std. Cost Price

	Eastern	Western
A	₹50 – ₹41 =₹9	₹50 – ₹41= ₹9
B	₹60 – ₹52 =₹8	₹60 – ₹52=₹8
C	₹80 – ₹57 =₹23	₹80 – ₹57=₹23

Std. Gross Profit for Zones:

Eastern		Western	
A 30,000 units × ₹9 =	₹2,70,000	16,000 units × ₹9 =	₹1,44,000
B 10,000 units × ₹8 =	₹80,000	15,000 units × ₹8=	₹1,20,000
C 4,000 units × ₹23=	₹92,000	6,000 units × ₹23=	₹1,38,000
	₹4,42,000		₹4,02,000
Gross Profit 10% of the above	₹ 44,200 (a)		₹ 40,200 (b)
		Total (a) + (b)	₹ 84,400

Illustration 14

S.G. Ltd. manufactures two products A and B. The summarised Balance Sheet of the company as at 31st March, 2012 is as under :–

Equity and Liabilities	*(₹)*
Shareholder's funds	
Share Capital	*12,00,000*
Reserve and Surplus	*96,000*
Current Liabilities	
Trade Payables	*48,000*
Short-Term Provisions	
Provision for Income Tax	*60,000*
	14,04,000

Assets	*(₹)*
Non-Current Assets	
Fixed Assets (Net)	*9,00,000*
Current Assets	
Inventories	*3,54,000*
Trade Receivables	*90,000*
Cash and Cash Equivalents	*60,000*
	14,04,000

The following information has been furnished to you for the preparation of the budget for the year ending 31st March, 2013:–

(i) *Sales forecast :–*

Product A 24,000 units at ₹ 30 per unit.

Product B 15,000 units at ₹ 40 per unit.

(ii) Raw materials :–

	Products	
	A	*B*
Material X @ ₹ 3 per kg.	*2 kgs.*	*4 kgs.*
Material Y @ ₹ 1 per kg.	*1 kg.*	*2 kgs.*

(iii) Direct Labour:–

Dep. P : 2 Hrs. @ ₹ 1 per hour for A.

1 Hr. @ ₹ 2 per hour for B.

Dep. Q: 1 Hr. @ ₹ 3 per hour for A

1 Hr. @ ₹ 3 per hour for B.

(iv) Overheads :–

	Dept. P ₹	*Dept. Q ₹*
Fixed overheads per annum :–		
Depreciation	*48,000*	*12,000*
Others	*96,000*	*30,000*
Variable overheads per hour	*0.50*	*1.50*

(v) Inventories :–

(a) Raw materials :

Opening stock

X	*36,000 kgs.*	*₹ 1,14,000*
Y	*6,000 kgs.*	

Closing stock

X	*48,000 kgs.*	
Y	*12,000 kgs.*	

(b) *Finished goods :*

Opening stock

A	*600 units*	*₹ 2,40,000*
B	*6,000 units*	

Closing stock

A	*6,600 units*
B	*3,000 units*

(vi) *Selling, Distribution and Administration expenses are estimated at ₹ 1,80,900 per annum.*

(vii) *The cost of raw material purchases, direct wages, factory overheads, selling, distribution and administration overheads of the year will be met in full in cash during the year. The estimated position of debtors and creditors as on 31st March, 2013 is ₹ 1,50,000 and ₹ 48,000 respectively. Income tax provision standing at the beginning of the year will be paid during the year. Rate of income tax is 30%. An equipment purchased at ₹ 1,20,000 will be paid during the year.*

You are required to prepare for the year ending 31st March, 2013 :

(a) *Cost of goods sold budget*

(b) *Cash budget*

(c) *Projected Balance Sheet as at 31st March, 2013 in the same format as given in the question.*

The detailed working for each of the above should be shown.

Solution

Working Notes:

1. Production Budget (Units)

Particulars	**A**	**B**
Sales	24,000	15,000
Add : Closing Stock	6,600	3,000
Total	30,600	18,000
Less : Opening Stock	600	6,000
Production	30,000	12,000

2. Direct Material Cost

Particulars	A (₹)	B (₹)	Total (₹)
Material X @ ₹ 3 per Kg.	6	12	
Material Y @ ₹ 1 per Kg.	1	2	
Material Cost (per unit)…(a)	7	14	
Production (units)…(b)	30,000	12,000	
Direct Material Cost (₹)…(a) x (b)	2,10,000	1,68,000	3,78,000

3. Direct Labour Cost

Particulars	A (₹)	B (₹)	Total (₹)
Dept. P : 2 hrs. @ ₹ 1 per hr. for A	2		
1 hr. @ ₹ 2 per hr. for B		2	
Dept. Q : 1 hr. @ ₹ 3 per hr. for A	3		
1 hr. @ ₹ 3 per hr. for B		3	
Direct Labour Cost (per unit)…(a)	5	5	
Production (units)…(b)	30,000	12,000	
Direct Labour Cost (₹)…(a) x (b)	1,50,000	60,000	2,10,000

4. Direct Labour Hours

Particulars	Dept. P	Dept. Q
A: P 30,000 × 2 hrs. Q 30,000 × 1 hr.	60,000	30,000
B: P 12,000 × 1 hrs. Q 12,000 × 1 hr.	12,000	12,000
	72,000	42,000

5. Overhead Recovery Rate

Particulars	Dept. P	Dept. Q
Fixed Overheads:	(₹)	(₹)
Depreciation	48,000	12,000
Others	96,000	30,000
Total	1,44,000	42,000
Direct Labour Hours	72,000	42,000
Fixed Overhead (rate per hr.) …(a)	2.00	1.00
Variable Overhead (rate per hr.)…(b)	0.50	1.50
Total Overhead (rate per hr.) ..(a)+(b)	2.50	2.50

6. Overhead Expenses

Particulars	Dept P (₹)	Dept Q (₹)	Total (₹)
Fixed (other than Depreciation)	96,000	30,000	
Variable [72,000 hrs. × ₹ 0.50; 42,000 hrs. × ₹ 1.50]	36,000	63,000	
Total Overheads (other than Depreciation) (a)	1,32,000	93,000	225,000
Depreciation (b)	48,000	12,000	60,000
Total Overheads (a) + (b)	1,80,000	1,05,000	285,000

7. Cost Sheet

Particulars	Products		
	A (₹)	B (₹)	Total (₹)
Direct Material (per unit)	7.00	14.00	
Direct Wages (per unit)	5.00	5.00	
Overhead (per unit) [Dept. P]	5.00	2.50	
[Dept. Q]	2.50	2.50	
Total Cost (per unit)..(a)	19.50	24.00	
Production…(b)	30,000	12,000	
Total Cost…(a) x (b)	5,85,000	2,88,000	8,73,000

8. Sales

Particulars			₹
A	24,000 units × ₹ 30		7,20,000
B	15,000 units × ₹ 40		6,00,000
	Total		13,20,000

9. Trade Receivables

Particulars	(₹)
Opening Balance	90,000
Add: Sales	13,20,000
Total	14,10,000
Less: Closing Balance	1,50,000
Cash Receipts	12,60,000

10. Raw Material

Particulars	Material		Total
	X (Kg.)	Y (Kg.)	(₹)
Consumption for 'A'	60,000	30,000	
Consumption for 'B'	48,000	24,000	
Total Consumption	1,08,000	54,000	
Add: Closing Stock	48,000	12,000	
Total	1,56,000	66,000	
Less: Opening Stock	36,000	6,000	
Material to be Purchase	1,20,000	60,000	
Purchase Price per Kg.	₹ 3	₹ 1	
Purchase Value (₹)	3,60,000	60,000	4,20,000

11. Trade Payables

Particulars	(₹)
Opening Balance	48,000
Add: Purchases	4,20,000
Total	4,68,000
Less: Closing Balance	48,000
Paid	4,20,000

12. Inventories as on 31.03.2013

Particulars	(₹)
Raw Material : 'X' 48,000 units × ₹ 3 = ₹1,44,000 'Y' 12,000 units × ₹ 1 = ₹12,000	1,56,000
Finished Goods : 'A' 6,600 × ₹ 19.50 = ₹1,28,700 'B' 3,000 × ₹ 24.00 = ₹72,000	2,00,700

13. Fixed Assets as at 31.03.2013

Particulars	₹
Opening Values of Fixed Assets	9,00,000
Add: Additions	1,20,000
Less: Depreciation	60,000
	9,60,000

Computation of Requirements of Question

(a) Cost of Goods Sold Budget

Particulars	(₹)
Direct Materials (Note 2)	3,78,000
Direct Wages (Note 3)	2,10,000
Overheads (Note 6)	2,85,000
Total	8,73,000
Add : Op. Stock (Balance Sheet)	2,40,000
Total	11,13,000
Less: Closing Stock (Note 12)	2,00,700
Cost of Goods sold	9,12,300

(b) Cash Budget

Particulars	(₹)
Opening Balance (Balance Sheet)	60,000
Receipts (Note 9)	12,60,000
Total Receipts (A)	13,20,000
Payments :	
Creditors (Note 11)	4,20,000
Direct Wages (Note 3)	2,10,000
Overheads (Note 6)	2,25,000
Selling, Distribution and Administration Expenses	1,80,900
Income Tax	60,000
Capital Expenditure	1,20,000
Total Payments (B)	12,15,900
Closing Balance (A) – (B)	1,04,100

(c) Projected Balance Sheet as at March, 31, 2013

Equity and Liabilities	(₹)
Shareholder's Funds	
Share Capital	12,00,000
Reserve and Surplus*	2,54,760
Current Liabilities	
Trade Payables	48,000
Short-term Provisions	
Provision for Income Tax	68,040
	15,70,800

Assets	(₹)
Non-Current Assets	
Fixed Assets (Net)	9,60,000
Current Assets	
Inventories	3,56,700
Trade Receivables	1,50,000
Cash and Cash Equivalents	1,04,100
	15,70,800

*** Reserve & Surplus**

Particulars	(₹)
Sales (Note 8)	13,20,000
Less: Cost of Goods Sold	9,12,300
Gross Profit	4,07,700
Less : Selling Dist. & Admn. Expenses	1,80,900
Profit before tax	2,26,800
Less Provisions for tax (30%)	68,040
Profit after tax	1,58,760
Add : Opening balance of Reserve & Surplus	96,000
Closing balance of Reserve Surplus	2,54,760

Budget Variance

Illustration 15

Nicefit manufactures readymade garments by a simple process of cutting the clothes in various shapes and then sewing the corresponding pieces together to form the finished product.

The sewing Department and the cutting department report to the production manager who along with Engineering Manager reports to the Director-Manufacturing. The Sales Manager, Publicity Manager and the Credit Manger report to the Director-Marketing, who along with Direct-Manufacturing reports to the Managing Director of the company.

The Accounts Department reports the following for the last quarter of 2012:

	Budgeted (₹)	***Actual (₹)***
Bad debt Losses	*5,000*	*3,000*
Cloth used	*31,000*	*36,000*
Advertising	*4,000*	*4,000*
Audit fees	*7,500*	*7,500*
Credit reports	*1,200*	*1,050*

Sales representative Travelling expenses	*9,000*	*10,200*
Sales commission	*7,000*	*7,000*
Cutting Labour	*6,000*	*6,600*
Thread	*500*	*450*
Sewing Labour	*17,000*	*18,400*
Credit Deptt. Salaries	*8,000*	*8,000*
Cutting utilities	*800*	*700*
Sewing utilities	*900*	*950*
Director Marketing salaries & Admn. Exp.	*20,000*	*21,400*
Production engineering expenses	*13,000*	*12,200*
Sales management office expenses	*16,000*	*15,700*
Production Manger office expenses	*18,000*	*17,000*
Direct Mfg. Salaries & Admn. Expenses	*21,000*	*20,100*

Using the above data, prepare Responsibility Accounting reports for the Director- marketing, the Director-manufacturing and the Production manager.

Solution

Responsibility Accounting Reports

For the Production Manager

Cutting Department	**Budgeted (₹)**	**Actual (₹)**	**Variance (₹)**
Cloth	31,000	36,000	5,000 (A)
Cutting Labour	6,000	6,600	600 (A)
Cutting Utilities	800	700	100 (F)
Total Cutting Deptt. (A)	37,800	43,300	5,500 (A)
Sewing Department:	**Budgeted (₹)**	**Actual (₹)**	**Variance (₹)**
Thread	500	450	50 (F)
Sewing Labour	17,000	18,400	1,400 (A)
Sewing Utilities	900	950	50(A)
Total Sewing Dept. (B)	18,400	19,800	1,400 (A)
Total (A + B)	56,200	63,100	6,900 (A)

For the Director-Manufacturing

	Budgeted (₹)	Actual (₹)	Variance (₹)
Production Department *	56,200	63,100	6,900 (A)
Production Engineering Expenses	13,000	12,200	800 (F)
Production Manager-Office Expenses	18,000	17,000	1,000 (F)
Total	87,200	92,300	5,100 (A)
*(*As per responsibility accounting report for the production manager)*			

For the Direct-Marketing

	Budgeted (₹)	Actual (₹)	Variance (₹)
Sales Representative:			
Travelling Expenses	9,000	10,200	1,200 (A)
Sales Commission	7,000	7,000	--
Total (A)	16,000	17,200	1,200 (A)
Sales Management:			
Office Expenses	16,000	15,700	300 (F)
Advertising	4,000	4,000	—
Total (B)	20,000	19,700	300 (F)
Credit Department:			
Salaries	8,000	8,000	—
Credit Reports	1,200	1,050	150 (F)
Bad Debt Losses	5,000	3,000	2,000 (F)
Total (C)	14,200	12,050	2,150 (F)
Total (A) + (B) + (C)	50,200	48,950	1,250 (F)

Note: 'F' denotes favourable variance while 'A' denotes adverse variance.

Illustration 16

The following data relate to a company which had a profit approved for selling 5,000 units per month at an average selling price of ₹ 10 per unit and budgeted variable cost of production was ₹ 4 per unit and fixed costs were budgeted at ₹ 20,000. Planned income being ₹ 10,000 per month. Because of shortage of raw—materials the plant could produce only 4,000 units and the cost of production was increased by 0.50 per unit. Consequently ₹ 1.00 raised the selling price per unit. To modify production processes in order to meet materials shortage, the Company incurred an expenditure of ₹ 1,000 in Research and Development. Set out a Performance budget and a summary report there.

Solution

Performance Budget

		Original Plan (₹)	Revised Budgeted (₹)	Actual Result (₹)	Variance (₹)
Revenue	(5,000×10)	50,000			
	(4,000×10)		40,000		
	(4,000×11)			44,000	4,000 (F)
Variable Costs	(5,000×4)	20,000			
	(4,000×4)		16,000		
	(4,000×4.5)			18,000	2,000 (A)
Contribution	(5,000×6)	30,000			
	(4,000×6)		24,000		
	(4,000×6.5)			26,000	2,000 (F)
Fixed costs		20,000	20,000	21,000	1,000 (A)
Net Profit		10,000	4,000	5,000	1,000 (F)

Summary Report on Profit Plan

	(₹)
Planned Income (from Project plan)	10,000
Activity Variance (lost contribution margin due to shortage of materials)	(6,000)
Selling Price Variance (increased Selling Price of ₹ 1/- per unit)	4,000
Variance Cost Variance (increased production Costs at 0.50 per unit)	(2,000)
Fixed Cost Variance (new research programme to *develop raw materials* and processes)	(1,000)
Actual Income (from income statement)	5,000

Budget Ratio

Illustration 17

A company manufactures two products X and Y, Product X requires 4 hours to produce while Y requires 6 hours. In Jun, 2012, of 26 effective working days of 8 hours a day, 600 units of X and 400 units of Y were produced. The company employs 20 workers in production department to produce X and Y. The budgeted hours are 36,000 for the year.

Calculate Capacity, Activity and efficiency ratio.

Solution

Standard Hours Produced

	Product X	*Product Y*	*Total*
Output (units)	600	400	
Hours (per unit)	4	6	
Standard Hours	2,400	2,400	4,800

Actual Hours Worked

20 Workers × 8 Hours × 26 Days = 4,160

Budgeted Hours *per month*

36,000/12 = 3,000

$$\text{Capacity Ratio} = \frac{\text{Actual Hours}}{\text{Budgeted Hours}} \times 100 = \frac{4,160}{3,000} = 138.67\%$$

$$\text{Efficiency Ratio} = \frac{\text{Standard Hours Produced}}{\text{Actual Hours}} \times 100 = \frac{4,800}{4,160} \times 100 = 115.38\%$$

$$\text{Activity Ratio} = \frac{\text{Standard Hours Produced}}{\text{Budget Hours}} \times 100 = \frac{4,800}{3,000} \times 100 = 160\%$$

5
Standard Costing

LEARNING OBJECTIVES
After studying this unit you will be able to understand:
• The meaning of standard costing and its definition
• How a standard costing system operates
• How to calculate material, labour, overhead, sales variances and reconcile actual profit with budgeted profit
• Distinguish between standard variable costing and standard absorption costing
• How to prepare a set of accounts for standard costing system.

5.1 Classification and Type of Variances

Cost variance is the difference between standard cost and the actual cost incurred. .

Variance analysis is the analysis of the cost variances into its component parts with appropriate justification of such variances, so that we can approach for corrective measures.

5.1.1 Classification of Variances: Variances can be established under material, labour & overheads. There are three distinct groups of variances that arise in standard costing which are

- **Variances of Efficiency***:* Variances due to the effective or ineffective use of materials quantities, labour hours, once actual quantities are compared with the predetermined standards.
- **Variances of Price Rates***:* Variances arising due to change in unit material prices, standard labour hour rates and standard allowances for indirect costs.
- **Variances Due to Volume:** Variance due to the effect of difference between actual activity and the level of activity assumed when the standard was set.

5.1.2 Why Standard Costing: Standard Costing main purpose is to

- Investigate the reasons
- Identify the problems
- Take corrective action.

Variances are broadly of two types, controllable and uncontrollable. Controllable variances are those which can be controlled by the departmental heads whereas uncontrollable variances are those which are beyond control.

For example, price variance is normally regarded as uncontrollable if the price increase is due to market fluctuations. It becomes controllable if the production controller has failed to place orders in time and urgent purchase was made at extra cost. In the former case, no responsibility is attached to any one whereas the departmental head has responsibility for the loss in the latter case. Since all price variances are uncontrollable and are of significant nature and are persistent, the standard may need revision.

The possible reasons for each type of variances and the suggested course of action are given below. This list is only illustrative and not exhaustive.

Type of Variance	Reasons of Variance	Suggestive Course of Action
Material		
Material Price	• Change in Basic Price • Fail to purchase the anticipated standard quantities at appropriate price	• Departmental head should take necessary action to purchase at right point of time • Cash discount or interest rate for payment of purchase should be consider at the time of such payment • Price check on the purchase of standard quality materials
Material Usage	• Use of sub-standard material • Ineffective use of materials • Pilferage • Non standardised mix	• Regular Inspection of quality of materials • Proper training of operators • Ensure best utilisation of resources
Labour		
Labour Efficiency	• Change in design and quality standard • Poor working conditions • Improper scheduling	• Proper planning • Proper training • Healthy working environment • Timelines for achieving set targets

Type of Variance	Reasons of Variance	Suggestive Course of Action
Labour Rate	• Improper placement of labour • Increments / high labour wages • Overtime	• Time scheduling for work performance • Proper job allocation according to capabilities of workers
Overheads		
Manufacturing	• Improper planning • Under or over absorption of fixed overheads • Reduction of sales • Breakdowns • Power failure • Labour trouble	• Efficient planning for better capacity utilization • Check on expenditure
Selling and Distribution	• Increase in delivery cost • Increase in stock holding period • Overtime	• Sales quotas • Sale targets
Administrative	• Over expenditure	• Comparison of budgets with actuals • Introduction of operating costing • Introduction of cost ratios

5.2 Computation of Variances

Let us now proceed to study with illustrations and the method of computation of major variances. In all the problems illustrated in the following pages, 'F' means favourable variance and 'A' means adverse variance.

5.2.1 Direct Material Variances : Direct material total variance (also known as material cost variance) for actual output can basically be divided into two types, namely (a) price variance and (b) usage variance. The method of calculating these variances is as under:

Direct Material Variances

Direct Material Total Variance[#]

[Standard Cost[*] – Actual Cost]

(The difference between the Standard Direct Material Cost *of the actual production volume* and the Actual Cost of Direct Material)

$[(SQ \times SP) - (AQ \times AP)]$

Direct Material Price Variance	Direct Material Usage Variance
[Standard Cost of Actual Quantity – Actual Cost]	[Standard Cost of Standard Quantity for Actual Production – Standard Cost of Actual Quantity]
(The difference between the Standard Price and Actual Price for the Actual Quantity)	(The difference between the Standard Quantity *specified for actual production* and the Actual Quantity *used*, at Standard Purchase Price)
$[(SP - AP) \times AQ]$ *Or* $[(SP \times AQ) - (AP \times AQ)]$	$[(SQ - AQ) \times SP]$ *Or* $[(SQ \times SP) - (AQ \times SP)]$

Direct Material Mix Variance	Direct Material Yield Variance
[Standard Cost of Actual Quantity in Standard Proportion – Standard Cost of Actual Quantity]	[Standard Cost of Standard Quantity for Actual Production – Standard Cost of Actual Quantity in Standard Proportion]
(The difference between the Actual Quantity *in standard proportion* and Actual Quantity *in actual proportion*, at Standard Purchase Price)	(The difference between the Standard Quantity *specified for actual production* and Actual Quantity *in standard proportion*, at Standard Purchase Price)
$[(RAQ - AQ) \times SP]$ *Or* $[(RAQ \times SP) - (AQ \times SP)]$	$[(SQ - RAQ) \times SP]$ *Or* $[(SQ \times SP) - (RAQ \times SP)]$
Alternative Formula [Total Actual Quantity (units) × {*Average* Standard Price *per unit* of Standard Mix Less *Average* Standard Price *per unit* of Actual Mix}]	***Alternative Formula*** [*Average* Standard Price *per unit* of Standard Mix × {Total Standard Quantity (units) Less Total Actual Quantity (units)}]

Note:

SQ	=	Standard Quantity = Expected Consumption for Actual Output
AQ	=	Actual Quantity *of Material Consumed*
RAQ	=	Revised Actual Quantity = Actual Quantity Rewritten in Standard Proportion
SP	=	Standard Price per Unit
AP	=	Actual Price per Unit
(*)	=	Standard Cost refers to 'Standard Cost of Standard Quantity for Actual Output'
(#)	=	Direct Material Total Variance (also known as material cost variance)

Material Purchase Price Variance

[Standard Cost of Actual Quantity – Actual Cost]

(The difference between the Standard Price and Actual Price *for the actual quantity of material purchased*)

[(SP – AP) × PQ]

Or

[(SP × PQ) – (AP × PQ)]

Note:

PQ	=	Purchase Quantity
SP	=	Standard Price
AP	=	Actual Price

Illustration 1

The standard quantity of material required is 4 kgs. per unit of actual output. The relevant figures are as under:

Material	*A*	*B*	*C*	*D*
Standard mix %	*30%*	*40%*	*20%*	*10%*
Price per kg. (₹)	*1.25*	*1.50*	*3.50*	*3.00*
Actual qty. used (Kg.)	*1,180*	*1,580*	*830*	*440*
Actual price per kg. (₹)	*1.30*	*1.80*	*3.40*	*3.00*
Actual output: 1,000 units				

Calculate price variance, mix variance, sub-usage variance and total material cost variance.

Solution

Basic Calculations:

Statement showing computation of Standard Cost/Actual Cost/Revised Actual Quantity

Material	Standard Cost of 1,000 Units			Actual Cost of 1,000 Units			Revised Actual Quantity [RAQ] (Kg.)
	Quantity [SQ] (Kg.)	Price [SP] (₹)	Amount [SQ × SP] (₹)	Quantity [AQ] (Kg.)	Price [AP] (₹)	Amount [AQ × AP] (₹)	
A	1,200	1.25	1,500	1,180	1.30	1,534	1,209
B	1,600	1.50	2,400	1,580	1.80	2,844	1,612
C	800	3.50	2,800	830	3.40	2,822	806
D	400	3.00	1,200	440	3.00	1,320	403
	4,000		7,900	4,030		8,520	4,030

Note:

SQ = Standard Quantity = Expected Consumption for Actual Output

AQ = Actual Quantity of Material Consumed

RAQ = Revised Actual Quantity = Actual Quantity Rewritten in Standard Proportion

SP = Standard Price per unit

AP = Actual Price per unit

Computation of Variances:

Statement showing Variances (₹)

	Material A	Material B	Material C	Material D	Total
Material Cost Variance = SQ × SP – AQ × AP	= 1,200 × 1.25 – 1,180 × 1.30 = 1,500 –1,534 = 34 (A)	= 1,600 × 1.50 – 1,580 × 1.80 = 2,400 – 2,844 = 444 (A)	= 800 × 3.50 – 830 × 3.40 = 2,800 – 2,822 = 22 (A)	= 400 × 3.00 – 440 × 3.00 = 1,200 – 1,320 = 120 (A)	620 (A)
Material Price Variance = AQ × (SP –AP)	= 1,180 × (1.25 – 1.30) = 59 (A)	= 1,580 × (1.50 × 1.80) = 474 (A)	= 830 × (3.50 – 3.40) = 83 (F)	= 440 × (3.00 – 3.00) = 0	450 (A)
Material Usage	= 1.25 × (1,200 –	= 1.50 × (1,600 – 1,580)	= 3.50 × (800 – 830)	= 3.00 × (400 – 440)	170 (A)

Variance = SP × (SQ – AQ)	1,180) = 25(F)	= 30 (F)	= 105 (A)	= 120 (A)	
Material Mix Variance = SP × (RAQ – AQ)	= 1.25 × (1,209 – 1,180) = 36.25 (F)	= 1.50 (1,612 – 1,580) = 48 (F)	= 3.50 × (806 – 830) = 84 (A)	= 3.00 × (403 – 440) = 111 (A)	110.75 (A)
Material Yield Variance = SP × (SQ – RAQ)	= 1.25 × (1,200 – 1,209) = 11.25 (A)	= 1.50 × (1,600 – 1,612) = 18 (A)	= 3.50 × (800 – 806) = 21 (A)	= 3.00 × (400 – 403) = 9 (A)	59.25 (A)

Illustration 2

The standard set for a chemical mixture of a firm is as under:

Material	***Standard Mix %***	***Standard Price Per Kg. (₹)***
A	*40*	*20*
B	*60*	*30*

The standard loss in production is 10 %. During a period, the actual consumption and price paid for a good output of 182 kg. are as under:

Material	***Quantity in Kg.***	***Actual Price Per Kg (₹)***
A	*90*	*18*
B	*110*	*34*

Calculate the variances.

Solution

Basic Calculations:

Take the good output of 182 Kg. The standard quantity of material required for 182 Kg. of output is

$$\frac{182}{90} \times 100 = 202.22 \text{ Kg.}$$

Statement showing computation of Standard Cost/Actual Cost/ Revised Actual Quantity

Material	Standard Cost			Actual Cost			Revised Actual Quantity [RAQ] (Kg.)
	Quantity [SQ] (Kg.)	Rate [SP] (₹)	Amount [SQ × SP] (₹)	Quantity [AQ] (Kg.)	Rate [AP] (₹)	Amount [AQ × AP] (₹)	
A (40% of 202.22 Kg.)	80.89	20	1,617.80	90	18	1,620	80
B (60% of. 202.22 Kg.)	121.33	30	3,639.90	110	34	3,740	120
	202.22		5,257.70	200		5,360	200

Note :

SQ = Standard Quantity = Expected Consumption for Actual Output

AQ = Actual Quantity of Material Consumed

RAQ = Revised Actual Quantity = Actual Quantity Rewritten in Standard Proportion

SP = Standard Price Per Unit

AP = Actual Price Per Unit

Computation of Variances:

Material Price Variance = AQ × (SP – AP)

A = 90 Kg. × (₹ 20 – ₹ 18) = ₹ 180 (F)

B = 110 Kg. × (₹ 30 – ₹ 34) = ₹ 440 (A)

Total = ₹ 180 (F) + ₹ 440 (A)

= ₹ 260 (A)

Material Usage Variance= SP × (SQ – AQ)

A = ₹ 20 × (80.89 Kg. – ₹ 90 Kg.) = ₹182.20 (A)

B = ₹ 30 × (121.33 Kg. – 110 Kg.) = ₹ 339.90 (F)

Total = ₹ 182.20 (A) + ₹ 339.90 (F)

= ₹ 157.70 (F)

Material Mix Variance = SP × (RAQ – AQ)

A = ₹ 20 × (80 Kg – 90 Kg) = ₹200 (A)

B = ₹ 30 × (120 Kg. – 110 Kg.) = ₹ 300 (F)

Total = ₹ 200 (A) + ₹ 300 (F)

= ₹ 100 (F)

Material Yield Variance = SP × (SQ – RAQ)

A = ₹ 20 × (80.89 Kg. – ₹ 80 Kg) = ₹ 17.80 (F)

B = ₹ 30 × (121.33 Kg. – 120 Kg.) = ₹ 39.90 (F)

Total = ₹ 17.80 (F) + ₹ 39.90 (F)

= ₹ 57.70 (F)

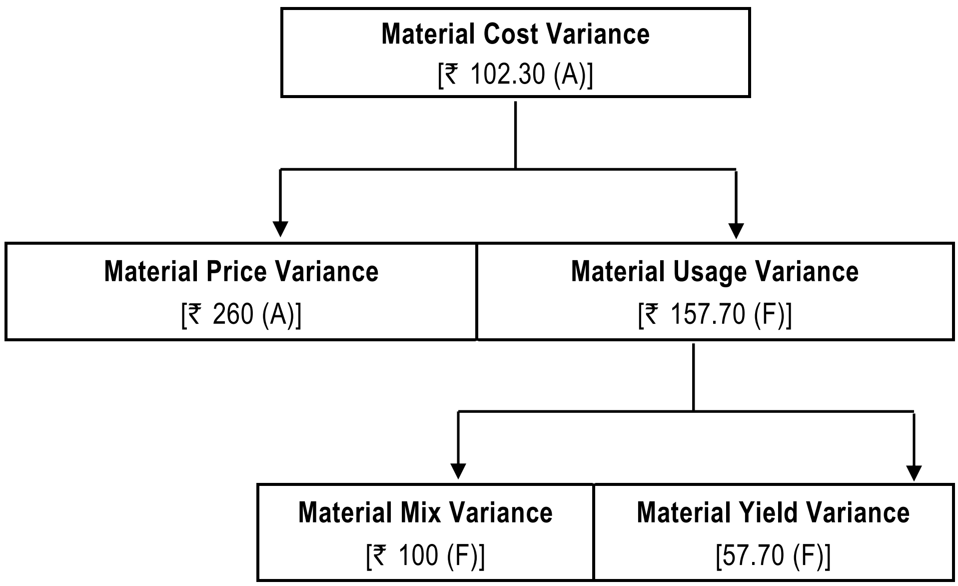
Material Cost Variance = SQ × SP – AQ × AP

A = ₹ 80.89 Kg. × ₹ 20– 90 Kg. × ₹ 18 = ₹ 2.20 (A)

B = ₹ 121.33 Kg. × 30 Kg. – 110 Kg. × ₹ 34 = ₹ 100.10 (A)

Total = ₹ 2.20 (A) + ₹ 100.10 (A)

= ₹ 102.30 (A)



5.2.2. Direct Labour Variances: The two basic variances that can be calculated in respect of direct labour total variance (also known as labour cost variance) are (a) rate variance and (b) efficiency variance. The formula's for calculating labour variances are as under:

Direct Labour Variances

Direct Labour Total Variance[1]

[Standard Cost[2] – Actual Cost]

(The difference between the Standard Direct Labour Cost and the Actual Direct Labour Cost *incurred for the production achieved*)

$[(SH \times SR) - (AH^* \times AR)]$

Direct Labour Rate Variance

[Standard Cost of Actual Time – Actual Cost]

(The difference between the Standard Rate *per hour* and Actual Rate *per hour* for the Actual Hours *paid*)

$[(SR - AR) \times AH^*]$

Or

$[(SR \times AH^*) - (AR \times AH^*)]$

Direct Labour Idle Time Variance

[Standard Rate per Hour x Actual Idle Hours]

(The difference between the Actual Hours *paid* and Actual Hours *worked* at Standard Rate)

$[(AH^* - AH^{\#}) \times SR]$

Or

$[(AH^* \times SR) - (AH^{\#} \times SR)]$

Direct Labour Efficiency Variance

[Standard Cost of Standard Time for Actual Production – Standard Cost of Actual Time]

(The difference between the Standard Hours *specified for actual production* and Actual Hours *worked* at Standard Rate)

$[(SH - AH^{\#}) \times SR]$

Or

$[(SH \times SR) - (AH^{\#} \times SR)]$

Direct Labour Mix Variance Or Gang Variance

[Standard Cost of Actual Time *Worked* in Standard Proportion – Standard Cost of Actual Time *Worked*]

(The difference between the Actual Hours *worked in standard proportion* and Actual Hours *worked in actual proportion*, at Standard Rate)

$[(RAH - AH^{\#}) \times SR]$

Or

$[(RAH \times SR) - (AH^{\#} \times SR)]$

Direct Labour Yield Variance Or Sub-Efficiency Variance

[Standard Cost of Standard Time for Actual Production – Standard Cost of Actual Time *Worked* in Standard Proportion]

(The difference between the Standard Hours *specified for actual production* and Actual Hours *worked in standard proportion*, at Standard Rate)

$(SH - RAH) \times SR$

Or

$(SH \times SR) - (RAH \times SR)$

Alternate Formula	***Alternate Formula***
[Total Actual Time *Worked* (hours) × {*Average* Standard Rate *per hour* of Standard Gang Less *Average* Standard Rate *per hour* of Actual Gang@}] @ *on the basis of hours worked*	[*Average* Standard Rate *per hour* of Standard Gang × {Total Standard Time (hours) Less Total Actual Time *Worked* (hours)}]

Note:

SH = Standard Hours = Expected time (Time allowed) for Actual Output

AH* = Actual Hours *paid for*

AH# = Actual Hours *worked*

RAH = Revised Actual Hours = Actual Hours (worked) rewritten in Standard Proportion

SR = Standard Rate per Labour Hour

AR = Actual Rate per Labour Hour Paid

(2) = Standard Cost refers to 'Standard Cost of Standard Time for Actual Output'

(1) = Direct Labour Total Variance (also known as labour cost variance)

In the absence of idle time

Actual Hours Worked = Actual Hours Paid

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Idle Time is a period for which a workstation is available for production but is not used due to e.g. shortage of tooling, material or operators. During Idle Time, Direct Labour Wages are being paid but no output is being produced. The cost of this can be identified separately in an Idle Time Variance, so that it is not 'hidden' in an adverse Labour Efficiency Variance.

Some organizations face Idle Time on regular basis. In this situation the Standard Labour Rate may include an allowance for the cost of the expected idle time. Only the impact of any unexpected or abnormal Idle Time would be included in the Idle Time Variance.

Illustration 3

Given the following data, compute the variances.

	Skilled	***Semi-Skilled***	***Unskilled***
Number of workers in standard gang	*16*	*6*	*3*
Standard rate per hour (₹)	*3*	*2*	*1*
Actual number of workers in the gang	*14*	*9*	*2*
Actual rate of pay per hour (₹)	*4*	*3*	*2*

In a 40- hour week, the gang as a whole produced 900 standard hours.

Solution

Basic Calculations:

In a 40 hour week, the standard gang should have produced 1,000 std. hours as shown below

Skilled	16 No. of workers × 40 hrs.	640
Semi – Skilled	6 No. of workers × 40 hrs.	240
Unskilled	3 No. of workers × 40 hrs.	120
		1,000 hours

However, the actual output is 900 standard hours. Hence to find out the total labour cost variance, the standard cost (or cost charged to production) is to be computed with reference to 900 standard hours. This is done in the following statement:

Statement showing the Standard Cost, Actual Cost and Standard Cost of Actual Time for Actual Output, i.e. 900

Gang	Standard Cost			Actual Cost			Std Cost of Actual Time		
	Hours	Rate	Amt	Hours	Rate	Amt	Hours	Rate	Amt
Skilled	576	3	1,728	560	4	2,240	560	3	1,680
Semi-Skilled	216	2	432	360	3	1,080	360	2	720
Unskilled	108	1	108	80	2	160	80	1	80
Total	**900**	**2.52**	**2,268**	**1000**	**3.48**	**3,480**	**1,000**	**2.48**	**2,480**

Computation of Variances:

Cost Variance = Std. Labour Cost – Actual Labour Cost

= ₹ 2,268 – ₹ 3,480 = ₹ 1,212 (A)

Rate Variance = Actual Time x (Std. Rate – Actual Rate)

= (Standard Cost of Actual Time – Actual Cost)

= ₹ 2,480 – ₹ 3,480 = ₹ 1,000 (A)

Efficiency variance = Std. Rate x (Std. Time – Actual Time)

= Standard Cost – Std. Cost of Actual Time

= ₹ 2,268 – ₹ 2,480 = ₹ 212 (A)

Gang Variance = Total Actual Time x (Std. Rate of Std. Gang– Std. Rate of Actual Gang)

= 1,000 x (₹ 2.52 – ₹ 2.48) = ₹ 40(F)

Sub-Efficiency Variance = Std. Rate of Std. Gang x (Total Std. Time – Total Actual Time)

= ₹ 2.52 x (900 Hours – 1,000 Hours) = ₹ 252 (A)

Illustration 4

A firm gives you the following data:

Standard time per unit	*2.5 hours*
Actual hours (paid)	*2,000 hours*
Standard rate of pay	*₹ 2 per hour*

25 % of the actual hours (paid) have been lost as idle time.

Actual output	*1,000 units*
Actual wages (paid)	*₹ 4,500*

Calculate the idle time variance.

Solution

Basic Calculations:

Standard Cost (1,000 units × 2.5 hours × ₹ 2)	₹ 5,000
Actual wages paid	₹ 4,500
Actual wage rate per hour (₹ 4,500 ÷2,000 hours)	₹ 2.25
Std. wage rate per hour	₹ 2.00
Abnormal idle time (25% of 2,000 hours)	500 hrs

Computation of Variances:

Rate Variance	= Actual Time x (Std. Rate – Actual Rate)
	= 2,000 Hours x (₹ 2 – ₹ 2.25) = ₹ 500 (A)
Efficiency Variance	= Std. Rate x (Std. Time – Actual time*)
	= ₹ 2 x (2,500 hrs. –1,500 hrs.) = ₹ 2,000 (F)
Idle time Variance	= Idle Time × Std. Rate
	= 500 hrs. × ₹ 2 = ₹ 1,000 (A)
Total Variance	= Std. Labour Cost – Actual Labour Cost
	= ₹ 5,000 – ₹ 4,500 = ₹ 500 (F)

**Actual time less idle time (Actual time worked).*

5.2.3 Overhead Variances: Overhead variances arise due to the difference between actual overheads and absorbed overheads.

The actual overheads can be known only at the end of the accounting period, when the expense accounts are finalised. The absorbed overheads are the overheads charged to each unit of production on the basis of a pre-determined overhead rate. This pre-determined rate is also known as standard overhead recovery rate, standard overhead absorption rate or standard burden rate. To calculate the standard overhead recovery rate, we have to first make an estimate of the likely overhead expenses for each department for the next year. The estimate of budget of the overheads is to be divided into fixed and variable elements. An estimate of the level of normal capacity utilisation is then made either in terms of production or machine hours or direct labour hours.

The estimated overheads are divided by the estimated capacity level to calculate the pre-determined overhead absorption rate as shown below:

$$\text{Standard Fixed Overhead Rate} = \frac{\text{Budgeted Fixed Overheads}}{\text{Normal Volume}}$$

$$\text{Standard Variable Overhead Rate} = \frac{\text{Budgeted Variable Overheads}}{\text{Normal Volume}}$$

Overhead variances can be classified in the following two major categories:

a) Fixed Overhead Variances

b) Variable Overhead Variances

(a) Fixed Overhead Variances:

Fixed overhead total variance (also known as fixed overhead cost variance) as may be broadly classified into:

Expenditure Variance: It represents the difference between the fixed overheads as per budget and the actual fixed overheads incurred.

Volume Variance: This variance represents the unabsorbed portion of the fixed costs because of underutilization of capacity. In case a firm exceeds capacity, this variance is favourable in nature. This can be divided into capacity variance and efficiency variance.

Fixed Production Overhead Variances

Fixed Overhead Total Variance@
(Absorbed Fixed Overheads) *Less* (Actual Fixed Overheads)

Fixed Overhead Expenditure Variance
(Budgeted Fixed Overheads)
Less
(Actual Fixed Overheads)

Fixed Overhead Volume Variance
(Absorbed Fixed Overheads)
Less
(Budgeted Fixed Overheads)

Fixed Overhead Capacity Variance
(Budgeted Fixed Overheads for Actual Hours#)
Less
(Budgeted Fixed Overheads)

Fixed Overhead Efficiency Variance
(Absorbed Fixed Overheads)
Less
(Budgeted Fixed Overheads for Actual Hours#)

Or

Fixed Overhead Capacity Variance
(Budgeted Fixed Overheads for Actual Hours#)
Less
(Possible Fixed Overheads)

Fixed Overhead Calendar Variance
(Possible Fixed Overheads)
Less
(Budgeted Fixed Overheads)

Fixed Overhead Efficiency Variance
(Absorbed Fixed Overhead)
Less
(Budgeted Fixed Overheads for Actual Hours#)

 # Actual Hours (Worked)

Note:

Standard Fixed Overheads for Production (Absorbed)

= Standard Fixed Overhead Rate per Unit × Actual Production in Units

= Standard Fixed Overhead Rate per Hour × Standard Hours for Actual Production

Budgeted Fixed Overheads

= It represents the amount of fixed overhead which should be spent according to the budget or standard during the period

= Standard Fixed Overhead Rate per Unit × Budgeted Production in Units

= Standard Fixed Overhead Rate per Hour × Budgeted Hours

Actual Fixed Overheads Incurred

Budgeted Fixed Overheads for Actual Hours

= Standard Fixed Overhead Rate per Hour × Actual Hours

Possible Fixed Overheads

= Expected Fixed Overhead for Actual Days Worked

$$= \frac{\text{Budgeted Fixed Overhead}}{\text{Budgeted Days}} \times \text{Actual Days}$$

(@)

= Fixed Overhead Total Variance also known as 'Fixed Overhead Cost Variance'

Fixed Overhead Efficiency Variance
(Absorbed Fixed Overheads) – (Budgeted Fixed Overheads for Actual Hours)
Or
(Standard Fixed Overhead Rate per Hour × Standard Hours for Actual Output) – (Standard Fixed Overhead Rate per Hour × Actual Hours)
Or
Standard Fixed Overhead Rate per Hour × (Standard Hours for Actual Output – Actual Hours)

Fixed Overhead Capacity Variance
(Budgeted Fixed Overheads for Actual Hours) – (Budgeted Fixed Overheads)
Or
(Standard Fixed Overhead Rate per Hour × Actual Hours) – (Standard Fixed Overhead Rate per Hour × Budgeted Hours)
Or
Standard Fixed Overhead Rate per Hour × (Actual Hours – Budgeted Hours)
Fixed Overhead Volume Variance-I
(Absorbed Fixed Overheads) – (Budgeted Fixed Overheads)

Or

(Standard Fixed Overhead Rate per Unit × Actual Output) – (Standard Fixed Overhead Rate per Unit × Budgeted Output)

Or

Standard Fixed Overhead Rate per Unit × (Actual Output – Budgeted Output)

Fixed Overhead Volume Variance-II
(Absorbed Fixed Overheads) – (Budgeted Fixed Overheads)
Or
(Standard Fixed Overhead Rate per Hour × Standard Hours for Actual Output) – (Standard Fixed Overhead Rate per Hour × Budgeted Hours)
Or
Standard Fixed Overhead Rate per Hour × (Standard Hours for Actual Output – Budgeted Hours)
Or
Standard Fixed Overhead Rate per Hour × (Standard Hours per Unit × Actual Output – Standard Hours per Unit × Budgeted Output)
Or
(Standard Fixed Overhead Rate per Hour × Standard Hours per Unit) × (Actual Output – Budgeted Output)
Or
Standard Fixed Overhead Rate per Unit × (Actual Output – Budgeted Output)

☞

Overhead Variances can also be affected by idle time. It is usually assumed that Overheads are incurred when labour is working, not when it is idle. Accordingly, <u>hours worked</u> has been considered for the calculation of Variable and Fixed Overheads Variances.

Illustration 5

You are given the following data:

	Budgeted	***Actual***
Fixed overhead for July	*₹ 10,000*	*₹ 10,200*
Units of production in July	*5,000*	*5,200*
Standard time for one unit	*4 hours*	
Actual hours worked		*20,100 hours*

Calculate all variances relating to fixed overheads

Solution

Basic Calculations:

1. Absorbed Fixed Overheads = Standard Rate per Hour × Standard Hours for Actual Output
   = ₹ 0.50# × (4 Hours × 5,200 Units)
   = ₹ 10,400
   OR
   = Standard Rate per unit@ × Actual Output
   = ₹ 2 × 5,200 Units
   = ₹ 10,400

Standard Rate per Hour = ₹ 10,000 (Budgeted Fixed Overheads) / [5,000 units (Budgeted Output) x 4 hours (Budgeted Hours)]
= ₹ 0.50

@ Standard Rate per Unit = ₹ 10,000 (Budgeted Fixed Overheads) / 5,000 units (Budgeted Output)
= ₹ 2

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2. Budgeted Fixed Overheads = ₹ 10,000
3. Actual Fixed Overheads = ₹ 10,200
4. Budgeted Fixed Overheads for Actual Hours = ₹ 0.50 × 20,100 Hrs
   = ₹ 10,050

Computation of Variances:

Fixed Overhead Cost Variance = Absorbed Fixed Overheads – Actual Fixed Overheads
= ₹ 10,400 – ₹ 10,200
= ₹ 200 (F)

Fixed Overhead Expenditure Variance = Budgeted Fixed Overheads – Actual Fixed Overheads
= ₹ 10,000 – ₹ 10,200
= ₹ 200 (A)

Fixed Overhead Volume Variance = Absorbed Fixed Overheads – Budgeted Fixed Overheads
= ₹ 10,400 – ₹ 10,000 = ₹ 400 (F)

Fixed Overhead Capacity Variance	= Budgeted Fixed Overheads for Actual Hours – Budgeted Fixed Overheads
	= ₹ 10,050 – ₹ 10,000
	= ₹ 50 (F)
Fixed Overhead Efficiency Variance	= Absorbed Fixed Overheads – Budgeted Fixed Overheads for Actual Hours
	= ₹ 10,400 – ₹ 10,050
	= ₹ 350 (F)

Calender Variance: Calender Variance arises due to the fact that the estimated fixed overheads are the same for each month or period irrespective of the actual number of working days. It is that portion of the volume variance which is due to the difference between the number of working days in the budget period and the number of actual working days in the period to which the budget is applied. The number of working days in the budget period are arrived at simply by dividing the number of annual days by twelve.

Illustration 6

Assuming the expenses to be fixed, calculate from the following data:

(a) Efficiency variance, (b) Volume variance, (c) Calendar variance and (d) Expense variance

	Budget	Actual
No. of working days per month	*20*	*22*
Man hours per day	*8,000*	*8,400*
Output per man hour in units	*1.0*	*1.2*
Standard overhead rate per man hour	*₹ 2*	
Actual fixed expenses per month		*₹ 3,25,000*

Solution

Basic Calculations:

1. Actual Output: 8,400 Hours × 22 Days × 1.2 Units per Hour = 2,21,760 Units
2. Standard Output per Man Hour: 1
3. Standard Hours Produced or Std. Hrs. for Actual Production: 2,21,760 Units x 1 Hr. = 2,21,760 Hrs.
4. Budgeted Hrs.: 8,000 Hours x 20 Days = 1,60,000 Hours
5. Possible Hours: 8,000 Hours x 22 Days = 1,76,000 Hours
6. Actual Hours Worked: 8,400 Hours x 22 Days = 1,84,800 Hours
7. Overheads as per Budget: 8,000 Hours × 20 Days x ₹ 2 per Hour = ₹ 3,20,000

Computation of Variances:

Efficiency variance	=	Std. Fixed Overhead Rate per Hour x (Std. Hrs. for Actual Production – Actual Hrs.)
	=	₹ 2 x (2,21,760 Hours – 1,84,800 Hours) = ₹ 73,920 (F)
Capacity variance	=	Standard Fixed Overhead Rate per Hour x (Actual Hours – Possible Hours)
	=	₹ 2 x (1,84,800 Hours – 1,76,000 hours) = ₹ 17,600 (F)
Calendar variance	=	Standard Fixed Overhead Rate per Hour x (Possible Hours – Budgeted Hours)
	=	₹ 2 x (1,76,000 Hours – 1,60,000 Hours) = ₹ 32,000 (F)
Volume variance	=	Standard Fixed Overhead Rate per Hour x (Standard Hours for Actual Output – Budgeted Hours)
	=	₹ 2 x (2,21,760 Hours – 1,60,000 Hours) = ₹ 1,23,520(F)
Expenses variance	=	Budgeted Expenses – Actual Expenses
	=	₹ 3,20,000 – ₹ 3,25,000 = ₹ 5,000 (A)
Total variance	=	Absorbed Overheads – Actual Overheads
	=	₹ 2,21,760 x ₹ 2 – ₹ 3,25,000
	=	₹ 4,43,520 – ₹ 3,25,000 = ₹ 1,18,520 (F)

(b) Variable Overhead Variances:

These variances arise due to the difference between the standard variable overheads for actual output and the actual variable overheads. The variable overhead total variance (also known as variable overhead cost variance) can be analysed further as:-

Variable Overhead Expenditure Variance: It is that part of variable overhead variance which arises due to the difference between the budgeted variable overhead and the actual variable overhead incurred.

Variable Overhead Efficiency Variance: It is that part of variable overhead variance which arises due to the difference between standard hours required for actual output and the actual hours worked. It can be computed by multiplying the difference of standard and actual hours by the standard variable overhead rate per hour. If standard hours exceed the actual hours worked, the variance will be favourable and vice versa.

Variable Production Overhead Variances

Variable Overhead Total Variance@
(Standard Variable Overheads for Production – Actual Variable Overheads)

Variable Overhead Expenditure (Spending) Variance	Variable Overhead Efficiency Variance
(Budgeted Variable Overheads for Actual Hours#) *Less* (Actual Variable Overheads)	(Standard Variable Overheads for Production) *Less* (Budgeted Variable Overheads for Actual Hours#)

Actual Hours (Worked)

Note:

Standard Variable Overheads for Production/Charged to Production

= Standard/Budgeted Variable Overhead Rate per Unit × Actual Production (Units)

= Standard Variable Overhead Rate per Hour × Standard Hours for Actual Production

Actual Overheads Incurred

Budgeted Variable Overheads for Actual Hours

= Standard Variable Overhead Rate per Hour × Actual Hours

(@)

= Variable Overhead Total Variance also known as 'Variable Overhead Cost Variance'

Variable Overhead Expenditure Variance
(Budgeted Variable Overheads for Actual Hours) – (Actual Variable Overheads)
Or
(Standard Rate per Hour × Actual Hours) – (Actual Rate per Hour × Actual Hours)
Or
Actual Hours × (Standard Rate per Hour – Actual Rate per Hour)

Variable Overhead Efficiency Variance
(Standard Variable Overheads for Production) – (Budgeted Overheads for Actual Hours)
Or
(Standard Variable Overhead Rate per Hour × Standard Hours for Actual Output) – (Standard Variable Overhead Rate per Hour × Actual Hours)
Or
Standard Variable Overhead Rate per Hour × (Standard Hours for Actual Output – Actual hours)

Illustration 7

XYZ Company has established the following standards for variable factory overhead.

Standard hours per unit :	*6*
Variable overhead per hour :	*₹ 2*
The actual data for the month are as follows:	
Actual variable overheads incurred	*₹ 2,00,000*
Actual output (units)	*20,000*
Actual hours worked	*1,12,000*

Calculate variable overhead variances viz

Solution

Basic Calculations:

1.	Standard Variable Overheads for Production	=	Standard Rate per Hour × Standard Hours for Actual Output
		=	₹ 2 × [6 Hours × 20,000 Units]
		=	₹ 2,40,000
2.	Actual Variable Overheads	=	₹ 2,00,000
3.	Budgeted Variable Overheads for Actual Hours	=	Standard Rate per Hour × Actual Hours
		=	₹ 2 × 1,12,000 hours
		=	₹ 2,24,000

Computation of Variances:

Variable Overhead Cost Variance	=	Standard Variable Overheads for Production – Actual Variable Overheads
	=	₹ 2,40,000 – ₹ 2,00,000
	=	₹ 40,000 (F)
Variable Overhead Expenditure Variance	=	Budgeted Variable Overheads for Actual Hours – Actual Variable Overheads
	=	₹ 2,24,000 – 2,00,000
	=	₹ 24,000 (F)
Variable Overhead Efficiency Variance	=	Standard Variable Overheads for Production – Budgeted Variable Overheads for Actual Hours
	=	₹ 2,40,000 – ₹ 2,24,000
	=	₹ 16,000 (F)

Illustration 8

The overhead expense budget for a cost centre is as under:

Indirect material	*₹ 0.40 per hour*
Indirect labour	*₹ 0.60 per hour*
Maintenance	*₹ 0.40 per hour*
Power	*₹ 0.30 per hour*
Sundries	*₹ 0.30 per hour*
Total variable expenses	*₹ 2.00 per hour*
Fixed overhead budgeted	*₹ 240*

Budgeted output = 9,600 units or 120 standard hours.

At the end of a period the actual rates given by the accounts department are as under:

Power ₹ 0.32; maintenance ₹ 0.45; indirect labour ₹ 0.60; indirect material ₹ 0.50 and sundry expenses ₹ 0.29 per hour; total variable expenses were ₹ 2.16 per hour. The actual output is 12,160 units for which the actual hours worked are 156. The fixed expenses amounted to ₹ 250. Compute the variances.

Solution

Basic Calculations:

Expenses	**Overhead Expenses Schedule**			
	Budget: 120 Std. Hours		**Actual: 156 Hours**	
	Rate per hour ₹	**Expenses ₹**	**Rate per hour ₹**	**Expenses ₹**
Indirect material	0.40	48	0.50	78
Indirect labour	0.60	72	0.60	94
Maintenance	0.40	48	0.45	70
Power	0.30	36	0.32	50
Sundries	0.30	36	0.29	45
Total Variable Overheads	2.00	240	2.16	337
Fixed overheads	2.00	240		250
Total overheads		480		587

Actual output = 12,160 units.

Hence standard hours produced or std. hours for actual production

$$= \frac{120 \text{ std. hours}}{9,600 \text{ units}} \times 12,160 \text{ actual output} = 152 \text{ hours}$$

<u>Computation of Variances:</u> Fixed Overheads

Working Notes:

1. Absorbed Fixed Overhead = Standard Rate per Hour × Standard Hours for Actual Output

$$= ₹\ 2 \times \left(\frac{120 \text{ Hours}}{9{,}600 \text{ Units}} \times 12{,}160 \text{ Units}\right)$$

= ₹ 2 × 152 Hours = ₹ 304

2. Budgeted Fixed Overhead = ₹ 240
3. Actual Fixed Overhead = ₹ 250
4. Budgeted Fixed Overheads for Actual Hours = Standard Rate per Hour × Actual Hours

= ₹ 2 × 156 Hours = ₹ 312

Computation of Variances:

Fixed Overhead Cost Variance = Absorbed Fixed Overheads – Actual Fixed Overheads

= ₹ 304 – ₹ 250 = ₹ 54 (F)

Fixed Overhead Expenditure Variance = Budgeted Fixed Overheads – Actual Fixed Overheads

= ₹ 240 – ₹ 250 = ₹ 10 (A)

Fixed Overhead Volume Variance = Absorbed Fixed Overheads – Budgeted Fixed Overheads

= ₹ 304 – ₹ 240 = ₹ 64 (F)

Fixed Overhead Capacity Variance = Budgeted Fixed Overheads for Actual Hours – Budgeted Fixed Overheads

= ₹ 312 – ₹ 240 = ₹ 72 (F)

Fixed Overhead Efficiency Variance = Absorbed Fixed Overheads – Budgeted Fixed Overheads for Actual Hours

= ₹ 304 – ₹ 312 = ₹ 8 (A)

<u>Computation of Variances:</u> Variable Overhead

Working Notes:

1. Standard Variable Overheads for Production = Standard Rate per Hour × Standard Hours for Actual Output

$$= ₹\ 2 \times \left(\frac{120 \text{ Hours}}{9{,}600 \text{ Units}} \times 12{,}160 \text{ Units}\right)$$

= ₹ 2 × 152 Hours = ₹ 304

2. Actual Variable Overheads = ₹ 337
3. Budgeted Variable Overheads for actual hours = Standard Rate per Hour × Actual Hours

= ₹ 2 × 156 Hrs. = ₹ 312

Computation of Variances:

Variable Overhead Cost Variance	=	Standard Variable Overheads for Production – Actual Variable Overheads
	=	₹ 304 – ₹ 337 = ₹ 33 (A)
Variable Overhead Expenditure Variance	=	Budgeted Variable Overheads for Actual Hours – Actual Variable Overheads
	=	₹ 312 – ₹ 337 = ₹ 25 (A)
Variable Overhead Efficiency Variance	=	Standard Variable Overheads for Production – Budgeted Variable Overheads for Actual Hours
	=	₹ 304 – ₹ 312 = ₹ 8 (A)

☞ Now Students should try to attempt Illustration No.15, 17 and 18

5.2.4 Sales Variances

The sales variances can be computed in two ways. They are:

(a) Sales Turnover or Value Method.

(b) Profit or Sales Margin Method.

(a) Sales Turnover or Sales Value Method: In the sales turnover method, the variances are computed on the basis of sales value. This method will give the sales manager an idea of the effect of various factors affecting sales such as prices, quantity and sales mix on the overall sales value.

Sales Value Variance: It is the difference between the *Actual Sales and Budgeted Sales*. The variance can be bifurcated into sales price variance and sales volume variance.

Sales Price Variance: It is difference between the *Actual Sales* and *Standard Sales.*

Sales Volume Variances: It is difference between the *Standard Sales* and *Budgeted Sales.*

As in the case of materials, the sales volume variance can be bifurcated into

— Sales Mix Variance and

— Sales Quantity Variance.

The former shows the difference in sales value due to the fact that the actual sales mix is different from what was expected as the budgeted mix. The latter shows the effect of total quantity being larger or smaller than what was budgeted.

For calculating the sales mix and quantity variances, we have to calculate the *average budgeted price per unit of budgeted mix* and the *average budgeted price per unit of actual mix*.

The sales mix variance can then be calculated as below:

Total Actual Sales Quantity x (Average Budgeted Price per Unit of Actual Mix – Average Budgeted Price per unit of Budgeted Mix)

The sales quantity variance can then be calculated as below:

Average Budgeted Price per unit of Budgeted Mix x (Total Actual Qty. – Total Budgeted Qty.)

Sales Variances (Turnover or Value)

Sales Variance

(Actual Sales) *Less* (Budgeted Sales)

[(AP × AQ) – (BP × BQ)]

Sales Price Variance	**Sales Volume Variance**
(Actual Sales) *Less* (Standard Sales)	(Standard Sales) *Less* (Budgeted Sales)
[(AP × AQ) – (BP × AQ)] *Or* [AQ × (AP – BP)]	[(BP × AQ) – (BP × BQ)] *Or* [BP × (AQ – BQ)]

Sales Mix Variance	**Sales Quantity Variance**
(Standard Sales) *Less* (Revised Standard Sales)	(Revised Standard Sales) *Less* (Budgeted Sales)
[(BP × AQ) – (BP × RAQ)] *Or* [BP × (AQ – RAQ)]	[(BP × RAQ) – (BP × BQ)] Or [BP × (RAQ – BQ)]
Alternative Formula [Total Actual Quantity (units) × {*Average* Budgeted Price *per unit* of Actual Mix Less *Average* Budgeted Price *per unit* of Budgeted Mix}]	***Alternative Formula*** [*Average* Budgeted Price *per unit* of Budgeted Mix × {Total Actual Quantity (units) Less Total Budgeted Qty (units)}]

Market Size Variance	**Market Share Variance**
[Budgeted Market Share % × (Actual Industry Sales Quantity *in units* – Budgeted Industry Sales Quantity *in units*) × (Average Budgeted Price *per unit*)]	[(Actual Market Share % – Budgeted Market Share %) × (Actual Industry Sales Quantity *in units*) × (Average Budgeted Price *per unit*)]

Note:

BQ	=	Budgeted Sales Quantity
AQ	=	Actual Sales Quantity
RAQ	=	Revised Actual Sales Quantity
	=	Actual Quantity Sold Rewritten in Budgeted Proportion
BP	=	Budgeted Selling Price per Unit
AP	=	Actual Selling Price per Unit

Market Size Variance

Budgeted Market Share % × (Actual Industry Sales Quantity *in units* – Budgeted Industry Sales Quantity *in units*) × (Average Budgeted Price *per unit*)

Or

(Budgeted Market Share % × Actual Industry Sales Quantity *in units* – Budgeted Market Share % × Budgeted Industry Sales Quantity *in units*) × (Average Budgeted Price *per unit*)

Or

(Required Sales Quantity *in units* –Total Budgeted Quantity *in units*) × (Average Budgeted Price *per unit*)

Market Share Variance

(Actual Market Share % – Budgeted Market Share %) × (Actual Industry Sales Quantity *in units*) × (Average Budgeted Price *per unit*)

Or

(Actual Market Share % × Actual Industry Sales Quantity *in units* – Budgeted Market Share % × Actual Industry Sales Quantity *in units*) × (Average Budgeted Price *per unit*)

Or

(Total Actual Quantity *in units*– Required Sales Quantity *in units*) × (Average Budgeted Price *per unit*)

Market Size Variance + Market Share Variance

(Required Sales Quantity *in units* – Total Budgeted Quantity *in units*) × (Average Budgeted Price per *unit*)

Add

(Total Actual Quantity *in units*– Required Sales Quantity *in units*) × (Average Budgeted Price *per unit*)

Equals to

(Total Actual Quantity *in units* – Total Budgeted Quantity *in units*) × (Average Budgeted Price *per unit*)

Sales Quantity Variance

Illustration 9

Compute the sales turnover variances from the following figures: -

Product	*Budget*		*Actual*	
	Quantity (Units)	*Price (₹)*	*Quantity (Units)*	*Price (₹)*
A	*2,000*	*2.50*	*2,400*	*3.00*
B	*1,500*	*5.00*	*1,400*	*4.50*
C	*1,000*	*7.50*	*1,200*	*7.00*
D	*500*	*10.00*	*400*	*10.50*

Solution

Basic Calculations:

Product	**Budgeted Price [BP] (₹)**	**Actual Price [AP] (₹)**	**Budgeted Quantity [BQ] (Units)**	**Actual Quantity [AQ] (Units)**	**Budgeted Sales [BP×BQ] (₹)**	**Standard Sales [BP×AQ] (₹)**	**Actual Sales [AP×AQ] (₹)**	**Revised Actual Quantity [RAQ] (Units)**
A	2.50	3.00	2,000	2,400	5,000	6,000	7,200	2,160
B	5.00	4.50	1,500	1,400	7,500	7,000	6,300	1,620
C	7.50	7.00	1,000	1,200	7,500	9,000	8,400	1,080
D	10.00	10.50	500	400	5,000	4,000	4,200	540
			5,000	5,400	25,000	26,000	26,100	5,400

Note:

BQ = Budgeted Sales Quantity

AQ = Actual Sales Quantity

RAQ = Revised Actual Sales Quantity

= Actual Quantity Sold Rewritten in Budgeted Proportion

BP = Budgeted Selling Price per Unit

AP = Actual Selling Price per Unit

Computation of Variances:

Statement Showing Sales Variance (₹)

	Product (A)	**Product (B)**	**Product (C)**	**Product (D)**	**Total**
Sales Variance = AP × AQ – BP × BQ	= 3.00 × 2,400 – 2.50 × 2,000 = 2,200 (F)	= 4.50 × 1,400 – 5.00 × 1,500 = 1,200 (A)	= 7.00 × 1,200 – 7.50 × 1,000 = 900 (F)	= 10.50 × 400 – 10.00 × 500 = 800 (A)	1,100 (F)

Sales Price Variance = AQ × (AP – BP)	= 2,400 × (3 –2.50) = 1,200 (F)	= 1,400 × (4.50 – 5) = 700 (A)	= 1,200 × (7.00 – 7.50) = 600 (A)	= 400 × (10.50 – 10.00) = 200 (F)	100 (F)
Sales Volume Variance = BP × (AQ – BQ)	= 2.50 × (2,400 – 2,000) = 1,000 (F)	= 5.00 × (1,400 – 1,500) = 500 (A)	= 7.50 × (1,200 – 1,000) = 1,500 (F)	= 10.00 × (400 – 500) = 1,000 (A)	1,000 (F)
Sales Mix Variance = BP × (AQ – RAQ)	= 2.50 × (2,400 – 2,160) = 600 (F)	= 5.00 × (1,400 – 1,620) = 1,100 (A)	= 7.50 × (1,200 – 1,080) = 900 (F)	10.00 × (400 – 540) = 1,400 (A)	1,000 (A)
Sales Quantity Variance = BP × (RAQ – BQ)	= 2.50 × (2,160 – 2,000) = 400 (F)	= 5.00 × (1,620 – 1,500) = 600 (F)	= 7.50 × (1,080 – 1,000) = 600 (F)	= 10.00 × (540 – 500) = 400 (F)	2,000 (F)

(b) Sales Margin Method: The purpose of measuring the variances under this method is to identify the effect of changes in sale quantities and selling prices on the profits of the company. The quantity and mix variances should be analysed in conjunction with each other because the sales manager is responsible for both of these variances. Where a company is engaged in the manufacture and sale of multiple products, the variances between budgeted sales and actual sales may arise due to the following reasons:

— Changes in unit price and cost.

— Changes in physical volume of each product sold. This is quantity variance.

— Changes in the physical volume of the more profitable or less profitable products.

There are five distinct variables that can cause actual performance to differ from budgeted performance. They are:

— Direct substitution of products.

— Actual quantity of the constituents of sales being different from the budgeted quantity.

— Actual total quantity being different from the budgeted total quantity.

— Difference between actual and budgeted unit cost.

— Difference between actual and budgeted unit sale price.

The sales management should consider particularly the interaction of more than one variable in making decisions. For example, decrease in selling price coupled with a favourable product quantity variance may help to assess the price elasticity of demand.

The formulae for the calculation of *Sales Margin Variances* are as under:

Total Sales Margin Variance: It is the difference between the *Actual Margin* and the *Budgeted Margin*.

Sales Margin Price Variance: This variance arises because of the difference between the budgeted price of the quantity actually sold and the actual price thereof.

Sales Margin Price Variance equals to Actual Quantity x (Actual Margin per Unit – Budgeted Margin per Unit.

Sales Margin Volume Variance:

This variance arises because of the difference between the actual and budgeted quantities of each product both evaluated at budgeted margin.

Sales Margin Volume Variance equals to Budgeted Margin per Unit x (Actual Units – Budgeted Units)

This can be further sub-divided into the following two variances:

Sales Margin Quantity Variance: This variance arises because of the difference between the actual total quantity and the budgeted total quantity and is ascertained by multiplying this difference by average budgeted margin per unit of budgeted mix.

Sales Margin Mix Variance: This variance arises because of the change in the quantities of actual sales mix from budgeted sale mix and can be computed as below:

Sales Margin Mix Variance equals to Total Actual Quantity × (Average Budgeted Margin per Unit of Actual Mix – Average Budgeted Margin per Unit of Budgeted Mix).

Sales Variances (Margin)

Sales Margin Variance
(Actual Margin) *Less* (Budgeted Margin)
[(AQ × AM) – (BQ × BM)]

Sales Margin Price Variance	**Sales Margin Volume Variance**
(Actual Margin) *Less* (Standard Margin)	(Standard Margin) *Less* (Budgeted Margin)
[(AM × AQ) – (BM × AQ)] *Or* [AQ × (AM – BM)]	[(BM × AQ) – (BM × BQ)] *Or* [BM × (AQ – BQ)]

Sales Margin Mix Variance	**Sales Margin Quantity Variance**
(Standard Margin) *Less* (Revised Standard Margin)	(Revised Standard Margin) *Less* (Budgeted Margin)
(AQ × BM) – (RAQ × BM) *Or* BM × (AQ – RAQ)	(RAQ × BM) – (BQ × BM) *Or* BM × (RAQ – BQ)

Alternative Formula	***Alternative Formula***
[Total Actual Qty (units) × {*Average* Budgeted Margin *per unit* of Actual Mix <u>Less</u> *Average* Budgeted Margin *per unit* of Budgeted Mix}]	[*Average* Budgeted Margin *per unit* of Budgeted Mix × {Total Actual Qty (units) <u>Less</u> Total Budgeted Qty (units)}]

Market Size Variance	**Market Share Variance**
[Budgeted Market Share % × (Actual Industry Sales Quantity *in units* – Budgeted Industry Sales Quantity *in units*) × (Average Budgeted Margin *per unit*)]	[(Actual Market Share % – Budgeted Market Share %) × (Actual Industry Sales Quantity *in units*) × (Average Budgeted Margin *per unit*)]

Note:

BQ	=	Budgeted Sales Quantity
AQ	=	Actual Sales Quantity
RAQ	=	Revised Actual Sales Quantity
	=	Actual Quantity Sold Rewritten in Budgeted Proportion
BM	=	Budgeted Margin
	=	Budgeted price per Unit – Standard Cost per Unit
AM	=	Actual Margin
	=	Actual Sales Price per Unit – Standard Cost per Unit

Market Size Variance

Budgeted Market Share % × (Actual Industry Sales Quantity *in units* – Budgeted Industry Sales Quantity *in units*) × (Average Budgeted Margin *per unit*)

Or

(Budgeted Market Share % × Actual Industry Sales Quantity *in units* – Budgeted Market Share % × Budgeted Industry Sales Quantity *in units*) × (Average Budgeted Margin *per unit*)

Or

(Required Sales Quantity *in units* –Total Budgeted Quantity *in units*) × (Average Budgeted Margin *per unit*)

Market Share Variance

(Actual Market Share % – Budgeted Market Share %) × (Actual Industry Sales Quantity *in units*) × (Average Budgeted Margin *per unit*)

Or

(Actual Market Share % × Actual Industry Sales Quantity *in units* – Budgeted Market Share % × Actual Industry Sales Quantity *in units*) × (Average Budgeted Margin *per unit*)

Or

(Total Actual Quantity *in units*– Required Sales Quantity *in units*) × (Average Budgeted Margin *per unit*)

Market Size Variance + Market Share Variance
(Required Sales Quantity *in units* – Total Budgeted Quantity *in units*) × (Average Budgeted Margin per *unit*)
Add
(Total Actual Quantity *in units*– Required Sales Quantity *in units*) × (Average Budgeted Margin *per unit*)
Equals to
(Total Actual Quantity *in units* – Total Budgeted Quantity *in units*) × (Average Budgeted Margin *per unit*)
Sales Margin Quantity Variance

Illustration 10

Compute the sales margin variances from the following data:

Products	*Budgeted Quantity (units)*	*Actual Quantity (units)*	*Budgeted Sale Price (₹)*	*Actual sale Price (₹)*	*Standard Cost per Unit (₹)*
A	*1,200*	*2,000*	*5.00*	*4.50*	*3.00*
B	*800*	*1,000*	*2.50*	*2.00*	*1.50*

Solution

Basic Calculations:

The margin for each product may be calculated as under:

Products	Budgeted Price (₹)	Actual Price (₹)	Std. Cost (₹)	Budgeted Margin (₹)	Actual Margin (₹)
A	5.00	4.50	3.00	2.00	1.50
B	2.50	2.00	1.50	1.00	0.50

For computing the various sales margin variances the following calculations be made:

Products	Margin		Quantity		Budgeted Margin	Standard Margin	Actual Margin	Revised Actual Quantity
	Budget [BM] (₹)	Actual [AM] (₹)	Budget [BQ]	Actual [AQ]	[BM×BQ] (₹)	[BM×AQ] (₹)	[AM×AQ] (₹)	[RAQ]
A	2.00	1.50	1,200	2,000	2,400	4,000	3,000	1,800
B	1.00	0.50	800	1,000	800	1,000	500	1,200
Total			2,000	3,000	3,200	5,000	3,500	3,000

Note:

BQ	=	Budgeted Sales Quantity
AQ	=	Actual Sales Quantity
RAQ	=	Actual Quantity Sold Rewritten in Budgeted Proportion
BM	=	Budgeted Margin
AM	=	Actual Margin

Computation of Variances:

Sales Margin Variance	=	Actual Margin – Budgeted Margin
	=	AQ × AM – BQ x BM
For Product A	=	₹ 3,000 – ₹ 2,400
	=	₹ 600 (F)
For Product B	=	₹ 500 – ₹ 800
	=	₹ 300 (A)
Total (A+ B)	=	₹ 600 (F) + ₹ 300 (A)
	=	₹ 300 (F)
Sales Margin Price Variance	=	Actual Margin –Standard Margin
	=	AQ × AM –AQ × BM
		Or
		AQ x (AM – BM)
For Product A	=	₹ 3,000 – ₹ 4,000
	=	₹ 1,000 (A)
For Product B	=	₹ 500 – ₹ 1,000
	=	₹ 500 (A)
Total (A + B)	=	₹ 1,000 (A) + ₹ 500 (A)
	=	₹ 1,500 (A)
Sales Margin Volume Variance	=	Standard Margin – Budgeted Margin
	=	AQ× BM –BQ x BM
		Or
		BM x (AQ – BQ)
For Product A	=	₹ 4,000 – ₹ 2,400
	=	₹ 1,600 (F)

For Product B	=	₹ 1,000 – ₹ 800
	=	₹ 200 (F)
Total (A + B)	=	₹ 1,600 (F) + ₹ 200 (F)
	=	₹ 1,800 (F)
Sales Margin Mix Variance	=	Standard Margin –Revised Standard Margin
	=	AQ × BM –RAQ × BM
		Or
		BM × (AQ – RAQ)
For Product A	=	₹ 2 × (2,000 Units – 1,800 Units)
	=	₹ 400 (F)
For Product B	=	₹ 1 × (1,000 Units – 1,200 Units)
	=	₹ 200 (A)
Total (A + B)	=	₹ 400 (F) + ₹ 200 (A)
	=	₹ 200 (F)
Sales Margin Quantity Variance	=	Revised Standard Margin –Budgeted Margin
	=	RAQ × BM –BQ × BM
		Or
		BM × (RAQ – BQ)
Product A	=	₹ 2 × (1,800 Units – 1,200 Units)
	=	₹ 1,200 (F)
Product B	=	₹ 1 × (1,200 Units – 800 Units)
	=	₹ 400 (F)
Total (A + B)	=	₹ 1,200 (F) + ₹ 400 (F)
	=	₹ 1,600 (F)

☞ Now Students should try to attempt Illustration No.16

Illustration 11

Super computers manufactures and sells three related PC models :

1. *PC — Sold mostly to college students*
2. *Portable PC— Smaller version of PC positioned as home computer*
3. *Super PC — Sold mostly to business executives*

Budgeted and actual data for 2012 is as follows:

Budgeted for 2012

	Selling price per unit ₹	*Variable cost per unit ₹*	*Contribution margin per unit ₹*	*Sales volume in units*
P C	*24,000*	*14,000*	*10,000*	*7,000*
Portable PC	*16,000*	*10,000*	*6,000*	*1,000*
Super PC	*1,00,000*	*60,000*	*40,000*	*2,000*
Total:				*10,000*

Actual for 2012

	Selling price per unit (₹)	*Variable cost per unit (₹)*	*Contribution margin per unit (₹)*	*Sales volume in units*
P C	*22,000*	*10,000*	*12,000*	*8,250*
Portable PC	*13,000*	*8,000*	*5,000*	*1,650*
Super PC	*70,000*	*50,000*	*20,000*	*1,100*
Total :				*11,000*

Super computers derived its total unit sales budget for 2012 from the internal management estimate of a 20% market share and an industry sales forecast by computer manufacturers association of 50,000 units. At the end of the year the association reported actual industry sales of 68,750 units.

Required:

(i) Compute the individual product and total sales volume variance.

(ii) Compute total sales quantity variance.

(iii) Compute the market size and market share variances.

Solution

Basic Calculations:

1. Statement of budgeted average contribution margin per unit for the year 2012

Products/ Different PC Models	**Budgeted Contribution Margin per Unit (₹)**	**Budgeted Sales Volume (Units)**	**Total Budgeted Contribution Margin (₹)**
PC	10,000	7,000	7,00,00,000
Portable PC	6,000	1,000	60,00,000
Super PC	40,000	2,000	8,00,00,000
Total		10,000	15,60,00,000

Budgeted Average Contribution Margin per Unit $= \dfrac{₹15,60,00,000}{10,000 \text{ Units}}$

$= ₹\ 15,600$

2. **Actual Market Share Percentage** $= \dfrac{\text{Actual Sales of 3 Models}}{\text{Actual Industry Sales}} \times 100$

$= \dfrac{11,000 \text{ Units}}{68,750 \text{ Units}} \times 100$

$= 16\%$

3. **Actual Sales Mix Percentage of Product** $= \dfrac{\text{Actual Sale of Product}}{\text{Total Actual Sale of 3 Models}} \times 100$

PC $= \dfrac{8,250 \text{ Units}}{11,000 \text{ Units}} \times 100 = 75\%$

Portable PC $= \dfrac{1,650 \text{ Units}}{11,000 \text{ Units}} \times 100 = 15\%$

Super PC $= \dfrac{1,100 \text{ Units}}{11,000 \text{ Units}} \times 100 = 10\%$

Computation of Variances:

(i) ***Computation of Individual Product and Total Sales Volume Variance***

$$\text{Sales Volume Variance} = \left[\begin{array}{c}\text{Actual Industry}\\\text{Sales in Units}\end{array} - \begin{array}{c}\text{Budgeted Industry}\\\text{Sales in Units}\end{array}\right] \times \begin{array}{c}\text{Budgeted Contribution}\\\text{Margin per Unit}\end{array}$$

Individual Product Sales Volume Variance:

PC :

= (8,250 Units – 7,000 Units) × ₹ 10,000 = ₹ 1,25,00,000 (F)

Portable PC :

= (1,650 Units – 1,000 Units) × ₹ 6,000 = ₹ 39,00,000 (F)

Super PC:

= (1,100 Units – 2,000 Units) × ₹ 40,000 = ₹ 3,60,00,000 (A)

Total Sales Volume Variance = ₹ 1,96,00,000 (A)

(ii) ***Computation of Total Sales Quantity Variance***

Total Sales Quantity Variance:

$$= \left[\begin{array}{c}\text{Total Actual}\\\text{Sales Units}\end{array} - \begin{array}{c}\text{Total Budgeted}\\\text{Sales Units}\end{array}\right] \times \begin{array}{c}\text{Budgeted Average Contribution}\\\text{Margin per Unit}\end{array}$$

= (11,000 Units – 10,000 Units) × ₹ 15,600 = ₹ 1,56,00,000 (F)

(iii) *Computation of the Market Size and Market Share Variances*

Market size variance:

$$= \begin{matrix}\text{Budgeted Market} \\ \text{Share Percentage}\end{matrix} \times \left[\begin{matrix}\text{Actual Industry} \\ \text{Sales in Units}\end{matrix} - \begin{matrix}\text{Budgeted Industry} \\ \text{Sales in Units}\end{matrix}\right] \times \begin{matrix}\text{Budgeted Average Contribution} \\ \text{Margin per Unit}\end{matrix}$$

= 0.20 × (68,750 Units – 50,000 Units) × ₹ 15,600

= ₹ 5,85,00,000 (F)

. *Market Share Variance:*

$$= \left[\begin{matrix}\text{Actual market} \\ \text{Share Percentage}\end{matrix} - \begin{matrix}\text{Budgeted Market} \\ \text{Shares Percentage}\end{matrix}\right] \times \left[\begin{matrix}\text{Actual Industry} \\ \text{Sale} \\ \text{Volume} \\ \text{in Units}\end{matrix}\right] \times \left[\begin{matrix}\text{Budgeted} \\ \text{Average} \\ \text{Contribution} \\ \text{Margin per Unit}\end{matrix}\right]$$

= (0.16 – 0.20) × 68,750 Units × ₹ 15,600

= ₹ 4,29,00,000 (A)

Note: Sales variances can also be calculated by using sales value approach.

5.3 Reporting of Variances

Computation of variances and their reporting is not the final step towards the control of various elements of cost. It infact demands an analysis of variances from the side of the executives, to ascertain the correct reasons for their occurance. After knowing the exact reasons, it becomes their responsibility to take necessary steps so as to stop the re-occurance of adverse variances in future. To enhance the utility of such a reporting system it is necessary that such a system of reporting should not only be prompt but should also facilitate the concerned managerial level to take necessary steps. Variance reports should be prepared after keeping in view its ultimate use and its periodicity. Such reports should highlight the essential cost deviations and possibilities for their improvements. In fact the variance reports should give due regard to the following points :-

(i) The concerned executives should be informed about what the cost performance should have been.

(ii) How close the actual cost performance is with reference to standard cost performance.

(iii) The analysis and causes of variances.

(iv) Reporting should be based on the principle of management by exception.

(v) The magnitude of variances should also be stated.

Preparation of Original Budget, Standard Product Cost Sheet and the Reconciliation of Budgeted Profit and Actual Profit: Generally, under variance analysis we compute various variances from the actual and the standard/budgeted data. Sometimes all or a few variances and actual data are made available and from that we are required to prepare standard product cost sheet, original budget and to reconcile the budgeted profit with the actual profit. Some important concept are given below:

Reconciliations (Budgeted / Standard Profit / Actual Profit)

☞

Reconciliation Statement-I
Budgeted Profit to Actual Profit

Budgeted Profit				❑
(Budgeted Quantity × Budgeted Margin)				
Effect of Variances				
Material Cost Variance				
Material Price Variance		❑		
Material Usage Variance				
Material Mix Variance	❑			
Material Yield Variance	❑	❑	❑	
Labour Cost Variance				
Labour Rate Variance		❑		
Labour Idle Time Variance		❑		
Labour Efficiency Variance				
Labour Mix Variance	❑			
Labour Sub-Efficiency Variance	❑	❑	❑	
Variable Overhead Cost Variances				
Variable Overhead Expenditure Variance		❑		
Variable Overhead Efficiency Variance		❑	❑	
Fixed Overhead Cost Variances				
Fixed Overhead Expenditure Variance		❑		
Fixed Overhead Volume Variance				
Fixed Overhead Capacity Variance	❑			
Fixed Overhead Efficiency Variance	❑	❑	❑	
Sales Margin Variance				
Sales Margin Price Variance		❑		
Sales Margin Volume Variance				
Sales Margin Mix Variance	❑			
Sales Margin Quantity Variance	❑	❑	❑	❑
Actual Profit				❑

Reconciliation Statement-II
Budgeted Profit to Actual Profit

Budgeted Profit				❑
(Budgeted Quantity ×Budgeted Margin)				
Effect of Variances				
Material Cost Variance				
Material Price Variance		❑		
Material Usage Variance				
Material Mix Variance	❑			
Material Yield Variance	❑	❑	❑	
Labour Cost Variance				
Labour Rate Variance		❑		
Labour Idle Time Variance		❑		
Labour Efficiency Variance				
Labour Mix Variance	❑			
Labour Sub-Efficiency Variance	❑	❑	❑	
Variable Overhead Cost Variances				
Variable Overhead Expenditure Variance		❑		
Variable Overhead Efficiency Variance		❑	❑	
Fixed Overhead Cost Variances				
Fixed Overhead Expenditure Variance		❑		
Fixed Overhead Volume Variance[1&2]				
Fixed Overhead Capacity Variance	NA			
Fixed Overhead Efficiency Variance	NA	NA	❑	
Sales Contribution Variances[3]				
Sales Contribution Price Variance		❑		
Sales Contribution Volume Variance				
Sales Contribution Mix Variance	❑			
Sales Contribution Quantity Variance	❑	❑	❑	❑
Actual Profit				❑

(1) Relation between Sales Margin Volume Variance & Sales Contribution Volume Variance		
Sales Margin Volume Variance	=	Budgeted Margin Per Unit × (Actual Quantity − Budgeted Quantity)
	Or	

Sales Margin Volume Variance	=	[Standard (or Budgeted) Contribution Per Unit − Standard Fixed Overheads Per Unit] × (Actual Quantity − Budgeted Quantity)
	Or	
Sales Margin Volume Variance	=	[Standard Contribution Per Unit × (Actual Quantity − Budgeted Quantity)] − [Standard Fixed Overheads Per Unit × (Actual Quantity − Budgeted Quantity)]
	Or	
Sales Margin Volume Variance	=	Sales Contribution Volume Variance − Fixed Overhead Volume Variance
	Or	
Sales Contribution Volume Variance	=	Sales Margin Volume Variance + Fixed Overhead Volume Variance

Note: Production units equals to Sales units for both actual & budget.

(2) Fixed Overhead Volume Variance does not arise in a Marginal Costing system-Why?

Fixed Overhead Volume Variance does not arise in a Marginal Costing system. In an Absorption Costing system it stands for the value of the under-or- over absorbed Fixed Overhead due to a change in production volume (Budgeted Vs Actual). When Marginal Costing is in use there is no Overhead Volume Variance, because Marginal Costing does not absorb Fixed Overhead. Fixed Overhead Expenditure Variance is the only variance for Fixed Overhead in a Marginal Costing system. It is calculated as in an Absorption Costing system. Sales Margin Volume Variance in Marginal Costing System also known as Sales Volume Contribution Variance. This variance calculates the Standard Contribution gained or lost as a result of an increase or decrease in sales volume.

(3) Sales Contribution Variance Formulas

Sales Contribution Volume Variance	=	[Standard Contribution Per Unit × (Actual Quantity − Budgeted Quantity)]
Sales Contribution Mix Variance	=	[Standard Contribution Per Unit × (Actual Quantity − Revised Actual Quantity)]
Sales Contribution Quantity Variance	=	[Standard Contribution Per Unit × (Revised Actual Quantity − Budgeted Quantity)]

(4) Relation between Sales Price Variance & Sales Contribution Price Variance

Sales Price Variance is equal to Sales Contribution Price Variance.

(This is because, for the actual quantity sold, standard cost remaining constant, change in selling price will have equal impact on turnover and contribution)

(5) Relation between Sales Volume Variance & Sales Contribution Volume Variance

Sales Contribution Volume Variance	=	Sales Volume Variance × Budgeted PV Ratio

Reconciliation Statement-III
Standard Profit to Actual Profit

Standard Profit				❑
(Actual Quantity × Budgeted Margin)				
Effect of Variances				
Material Cost Variance				
Material Price Variance		❑		
Material Usage Variance				
Material Mix Variance	❑			
Material Yield Variance	❑	❑	❑	
Labour Cost Variance				
Labour Rate Variance		❑		
Labour Idle Time Variance		❑		
Labour Efficiency Variance				
Labour Mix Variance	❑			
Labour Sub-Efficiency Variance	❑	❑	❑	
Variable Overhead Cost Variances				
Variable Overhead Expenditure Variance		❑		
Variable Overhead Efficiency Variance		❑	❑	
Fixed Overhead Cost Variances				
Fixed Overhead Expenditure Variance		❑		
Fixed Overhead Volume Variance				
Fixed Overhead Capacity Variance	❑			
Fixed Overhead Efficiency Variance	❑	❑	❑	
Sales Margin Variance				
Sales Margin Price Variance		❑		
Sales Margin Volume Variance				
Sales Margin Mix Variance	NA			
Sales Margin Quantity Variance	NA	NA	❑	❑
Actual Profit				❑

Illustration 12

New Jumbo Enterprises manufactures one product, and the entire product is sold as soon as it is produced. There are no opening or closing stocks and work in progress is negligible. The company operates a standard costing system and analysis of variances is made every month. The standard cost card for the product is as follows:

		₹
Direct material	*0.5 kgs at ₹ 4 per kg.*	*2.00*
Direct Wages	*2 hrs. at ₹ 2 per hour*	*4.00*
Variable overheads	*2 hrs at ₹ 0.30 per hour*	*0.60*
Fixed overheads	*2 hours at ₹ 3.70 per hour*	*7.40*
Standard cost		*14.00*
Standard profit		*6.00*
Standard selling price		*20.00*

Budgeted output for April 2012 was 5,100 units.

Actual results for April 2012 were as follows:

Production of 4,850 units was sold for ₹ 95,600.

Materials consumed in production amounted to 2,300 kgs. At a total cost of 9,800.

Labour hours paid for amounted to 8,500 hours at a cost of ₹ 16,800.

Actual operating hours amounted to 8,000 hours.

Variable overheads amounted to ₹ 2,600.

Fixed overheads amounted to ₹ 42,300

You are required to

(a) *Calculate Material, Labour, Variable Overhead, Fixed Overhead, Sales Value & Sales Margin Variances.*

(b) *Prepare an operating statement for the month ended 30th April 2012.*

(c) *Prepare an reconciliation Statement between 'Budgeted Profit & Actual Profit' under 'Absorption Costing Method'*

(d) *Prepare an reconciliation Statement between 'Budgeted Profit & Actual Profit' under 'Marginal Costing Method'*

(e) *Prepare an reconciliation Statement between 'Standard Profit & Actual Profit' under 'Absorption Costing Method'*

Solution

(a) Calculation of Variances:

1. Material Variances

[SP × SP]	[AQ × AP]	[AQ × SP]
(4.850 × 0.50) Kg. × ₹ 4 = ₹ 9,700	2,300 Kg. × ₹ 4.26* = ₹ 9,800	2,300 Kg. × ₹ 4 = ₹ 9,200

** ₹ 9,800 / 2,300Kg.*

Note:

SQ = Standard Quantity = Expected Consumption for Actual Output
AQ = Actual Quantity of Material Consumed
SP = Standard Price per Unit
AP = Actual Price per Unit

Material Cost Variance	=	Standard Cost – Actual Cost
	=	SQ × SP – AQ × AP
	=	₹ 9,700 – ₹ 9,800
	=	₹ 100 (A)
Material Price Variance	=	Standard Cost of Actual Quantity – Actual Cost
	=	AQ × SP – AQ × AP
	=	₹ 9,200 – ₹ 9,800
	=	₹ 600 (A)
Material Usage Variance	=	Standard Cost of Standard Quantity for Actual Output –
	=	Standard Cost of Actual Quantity
	=	SQ × SP – AQ × SP
	=	₹ 9,700 – ₹ 9,200
	=	₹ 500 (F)

2. Labour Variances

[SH × SR]	[AH × AR]	[AH × SR]
(4,850 × 2) hrs. × ₹ 2 = ₹ 19,400	8,500 hrs. × ₹ 1.976* = ₹ 16,800	8,500 hrs. × ₹ 2 = ₹ 17,000

** ₹ 16,800/8,500 hrs.*

Note:

SH = Standard Hours = Expected Time Allowed for Actual Output
AH = Actual Hours *paid for*
SR = Standard Rate per Labour Hour
AR = Actual Rate per Labour Hour Paid

Labour Cost Variance	=	Standard Wages – Actual Wages
	=	SH × SR – AH × AR
	=	₹ 19,400 – ₹ 16,800 = ₹ 2,600 (F)
Labour Rate Variance	=	Standard Cost of Actual Time – Actual Cost
	=	SR × AH – AR × AH
	=	₹ 17,000 – ₹ 16,800 = ₹ 200 (F)
Labour Efficiency Variance	=	Standard Cost of Standard Time for Actual Output – Standard Cost of Actual time
	=	SH × SR – AH# × SR
	=	₹ 19,400 – ₹ 16,000 = ₹ 3,400 (F)

AH# refers to actual hours worked

Labour Idle Time Variance	=	Actual Idle Hrs. × Standard Rate per Hour
	=	500 Hrs. × ₹ 2 = ₹ 1,000

3. Fixed Overhead Variance

Absorbed Fixed Overheads [SR@ × AO]	Budgeted Fixed Overheads [BO × SR@]	Actual Fixed Overheads [AO × AR]	Budgeted Overheads for Actual Hours [SR# × AH]
₹ 7.40 × 4,850 Units = ₹ 35,890	5,100 Units × ₹ 7.40 = ₹ 37,740	4,850 Units × ₹ 8.722* = ₹ 42,300	₹ 3.70 × 8,000 Hrs = ₹ 29,600

**₹ 42,300 / 4,850 Units*

Note:

SR@	= Standard Fixed Overhead Rate per Unit
AO	= Actual Output
BO	= Budgeted Output
AR	= Actual Fixed Overhead Rate per Unit
SR#	= Standard Fixed Overhead Rate per Hour
AH	= Actual Hours

Fixed Overhead Cost Variance	=	Absorbed Fixed Overheads – Actual Fixed Overheads
	=	₹ 35,890 – ₹ 42,300 = ₹ 6,410 (A)
Fixed Overhead Expenditure Variance	=	Budgeted Fixed Overheads – Actual Fixed Overheads
	=	₹ 37,740 – ₹ 42,300 = ₹ 4,560 (A)

Fixed Overhead Capacity Variance	=	Budgeted Fixed Overheads for Actual Hours – Budgeted Fixed Overheads
	=	₹ 29,600 – ₹ 37,740 = ₹ 8,140 (A)
Fixed Overhead Efficiency Variance	=	Absorbed Fixed Overheads – Budgeted Fixed Overheads for Actual hours
	=	₹ 35,890 – ₹ 29,600 = ₹ 6,290 (F)
Fixed Overhead Volume Variance	=	Absorbed Fixed Overheads – Budgeted Fixed Overheads
	=	₹ 35,890 – ₹ 37,740 = ₹ 1,850 (A)

4. Variable Overhead Variance

Standard Variable Overheads [SR@ × AO]	**Actual Variable Overheads [AO × AR]**	**Budgeted Variable Overheads for Actual Hours [SR# × AH]**
₹ 0.60 × 4,850 Units = ₹ 2,910	4,850 Units × ₹ 0.536* = ₹ 2,600	₹ 0.30 × 8,000 Hours = ₹ 2,400

**₹ 2,600/4,850 Units*

Note:

SR@	= Standard Variable Overhead Rate per Unit
AO	= Actual Output
AR	= Actual Variable Overhead Rate per Unit
SR#	= Standard Variable Overhead Rate per Hour
AH	= Actual Hours

Variable Overhead Cost Variance	=	Standard Variable Overheads – Actual Variable Overheads
	=	₹ 2,910 – ₹ 2,600 = ₹ 310 (F)
Variable Overhead Expenditure Variance	=	Budgeted Variable Overheads for Actual Hours – Actual Variable Overheads
	=	₹ 2,400 – 2,600 = ₹ 200 (A)
Variable Overhead Efficiency Variance	=	Standard Variable Overheads – Budgeted Variable Overheads for Actual hours
	=	₹ 2,910 – ₹ 2,400 = 510 (F)

5. *Sales Value Variances*

BQ × BP	AQ × AP	AQ ×BP
5,100 Units × ₹ 20 = ₹ 1,02,000	4,850 units × ₹ $\boxed{19.71^*}$ = ₹ 95,600	4,850 × ₹ 20 = ₹ 97,000

* ₹ 95,600/ 4,850 units

Note:

BQ = Budgeted Sales Quantity
AQ = Actual Sales Quantity
BP = Budgeted Selling Price per Unit
AP = Actual Selling Price per Unit

Sales Variance = Actual Sales – Budgeted Sales
= AP × AQ – BP × BQ
= ₹ 95,600 – ₹ 1,02,000
= ₹ 6,400 (A)

Sale Price Variance = Actual Sales – Standard Sales
= AP × AQ – BP × AQ
= ₹ 95,600 – ₹ 97,000
= ₹ 1,400 (A)

Sales Volume Variance = Standard Sales – Budgeted Sales
= BP × AQ – BP × BQ
= ₹ 97,000 – ₹ 1,02,000
= ₹ 5,000 (A)

6. *Sales Margin Variances*

BQ × BM	AQ × AM	AQ × BM
5,100 Units × ₹ 6 = ₹ 30,600	4,850 units × $\left[\frac{₹\,95{,}600}{4{,}850\text{ units}} - ₹\,14\right]$ = ₹ 27,700	4,850 units × ₹ 6 = ₹ 29,100

Note:

BM = Budgeted Margin
= (Budgeted Price per Unit – Standard Cost per Unit)
AM = Actual Margin
= (Actual Sales Price per Unit – Standard Cost per Unit)
BQ = Budgeted Sales Quantity
AQ = Actual Sales Quantity

Sales Margin Variance = Actual Margin – Budgeted Margin
= AQ × AM – BQ × BM
= ₹ 27,700 – ₹ 30,600
= ₹ 2,900 (A)

Sales Margin Price Variance = Actual Margin – Standard Margin
= AM × AQ – BM × AQ
= ₹ 27,700 – ₹ 29,100
= ₹ 1,400 (A)

Sales Margin Volume Variance = Standard Margin – Budgeted Margin
= BM × AQ – BM × BQ
= ₹ 29,100 – ₹ 30,600
= ₹ 1,500 (A)

OR

Sales Margin Volume Variance = [Sales Volume Variance × Budgeted Net Profit Ratio]

$$= ₹\ 5{,}000\ (A) \times \left[\frac{₹\ 6}{₹\ 20} \times 100\right] = ₹\ 1{,}500\ (A)$$

(b) Operating Statement for the month ended 30th April 2012:

Operating Statement	₹	₹
Sales		95,600
Less: Cost of Materials	9,800	
Labour	16,800	
Variable Overhead	2,600	
Fixed Overhead	42,300	71,500
Net Profit		24,100

(c) Reconciliation Statement between 'Budgeted Profit & Actual Profit' under 'Absorption Costing' method

Reconciliation Statement (Absorption Costing)

Budgeted Profit ⟶ Actual Profit

	₹	₹	₹	₹
Budgeted Profit				30,600
(Budgeted Quantity x Budgeted Margin)				
Effect of Variances				
Material Cost Variances:				
Material Price Variance		(600)		
Material Usage Variance		500	(100)	
Labour Cost Variances:				

Labour Rate Variance		200		
Labour Efficiency Variance		3,400		
Labour Idle Time Variance		(1,000)	2,600	
Variable Overhead Cost Variances:				
Variable Overhead Expenditure Variance		(200)		
Variable Overhead Efficiency Variance		510	310	
Fixed Overhead Cost Variances:				
Fixed Variable Overhead Expenditure Variance		(4,560)		
Fixed Overhead Volume Variance				
Fixed Overhead Capacity Variance	(8,140)			
Fixed Overhead Efficiency Variance	6,290	(1,850)	(6,410)	
Sales Margin Variance:				
Sales Margin Price Variance		(1,400)		
Sales Margin Volume Variance		(1,500)	(2,900)	(6,500)
Actual Profit				24,100

Adverse shown by (–) symbol

☞ Overheads can also be affected by Idle time. It is usually assume that overhead expenditure s incurred in active hours only.

(d) Reconciliation between 'Budgeted Profit & Actual Profit' under 'Marginal Costing' Method

Reconciliation Statement (Marginal Costing)

Budgeted Profit ⟶ **Actual Profit**

	₹	₹	₹	₹
Budgeted Profit				30,600
(Budgeted Quantity x Budgeted Margin)				
Effect of Variances				
Material Cost Variances:				
Material Price Variance		(600)		
Material Usage Variance		500	(100)	
Labour Cost Variances:				
Labour Rate Variance		200		

Labour Efficiency Variance		3,400		
Labour Idle Time Variance		(1,000)	2,600	
Variable Overhead Cost Variances:				
Variable Overhead Expenditure Variance		(200)		
Variable Overhead Efficiency Variance		510	310	
Fixed Overhead Cost Variances:				
Fixed Variable Overhead Expenditure Variance		(4,560)		
Fixed Overhead Volume Variance				
Fixed Overhead Capacity Variance	NA			
Fixed Overhead Efficiency Variance	NA	NA	(4,560)	
Sales Margin Variance:				
Sales Contribution Price Variance		(1,400)		
Sales Contribution Volume Variance*		(3,350)	(4,750)	(6,500)
Actual Profit				24,100

Adverse shown by (–) symbol

☞ **Calculation of Sales Contribution Volume Variance**

* Sales Contribution Volume Variance = Standard Contribution per Unit × (Actual Quantity – Budgeted Quantity)

= ₹ 13.40 × (4,850 Units – 5,100 Units)

= 3,350 (A)

Or

* Sales Contribution Volume Variance = Sales Margin Volume Variance + Fixed Overhead Volume Variance

= ₹ 1,500(A) + ₹ 1,850 (A)

= 3,350 (A)

Or

* Sales Contribution Volume Variance = Sales Volume Variance × Budgeted PV Ratio

= ₹ 5,000 (A) × (₹ 13.40/20.00 × 100) %

= 3,350 (A)

(e) Reconciliation between 'Standard Profit & Actual Profit' under 'Absorption Costing' method

Reconciliation Statement (Absorption Costing)

Standard Profit ⟶ Actual Profit

	₹	₹	₹	₹
Standard Profit				29,100

(Actual Quantity x Budgeted Margin)				
Effect of Variances				
Material Cost Variances:				
Material Price Variance		(600)		
Material Usage Variance		500	(100)	
Labour Cost Variances:				
Labour Rate Variance		200		
Labour Efficiency Variance		3,400		
Labour Idle Time Variance		(1,000)	2,600	
Variable Overhead Cost Variances:				
Variable Overhead Expenditure Variance		(200)		
Variable Overhead Efficiency Variance		510	310	
Fixed Overhead Cost Variances:				
Fixed Variable Overhead Expenditure Variance		(4,560)		
Fixed Overhead Volume Variance				
Fixed Overhead Capacity Variance	(8,140)			
Fixed Overhead Efficiency Variance	6,290	(1,850)	(6,410)	
Sales Margin Variance:				
Sales Margin Price Variance		(1,400)		
Sales Margin Volume Variance		NA	(1,400)	(5,000)
Actual Profit				24,100

Adverse shown by (–) symbol

☞ Now Students should try to attempt Illustration No.25

5.4 Accounting Procedure For Standard Cost

The standard cost operations can be recorded in the books of account. Two important accounting procedures for standard costs are:

Partial Plan: This system uses current standards in which the inventory will be valued at current standard cost figure. Under this method the work-in-progress account is charged at the actual cost of production for the month and is credited with the standard cost of the month's production of finished product. The closing balance of work-in-progress is also shown at standard cost. The balance after making the credit entries represent the variance from standard for the month. The analysis of the variance is done after the end of the month. This method is simple in operation because variances are analysed after the end of month but may present difficulties if the firm makes a variety of products. The following illustration will explain the operation of the recording of standard cost under this method.

Single Plan: The main purpose of standard costing is cost control. To achieve this purpose, the variances should be analysed according to their causes. Analysis should be timely so that much time is not lost in taking corrective action wherever needed. In the partial plan, we have seen that the variances are analysed at the end of period. The single plan system envisages the posting of all items in the debit side of the work-in-progress account at the standard cost leaving the credit side to represent the standard cost of finished production and work-in-progress. This system enables the ascertainment of variances as and when the transaction is posted to work-in-progress account. In other words, the analysis of variances is done from the original documents like invoices, labour sheets, etc., and this method of analysis is known as analysis at source. Since, the single plan system contemplates the analysis of variances at source, the installation of this system requires more planning so that effective documentation at each stage is introduced for proper recording and analysis of variance. Thus for example, the issue of bill of materials to the stores enables the storekeeper to calculate the standard value of materials. If any material is requisitioned beyond the standard, he can mark the same for material usage variance account. In the production department, as and when the finished output is recorded, the standard waste and actual waste can be compared and necessary entries can be made by the shop supervisors for posting the excessive usage to appropriate variance accounts.

Illustration 13

XYZ & Co. manufactures product 'Gamma'. It uses a standard costing system in which material price variance and labour rate variance are segregated at the point of purchase of material and the incurrence of labour cost respectively.

The standard cost card for product 'Gamma' shows the following details:-

	Per unit (₹)
Material – 1 Kg. at 6 per Kg.	*6*
Labour – 2.5 Hrs. at ₹ 4 per Hr.	*10*
Overhead – 2.5 Hrs. at ₹ 2 per Hr.	*5*
Standard cost	*21*

Overhead rate is ₹ 2 per hour, the budgeted overhead being ₹ 2,000 for 1,000 budgeted hours.

Other information for the month of Nov'2012 is as follows:-

Materials:

Opening stock	*400 Kgs. at ₹ 6.00 per Kg.*
Purchase	*500 Kgs. at ₹ 7.00 per Kg.*

Issued to production	*450 Kgs.*
Direct labour:	*925 Hours at ₹ 4.40 per Hour.*
Overhead:	*₹ 2,100*

During this month, 360 units are completed and in respect of 40 units, it is estimated that they are complete as to materials, but half complete as to labour and overhead. 300 units are sold at ₹ 30 per unit during the month. Prepare:-

(a) Cost Control Accounts

(b) Variance Accounts

(c) Trial Balance at the end of the month.

Also prepare the Cost Control Accounts, Variance Accounts and Trial Balance if the Company had implemented the 'Partial Plan' of accounting for variance.

Solution

Single Plan

(a) Cost Control Accounts

Raw Material Control A/c

	(₹)		(₹)
To Balance b/d (400 Kg. at ₹ 6)	2,400	By Work – in – Progress Control A/c (Issued 400 Kg. at ₹ 6)	2,400
To GL Adjustment A/c (500 Kg. at ₹ 6)	3,000	By Material Usage Variance A/c (50 Kg. @ ₹ 6)	300
		By Balance c/d (450 Kg. at ₹ 6)	2,700
	5,400		5,400

Wages Control A/c

	(₹)		(₹)
To GL Adjustment A/c (925 hours at ₹ 4)	3,700	By Work – in – Progress Control A/c (950 hours at ₹ 4)	3,800
To Efficiency Variance A/c (25 hours at ₹ 4)	100		
	3,800		3,800

Overhead Control A/c

	(₹)		(₹)
To GL Adjustment A/c	2,100	By Work – in – Progress Control A/c	1,900
To Overhead Efficiency Variance A/c	50	By Overhead Expenditure Variance A/c	100
		By Overhead Capacity Variance A/c	150
	2,150		2,150

Work-in-Progress Control A/c

	(₹)		(₹)
To Raw Material Control A/c (400 units at ₹ 6)	2,400	By FG Control A/c (360 units at ₹ 21)	7,560
To Wage Control A/c (380 units at ₹ 10)	3,800	By Balance c/d (40 units)	540
To Overhead Control A/c (380 units at ₹ 5)	1,900		
	8,100		8,100

***Work-in-Progress Control A/c (Closing Balance)** ₹ 540

[Material: 40 units x ₹ 6 = 240, Labour: 20 units x ₹ 10 = 200, Overhead 20 units x ₹ 5]

Finished Goods (FG) Control A/c

	(₹)		(₹)
To Work-in-Progress Control A/c (360 units at ₹ 21)	7,560	By Costing P & L A/c (300 units at ₹ 21)	6,300
		By Balance c/d (60 units at ₹ 21)	1,260
	7,560		7,560

Costing P & L A/c

	(₹)		(₹)
To Finished Goods Control A/c	6,300	By GL Adjustment A/c (Sales)	9,000
To Material Purchase Price Variance A/c	500	By Labour Efficiency Variance A/c	100
To Material Usage Variance A/c	300	By Overhead Efficiency Variance A/c	50
To Labour Rate Variance A/c	370		

To Overhead Expenditure Variance A/c	100		
To Overhead Capacity Variance A/c	150		
To GL Adjustment A/c (Profit)	1,430		
	9,150		9,150

General Ledger (GL) Adjustment A/c

	(₹)		(₹)
To Costing P & L A/c	9,000	By Balance b/d	2,400
To Balance c/d	4,500	By Material Control A/c	3,000
		By Material Purchase Price Variance A/c	500
		By Wages Control A/c	3,700
		By Labour Rate Variance A/c	370
		By Overhead Control A/c	2,100
		By Costing P & L A/c (Profit)	1,430
	13,500		13,500

(b) Variance Accounts:

Material Purchase Price Variance A/c

	(₹)		(₹)
To GL Adjustment A/c	500	By Costing P & L A/c	500
	500		500

Material Usage Variance A/c

	(₹)		(₹)
To Raw Material Control A/c	300	By Costing P & L A/c	300
	300		300

Labour Rate Variance A/c

	(₹)		(₹)
To GL Adjustment A/c	370	By Costing P & L A/c	370
	370		370

Labour Efficiency Variance A/c

	(₹)		(₹)
To Costing P & L A/c	100	By Wages Control A/c	100
	100		100

Overhead Expenditure Variance A/c

	(₹)		(₹)
To Overhead Control A/c	100	By Costing P & L A/c	100
	100		100

Overhead Capacity Variance A/c

	(₹)		(₹)
To Overhead Control A/c	150	By Costing P & L A/c	150
	150		150

Overhead Efficiency Variance A/c

	(₹)		(₹)
To Costing P & L A/c	50	By Overhead Control A/c	50
	50		50

(c) Trial Balance at the end of the Month

	Dr. (₹)	Cr. (₹)
GL Adjustment A/c	--	4,500
Raw Material Control	2,700	--
Work-in-Progress Control	540	--
Finished Goods Control	1,260	--
	4,500	4,500

Partial Plan

(a) Cost Control Accounts

Raw Material Control A/c

	(₹)		(₹)
To Balance b/d (400 Kg. at ₹ 6)	2,400	By Work-in-Progress Control A/c	2,750
To GL Adjustment A/c (500 Kg. at ₹ 7)	3,500	By Balance c/d (450 Kg. at ₹ 7)	3,150
	5,900		5,900

Wages Control A/c

	(₹)		(₹)
To GL Adjustment A/c	4,070	By Work-in-Progress Control A/c	4,070
	4,070		4,070

Overhead Control A/c

	(₹)		(₹)
To GL Adjustment A/c	2,100	By Work-in-Progress Control A/c	2,100
	2,100		2,100

Work-in-Progress Control A/c

	(₹)		(₹)
To Raw Material Control A/c	2,750	By FG Control A/c (360 units at ₹ 21)	7,560
To Wage Control A/c	4,070	By Material Price Variance A/c	50
To Overhead Control A/c	2,100	By Material Usage Variance A/c	300
To Labour Efficiency Variance A/c	100	By Overhead Capacity Variance A/c	150
To Overhead Efficiency Variance A/c	50	By Overhead Expenditure Variance A/c	100
		By Labour Rate Variance A/c	370
		By Balance c/d (40 units)*	540
	9,070		9,070

***Work-in-Progress Control A/c (Closing Balance)** ₹ 540

[Material: 40 units x ₹ 6 = 240, Labour: 20 units x ₹ 10 = 200, Overhead 20 units x ₹ 5]

Finished Goods (FG) Control A/c

	(₹)		(₹)
To Work-in-Progress Control A/c (360 units at ₹ 21)	7,560	By Costing P&L A/c	6,300
		By Balance c/d (60 units at ₹ 21)	1,260
	7,560		7,560

Costing P & L A/c

	(₹)		(₹)
To FG Control A/c	6,300	By GL Adjustment A/c (Sales)	9,000
To Material Price Variance A/c	50	By Labour Efficiency Variance A/c	100
To Material usage Variance A/c	300	By Overhead Efficiency Variance A/c	50
To Labour Rate Variance A/c	370		

To Overhead Expenditure Variance A/c	100		
To Overhead Capacity Variance A/c	150		
To GL Adjustment A/c (Profit)	1,880		
	9,150		9,150

General Ledger (GL) Adjustment A/c

	(₹)		(₹)
To Costing P & L A/c	9,000	By Balance b/d	2,400
To Balance c/d	4,950	By Material Control A/c	3,500
		By Wages Control A/c	4,070
		By Overhead Control A/c	2,100
		By Costing P & L A/c (Profit)	1,880
	13,950		13,950

(b) Variance Accounts:

Material Price Variance A/c

	(₹)		(₹)
To Work-in-Progress Control A/c	50	By Costing P & L A/c	50
	50		50

Material Usage Variance A/c

	(₹)		(₹)
To Work-in-Progress Control A/c	300	By Costing P & L A/c	300
	300		300

Labour Rate Variance A/c

	(₹)		(₹)
To Work-in-Progress Control A/c	370	By Costing P & L A/c	370
	370		370

Labour Efficiency Variance A/c

	(₹)		(₹)
To Costing P & L A/c	100	By Work-in-Progress Control A/c	100
	100		100

Overhead Expenditure Variance A/c

	(₹)		(₹)
To Work-in-Progress Control A/c	100	By Costing P & L A/c	100
	100		100

Overhead Capacity Variance A/c

	(₹)		(₹)
To Work-in-Progress Control A/c	150	By Costing P & L A/c	150
	150		150

Overhead Efficiency Variance A/c

	(₹)		(₹)
To Costing P & L A/c	50	By Work-in-Progress Control A/c	50
	50		50

(c) Trial Balance at the end of the Month

	Dr. (₹)	Cr. (₹)
GL Adjustment	--	4,950
Raw Material Control	3,150	--
Work-in-Progress Control	540	--
Finished Goods Control	1,260	--
	4,950	4,950

Working Notes:

(1) Equivalent Production:

As regards Material (360 + 40) = 400 Units

As regards Labour and Overhead (360 + ½ X 40) = 380 Units

(2) Material Variances:

SQ x SP	AQ x AP	AQ x SP
400 Units x ₹ 6 = ₹ 2,400	400 Units x ₹ 6 + 50 Units x ₹ 7 = ₹ 2,750	450 Units x ₹ 6 = ₹ 2,700

Material Price Variance = SP x AQ – AP x AQ

= ₹ 2,700 – ₹ 2,750 = ₹ 50 (A)

Material Usage Variance = SQ x SP – AQ x SP
= ₹ 2,400 – ₹ 2,700 = ₹ 300 (A)

Material Purchase Price Variance = (SP– AP) x PQ
=(₹ 6 – ₹ 7)× 500 Units = ₹ 500(A)

(3) Labour Variances:

SH x SR	AH x AR	AH x SR
(380 x 2.5) hrs x ₹ 4 = ₹ 3,800	925 hrs x ₹ 4.40 = ₹ 4,070	925 hrs x ₹ 4 = ₹ 3,700

Labour Rate Variance = SR x AH – AR x AH
= ₹ 3,700 – ₹ 4,070 = ₹ 370 (A)

Labour Efficiency Variance = SH x SR – AH x SR
= ₹ 3,800 – ₹ 3,700 = ₹ 100 (F)

(4) Overhead Variances:

Absorbed Overheads	Budgeted Overheads	Actual Overheads	Budgeted Overheads for Actual Hours
380 Units x ₹ 5 ₹ 1,900	₹ 2,000 (Given)	₹ 2,100 (Given)	₹ 925 hrs x Rs. 2 = ₹ 1,850

Fixed Overhead Expenditure Variance = Budgeted Overheads – Actual Overheads
= ₹ 2,000 – ₹ 2,100 = ₹ 100 (A)

Fixed Overhead Volume Variance = Absorbed Overheads – Budgeted Overheads
= ₹ 1,900 – ₹ 2,000
= ₹ 100 (A)

Fixed Overhead Capacity Variance = Budgeted Overheads for Actual Hours – Budgeted Overheads
= ₹ 1,850 – ₹ 2,000 = ₹ 150 (A)

Fixed Overhead Efficiency Variance = Absorbed Overheads – Budgeted Overheads for Actual Hours
= ₹ 1,900 – ₹ 1,850 = ₹ 50 (F)

Illustration 14

A company following standard costing system has the following information for the quarter ending 30th June, 2012:

Material purchased	*12,000 pieces at ₹ 1.32*	*₹ 15,840.00*
Materials consumed	*11,400 pieces at ₹1.32*	*₹ 15,048.00*
Actual wages paid	*2,970 hours at ₹ 4.20*	*₹12,474.00*

Actual factory expenses incurred ₹ 20,400 (Budgeted ₹ 19,800)

Units produced: 1,080 units and sold at ₹ 72 per unit

The standard rates and prices are as under:

Direct materials ₹ 1.20 per unit
Standard input 12 pieces per unit
Direct labour rate ₹ 3.60 per hour
Standard requirement 3.00 hours per unit
Overheads ₹ 7.20 per labour hour

You are required to:

(a) Calculate Material, Labour and Overhead Variances.

(b) Prepare Material Control Account and Work in Progress Control Account if the company had adopted the Partial Plan for accounting of variances. Also give Journal Entries for the same.

Solution

(A) Calculation of Variances:

The cost sheet for 1,080 units will appear as under:

Cost	Std. Qty.	Std. Rate (₹)	Std. Cost (₹)
Direct Material	12,960	1.20	15,552
Direct Labour	3,240	3.60	11,664
Overheads	3,240	7.20	23,328
			50,544

Material Variances

Material Price Variance:

= 11,400 pcs. x (₹ 1.20 – ₹1.32) = ₹ 1,368 (A)

Material Usage Variance:

= ₹ 1.20 x (12,960 pcs. – 11,400 pcs.)

= ₹ 1,872 (F)

Labour Variances

Labour Rate Variance:

= 2,970 hrs. x (₹ 3.60 – ₹ 4.20)= ₹ 1,782 (A)

Labour Efficiency Variance:

= ₹ 3.60 x (3,240 hrs. – 2,970 hrs.) = ₹ 972(F)

Overhead Variances:

(a)	Charged to Production as per Cost Sheet (Absorbed):	₹ 23,328
(b)	Actual Hours × Std. Rate (2,970 hrs. × ₹ 7.20):	₹ 21,384

(c)	Overheads as per Budget	₹ 19,800
(d)	Actual Overheads	₹ 20,400
	Efficiency Variance: (a) – (b)	₹ 1,944 (F)
	Capacity Variance: (b) – (c)	₹ 1,584 (F)
	Expense Variance: (c) – (d)	₹ 600 (A)
	Total Variance: (a) – (d)	₹ 2,928 (F)

(B) Ledger Accounts:

Material Control A/c

Dr. Cr.

	(₹)		(₹)
To Opening balance	—	By Work-in-Progress Control A/c	15,048
To General Ledger Adjustment A/c	15,840	By Balance c/d	792
	15,840		15,840

Work-in-Progress Control A/c

Dr. Cr.

	(₹)		(₹)
To Opening balance	–	By Finished Stock Control A/c	50,544.00
To Material Control A/c	15,048.00	By Material Price Variance A/c	1,368.00
To Wages Control A/c	12,474.00	By Labour Rate Variance A/c	1,782.00
To Overheads Control A/c	20,400.00	By Overhead Exp Variance A/c	600.00
To Material Usage Variance A/c	1,872.00		
To Labour Efficiency Variance A/c	972.00		
To Overhead Efficiency Variance A/c	1,944.00		
To Overhead Capacity Variance A/c	1,584.00		
	54,294.00		54,294.00

Note: Assumed that there is no closing balance of work-in progress.

(C) Journal Entries:

		(₹)	(₹)
(i)	Material Control A/c Dr.	15,840	
	To General Ledger Adjustment A/c		15,840
	(Being the purchase value of 12,000 pieces of materials at ₹ 1.32 each)		

(ii) Work-in-Progress A/c Dr.	15,048	
To Material Control A/c		15,048
(Being the cost of 11,400 pieces of materials actually issued to production at the actual price of ₹ 1.32 each)		
(iii) Work-in-Progress A/c Dr.	12,474	
To Wages Control A/c		12,474
Being the actual amount of direct wages paid for 2,970 hours at ₹ 4.20 per hour)		
(iv) Work-in-Progress A/c Dr	20,400	
To Overhead Expense Control A/c		20,400
(Being the actual overhead expenses incurred)		
(v) Finished Stock Control A/c Dr.	50,544	
To Work-in-Progess A/c		50,544
(Being the standard cost of production transferred to finished goods account)		
(vi) Material Price Variance A/c Dr.	1,368	
Labour Rate Variance A/c Dr.	1,782	
Overhead Expense Variance A/c Dr.	600	
Work-in-Progress A/c Dr.	2,622	
To Material Usage Variance A/c		1,872
To Labour Efficiency Variance A/c		972
To Overhead Efficiency Variance A/c		1,944
To Overhead Capacity Variance A/c		1,584
(Being variance charged)		

5.5 Miscellaneous Illustrations:

☞ *Note: In some solutions alternate formulas are given with the basic formulas for the better understanding of students.*

Computation of All Variances

Illustration 15

The Standard Cost Card of producing one unit of Item 'Q' is as under:

			₹
Direct material —	*A —*	*12 Kg. @ ₹ 10/-*	*120*
	B —	*5 Kg. @ ₹ 6/-*	*30*
Direct wages —		*5 Hrs. @ ₹ 3/-*	*15*

Fixed production overheads		*35*
Total standard cost		*200*

Fixed Production overhead is absorbed on expected annual output of 13,200 units. Actual result for the month of September, 2012 are as under:

Actual production :	*1,000 units*	
		₹
Direct material A	*11,000 Kg.*	*1,21,000*
B	*5,200 Kg.*	*28,600*
Direct wages	*5,500 Hrs.*	*17,500*
Fixed Overheads		*39,000*

You are required to calculate all variances.

Solution

Basic Calculations:

Statement showing Standard and Actual Costs of Material for 1,000 Units of Output and Standard Cost of Actual Input (Standard Proportion)

Material	**Standard Cost**			**Actual Cost**			**Standard Cost of Actual Input in Std. Proportion**		
	Qty. [SQ] (Kg.)	**Price [SP] (₹)**	**Amount [SQ × SP] (₹)**	**Qty. [AQ] (Kg.)**	**Price [AP] (₹)**	**Amount [AQ × AP] (₹)**	**Qty. [RAQ] (Kg.)**	**Price [SP] (₹)**	**Amount [RAQ × SP] (₹)**
A	12,000	10	1,20,000	11,000	11	1,21,000	11,435	10	1,14,350
B	5,000	6	30,000	5,200	5.50	28,600	4,765	6	28,590
	17,000		1,50,000	16,200		1,49,600	16,200		1,42,940

Note :

SQ = Standard Quantity = Expected Consumption for Actual Output

AQ = Actual Quantity of Material Consumed

RAQ = Revised Actual Quantity = Actual Quantity Rewritten in Standard Proportion

SP = Standard Price Per Unit

AP = Actual Price Per Unit

Statement showing Standard and Actual Labour Cost of 1,000 Units Produced

Standard Cost			Actual Cost		
Hours [SH]	Rate [SR] (₹)	Amount [SH x SR] (₹)	Hours [AH]	Rate [AR] (₹)	Amount [AH x AR] (₹)
5,000	3	15,000	5,500	3.1818	17,500

Note :

SH = Standard Hours = Expected time (Time allowed) for Actual Output

AH = Actual Hours paid for

SR = Standard Rate per Labour Hour

AR = Actual Rate per Labour Hour Paid

Statement showing Overheads

Absorbed Overheads [SR** × AO]	Budgeted Overheads [SR* × BH]	Actual Overheads [AR × AH]	Budgeted Overheads for Actual Hours [SR* × AH]
₹35 × 1,000 Units = ₹35,000	₹7* × (1,100 Units × 5 Hrs.) = ₹38,500	₹39,000 (Given)	₹7* × 5,500 Hrs. =₹ 38,500

* Standard Rate per hour = $\left(\frac{₹\ 35}{5\,\text{Hours}} = ₹\ 7\text{/hr.}\right)$

** Standard Rate per Unit (Given)

<u>**Computation of Variances:**</u>

Material Cost Variance = Standard Cost –Actual Cost

= SQ × SP –AQ × AP

(A) = ₹ 1,20,000 – ₹ 1,21,000

= ₹ 1,000 (A)

(B) = ₹ 30,000 – ₹ 28,600

= ₹ 1,400 (F)

Total = ₹ 1,000 (A) + ₹ 1,400 (F)

= ₹ 400 (F)

Material Price Variance **=** Standard Cost of Actual Quantity – Actual Cost

= AQ × SP –AQ × AP

Or

= AQ × (SP – AP)

(A) = 11,000 Kgs. × (₹ 10 – ₹ 11)

	=	₹ 11,000 (A)
(B)	=	5,200 Kgs. × (₹ 6 – ₹ 5.50)
	=	₹ 2,600 (F)
Total	=	₹ 11,000 (A) + ₹ 2,600 (F)
	=	₹ 8,400 (A)
Material Usage Variance	=	Standard Cost of Standard Quantity for Actual Output – Standard Cost of Actual Quantity
	=	SQ × SP – AQ × SP
		Or
	=	SP × (SQ – AQ)
(A)	=	₹ 10 × (12,000 Kgs. – 11,000 Kgs.)
	=	₹ 10,000 (F)
(B)	=	₹ 6 × (5,000 Kgs. – 5,200 Kgs)
	=	₹ 1,200 (A)
Total	=	₹ 10,000 (F) + ₹ 1,200 (A)
	=	₹ 8,800 (F)
Material Mix Variance	=	Standard Cost of Actual Quantity in Standard Proportion – Standard Cost of Actual Quantity
	=	RAQ × SP – AQ × SP
(A)	=	11,435 Kgs. × ₹ 10 – 11,000 Kgs. × ₹ 10
	=	₹ 4,350 (F)
(B)	=	4,765 Kgs. × ₹ 6 – 5,200 Kgs. × ₹ 6 = ₹ 2,610 (A)
Total	=	₹ 1,740 (F)
Material Yield Variance	=	Standard Cost of Standard Quantity for Actual Output – Standard Cost of Actual Quantity in Standard Proportion
	=	SQ × SP –RAQ × SP
		Or
	=	SP × (SQ – RAQ)
(A)	=	₹ 10 × (12,000 – 11,435) = ₹ 5,650 (F)
(B)	=	₹ 6 × (5,000 – 4,765) = ₹ 1,410 (F)
Total	=	₹ 5,650 (F) + ₹ 1,410 (F) = ₹ 7,060 (F)
Labour Cost Variance	=	Standard Cost –Actual Cost
	=	SH × SR –AH × AR
	=	5,000 hrs. × ₹ 3 – 5,500 hrs. × ₹ 3.818....
	=	₹ 2,500 (A)
Labour Rate Variance	=	Standard Cost of Actual Time –Actual Cost

	= AH × SR –AH × AR Or AH × (SR – AR)
	= 5,500 hrs. × ₹ 3 – 5,500 hrs. × ₹ 3.1818.......
	= ₹ 1,000 (A)
Labour Efficiency Variance	= Standard Cost of Standard Time for Actual Output –Standard Cost of Actual Time SH × SR –AH × SR Or SR × (SH – AH)
	= 5,000 hrs × ₹ 3 – 5,500 hrs. × ₹ 3
	= ₹ 1,500 (A)
Fixed Overhead Cost Variance	= Absorbed Fixed Overheads –Actual Fixed Overheads
	= SR** × AO – AR × AH
	= ₹ 35 × 1,000 units – ₹ 39,000
	= ₹ 4,000 (A)

*** Standard Rate per Unit*

Fixed Overhead Expenditure Variance	= Budgeted Fixed Overheads –Actual Fixed Overheads
	= BH x SR* – AR × AH
	= ₹ 7 × (1,100 Units × 5 hrs.) – ₹ 39,000
	= ₹ 500 (A)

** Standard Rate per Hour*

Fixed Overhead Volume Variance	= Absorbed Fixed Overheads –Budgeted Fixed Overheads
	= SR** × AO –SR* × BH
	= ₹ 35 × 1,000 Units – ₹ 7 × (1,100 units × 5 hrs.)
	= ₹3,500 (A)

*** Standard Rate per Unit*   **Standard Rate per Hour*

Fixed Overhead Capacity Variance	= Budgeted Fixed Overheads for Actual Hours –Budgeted Fixed Overheads
	= SR* × AH –SR* × BH
	= ₹ 7 × 5,500 hrs. – ₹ 7 × (1,100 units × 5 hrs.)
	= NIL

**Standard Rate per Hour*

Fixed Overhead Efficiency Variance	=	Absorbed Fixed Overheads –Budgeted Fixed Overheads for Actual Hours
	=	SR** × AO –SR* × AH
	=	₹ 35 × 1,000 units – ₹ 7 × 5,500 hrs.
	=	₹ 3,500 (A)

** *Standard Rate per Unit* **Standard Rate per Hour*

Illustration 16

HK Corporation produces three products A,B and C. The master budget called for the sale of 10,000 units of A at ₹ 12 6,000 units of B at ₹ 15 and 8,000 units of C at ₹ 9. In addition, the standard variable cost for each product was ₹ 7 for A, ₹ 9 for B and ₹ 6 for C. Infact, the firm actually produced and sold 11,000 units of A at ₹ 11.50, 5,000 units of B at ₹ 15.10 and 9,000 units of C at ₹ 8.55.

The firm uses two input to produce each of the products X and Y. The standard price of material X is ₹ 2 and for a unit of material Y is ₹ 1. The materials budgeted to be used for each product were:

Products	***Materials***	
	X (units)	***Y (units)***
A	*2*	*3*
B	*4*	*1*
C	*1*	*4*

The firm actually used 54,000 units of X at a cost of ₹ 1,09,620 and 72,000 units of Y at a cost of ₹ 73,000.

Required:

Determine the mix, quantity and rate variances for sales as well as the yield, mix and price variance for materials.

Solution

<u>Basic Calculations Sales Variances</u> (Sales Value Method):

Product	**Budgeted Sales**			**Actual Sales**			**Standard Sales**	**Revised**
	Qty. [BQ] (Units)	**Rate [BP] (₹)**	**Amount [BQ × BP] (₹)**	**Qty. [AQ] (Units)**	**Rate [AP] (₹)**	**Amount [AQ×AP] (₹)**	**Actual Quantity ×Budgeted price [AQ×BP] (₹)**	**Actual Quantity [RAQ] (Units)**
A	10,000	12	1,20,000	11,000	11.50	1,26,500	1,32,000	10,417
B	6,000	15	90,000	5,000	15.10	75,500	75,000	6,250
C	8,000	9	72,000	9,000	8.55	76,950	81,000	8,333
	24,000		2,82,000	25,000		2,78,950	2,88,000	25,000

Note:

BQ	=	Budgeted Sales Quantity
AQ	=	Actual Sales Quantity
RAQ	=	Revised Actual Sales Quantity
	=	Actual Quantity Sold Rewritten in Budgeted Proportion
BP	=	Budgeted Selling Price per Unit
AP	=	Actual Selling Price

Computation of Sales Variances (Sales Value Method):

Sales Value Variance	=	Actual Sales –Budgeted Sales
	=	AP × AQ –BP × BQ
For Product A	=	₹ 1,26,500 – ₹ 1,20,000 = ₹ 6,500 (F)
For Product B	=	₹ 75,500 – ₹ 90,000 = ₹ 14,500 (A)
For Product C	=	₹ 76,950 – ₹ 72,000 = ₹ 4,950 (F)
Total	=	₹ 6,500 (F) + ₹ 14,500 (A) + ₹ 4,950 (F)
	=	₹ 3,050 (A)
Sales Price Variance	=	Actual Sales –Standard Sales
	=	AP x AQ –BP x AQ
		Or
	=	AQ × (AP – BP)
For Product A	=	11,000 Units × (₹ 11.50 – ₹ 12.00) = ₹ 5,500 (A)
For Product B	=	5,000 Units × (₹ 15.10 – ₹ 15.00) = ₹ 500 (F)
For Product C	=	9,000 Units × (₹ 8.55 – ₹ 9.00) = ₹ 4,050 (A)
Total	=	₹ 5,500 (A) + ₹ 500 (F) + ₹ 4,050 (A)
	=	₹ 9,050 (A)
Sales Volume Variance	=	Standard Sales –Budgeted Sales
	=	BP x AQ –BP x BQ
		Or
	=	BP × (AQ – BQ)
For Product A	=	₹ 12 × (11,000 Units – 10,000 Units) = ₹ 12,000 (F)
For Product B	=	₹ 15 × (5,000 Units – 6,000 Units) = ₹ 15,000 (A)
For Product C	=	₹ 9 × (9,000 Units – 8,000 Units) = ₹ 9,000 (F)
Total	=	₹ 12,000 (F) + ₹ 15,000 (A) + ₹ 9,000 (F)
	=	₹ 6,000 (F)
Sales Mix Variance	=	Standard Sales –Revised Standard Sales
		BP x AQ–BP x RAQ
		Or
		BP × (AQ – RAQ)
For Product A	=	₹ 12 × (11,000 Units – 10,417* Units) = 6,996 (F)
For Product B	=	₹ 15 × (5,000 Units – 6,250 Units) = 18,750 (A)

For Product C = ₹ 9 × (9,000 Units – 8,333* Units) = 6,003 (F)

Total = ₹ 6,996 (F) + ₹ 18,750 (A) + ₹ 6,003 (F)

= ₹ 5,751 (A) #

OR

Sales Mix Variance = Total Actual Qty (units) × (*Average* Budgeted Price *per unit* of Actual Mix – *Average* Budgeted Price *per unit* of Budgeted Mix)

$$= 25{,}000 \text{ Units} \times \left[\left(\frac{₹\ 2{,}88{,}000}{25{,}000 \text{ Units}}\right) - \left(\frac{₹\ 2{,}82{,}000}{24{,}000 \text{ Units}}\right)\right] = 5{,}750(A)^{\#}$$

*Note: ₹ 1 difference is due to * marked figures, rounded nearest to one.*

Sales Quantity Variance = Revised Standard Sales – Budgeted Sales

= BP x RAQ – BP x BQ

Or

= BP × (RAQ – BQ)

For Product A = ₹ 12 × (10,417* Units – 10,000 Units) = 5,004 (F)

For Product B = ₹ 15 × (6,250 Units – 6,000 Units) = 3,750 (F)

For Product C = ₹ 9 × (8,333* Units – 8,000 Units) = 2,997 (F)

Total = ₹ 5,004 (F) + ₹ 3,750 (F) + ₹ 2,997 (F)

= ₹ 11,751 (F) #

OR

Sales Quantity Variance = *Average* Budgeted Price *per unit* of Budgeted Mix × [Total Actual Qty (units) – Total Budgeted Qty (units)]

$$= \left(\frac{₹\ 2{,}82{,}000}{24{,}000 \text{ Units}}\right) \times (25{,}000 \text{ Units} - 24{,}000 \text{ Units})$$

= ₹ 11.75 × (1,000 Units)

= ₹ 11,750 (F) #

*Note: ₹ 1 difference is due to * marked figure we have rounded nearest to one.*

Basic Calculations Sales Variances (Sales Margin Method):

Product	Budgeted Margin			Actual Margin			Actual Quantity × Budgeted Margin	Revised Actual Quantity
	Qty. [BQ] (Units)	Rate [BM] (₹)	Amount [BQ × BM] (₹)	Qty. [AQ] (Units)	Rate [AM] (₹)	Amount [AQ×AM] (₹)	[AQ×BM] (₹)	[RAQ] (Units)
A	10,000	5	50,000	11,000	4.50	49,500	55,000	10,417
B	6,000	6	36,000	5,000	6.10	30,500	30,000	6,250
C	8,000	3	24,000	9,000	2.55	22,950	27,000	8,333
	24,000		1,10,000	25,000		1,02,950	1,12,000	25,000

Note:

BQ	=	Budgeted Sales Quantity
AQ	=	Actual Sales Quantity
RAQ	=	Actual Quantity Sold Rewritten in Budgeted Proportion
BM	=	Budgeted Margin
AM	=	Actual Margin

Computation of Sales Variances (Sales Margin Method):

Sales Margin Variance	=	Actual Margin –Budgeted Margin
	=	AQ × AM –BQ x BM
For Product A	=	₹ 49,500 – ₹ 50,000 = ₹ 500 (A)
For Product B	=	₹ 30,500 – ₹ 36,000 = ₹ 5,500 (A)
For Product C	=	₹ 22,950 – ₹ 24,000 = ₹ 1,050 (A)
Total	=	₹ 500 (A) + ₹ 5,500 (A) + ₹ 1,050 (A)
	=	₹ 7,050 (A)
Sales Margin Price Variance	=	Actual Margin –Standard Margin
	=	AQ × AM –AQ × BM
For Product A	=	11,000 Units × (₹ 4.50 – ₹ 5.00) = 5,500 (A)
For Product B	=	5,000 Units × (₹ 6.10 – ₹ 6.00) = ₹ 500 (F)
For Product C	=	9,000 Units × (₹ 2.55 – 3.00) = 4,050 (A)
Total	=	₹ 5,500 (A) + ₹ 500 (F) + ₹ 4,050 (A)
	=	₹ 9,050 (A)
Sales Margin Volume Variance	=	Standard Margin –Budgeted Margin
	=	AQ× BM – BQ x BM Or
	=	BM x (AQ – BQ)
For Product A	=	₹ 5.00 × (11,000 Units – 10,000 Units) = ₹ 5,000 (F)
For Product B	=	₹ 6.00 × (5,000 Units – 6,000 Units) = ₹ 6,000 (A)
For Product C	=	₹ 3.00 × (9,000 Units – 8,000 Units) = ₹ 3,000 (F)
Total	=	₹ 5,000 (F) + ₹ 6,000 (A) + ₹ 3,000 (F)
	=	₹ 2,000 (F)
Sales Margin Mix Variance	=	Standard Margin – Revised Standard Margin
	=	AQ × BM – RAQ × BM Or BM × (AQ – RAQ)
For Product A	=	₹ 5.00 × (11,000 Units – 10,417* Units) = ₹ 2,915 (F)
For Product B	=	₹ 6.00 × (5,000 Units – 6,250 Units) = ₹ 7,500 (A)

For Product C	=	₹ 3.00 × (9,000 Units – 8,333* Units) = ₹ 2001 (F)
Total	=	₹ 2,915 (F) + ₹ 7,500 (A) + ₹ 2,001 (F) = ₹ 2,584# (A)
		Or
Sales Margin Mix Variance	=	Total Actual Quantity (units) × (*Average* Budgeted Margin *per unit* of Actual Mix – *Average* Budgeted Margin *per unit* of Budgeted Mix)
	=	$25{,}000 \text{ Units} \times \left(\frac{₹\ 1{,}12{,}000}{25{,}000 \text{ units}} - \frac{₹\ 1{,}10{,}000}{24{,}000\text{units}}\right) = ₹\ 2{,}583^{\#}$ (A)

***Note:* #₹ 1 difference is due to * marked figures, rounded nearest to one.**

Sales Margin Quantity Variance	=	Revised Standard Margin – Budgeted Margin
	=	RAQ × BM –BQ × BM
		Or
	=	BM × (RAQ – BQ)
For Product A	=	₹ 5 × (10,417* Units – 10,000 Units) = ₹ 2,085 (F)
For Product B	=	₹ 6 × (6,250 Units – 6,000 Units) = ₹ 1,500 (F)
For Product C	=	₹ 3 × (8,333* Units – 8,000 Units) = ₹ 999 (F)
Total	=	₹ 2,085 (F) + ₹ 1,500 (F) + ₹ 999 (F)
	=	4,584# (F)
		Or
Sales Margin Quantity Variance	=	*Average* Budgeted Margin *per unit* of Budgeted Mix × [Total Actual Quantity (units) – Total Budgeted Quantity (units)]
	=	$\left(\frac{₹\ 1{,}10{,}000}{24{,}000 \text{ units}}\right)$ × (25,000 Units – 24,000 Units)= ₹ 4,583# (F)

***Note:* #₹ 1 difference is due to * marked figures, rounded nearest to one.**

<u>Basic Calculations:</u> Material Variance

Material	Standard Cost			Actual Cost			Actual Quantity × Standard Price	Revised Actual Quantity
	Qty. [SQ] (Units)	Price [SP] (₹)	Amount [SQ × SP] (₹)	Qty. [AQ] (Units)	Amount [AQ×AP] (₹)	Price [AP] (₹)	[AQ×SP] (₹)	[RAQ] (Units)
X	51,000*	2	1,02,000	54,000	1,09,620	2.03	1,08,000	51,408
Y	74,000**	1	74,000	72,000	73,000	1.013	72,000	74,592
	1,25,000		1,76,000	1,26,000	1,82,620		1,80,000	1,26,000

* (11,000 × 2 + 5,000 × 4 +9,000 × 1 = 51,000)
** (11,000 × 3 + 5,000 × 1 + 9,000 × 4 = 74,000)

Note:

SQ = Standard Quantity = Expected Consumption for Actual Output

AQ = Actual Quantity of Material Consumed

RAQ = Revised Actual Quantity = Actual Quantity Rewritten in Standard Proportion

SP = Standard Price per unit

AP = Actual Price per unit

Computation of Material Variances:

Material Cost Variance	=	Standard Cost –Actual Cost
	=	SQ × SP –AQ × AP
For Material 'X'	=	₹ 1,02,000 – ₹ 1,09,620
	=	₹ 7,620 (A)
For Material 'Y'	=	₹ 74,000 – ₹ 73,000
	=	₹ 1,000 (F)
Total	=	₹ 7,620 (A) + ₹ 1,000 (F)
	=	₹ 6,620 (A)
Material Price Variance	=	Standard Cost of Actual Quantity– Actual Cost
	=	AQ × SP –AQ × AP
		Or
		AQ × (SP – AP)
For Material 'X'	=	54,000 Units × (₹ 2.00 – ₹ 2.03) = ₹ 1,620 (A)
For Material 'Y'	=	72,000 Units × (₹ 1.00 – ₹ 1.0139) = ₹ 1,000 (A)
Total	=	₹ 1,620 (A) + ₹ 1,000 (A) = ₹ 2,620 (A)
Material Usage Variance	=	Standard Cost of Standard Quantity for Actual Output – Standard Cost of Actual Quantity
	=	SQ × SP –AQ × SP
		Or
		SP × (SQ –AQ)
For Material 'X'	=	₹ 2 × (51,000 Units – 54,000 Units) = ₹ 6,000 (A)
For Material 'Y'	=	₹ 1 × (74,000 Units – 72,000 Units) = ₹ 2,000 (F)
Total	=	₹ 6,000 (A) + ₹ 2,000 (F) = ₹ 4,000 (A)
Material Mix Variance	=	Standard Cost of Actual Quantity in Standard Proportion – Standard Cost of Actual Quantity
		RAQ x SP –AQ x SP
		Or
	=	SP × (RAQ – AQ)

For Material 'X'	=	₹ 2 × (51,408 Units – 54,000 Units) = ₹ 5,184 (A)
For Material 'Y'	=	₹ 1 × (74,592 Units – 72,000 Units) = ₹ 2,592 (F)
Total	=	₹ 5,184 (A) + ₹ 2,592 (F) = ₹ 2,592 (A)
		Or
Material Mix Variance	=	Total Actual Quantity (units) × (*Average* Standard Price *per unit* of Standard Mix – Average Standard Price *per unit* of Actual Mix)
	=	1,26,000 Units x $\left(\frac{₹\ 1,76,000}{1,25,000 \text{ units}} - \frac{₹\ 1,80,000}{1,26,000 \text{ units}}\right)$
	=	₹ 2,592 (A)
Material Yield Variance	=	Standard Cost of Standard Quantity for Actual Output – Standard Cost of Actual Quantity in Standard Proportion
		SQ x SP –RAQ x SP
		Or
		SP × (SQ – RAQ)
For Material 'X'	=	₹ 2 × (51,000 Units – 51,408 Units) = ₹ 816 (A)
For Material 'Y'	=	₹ 1 × (74,000 Units – 74,592 Units) = ₹ 592 (A)
Total	=	₹ 816 (A) + ₹ 592 (A)
	=	₹ 1,408 (A)
		Or
Material Yield Variance	=	*Average* Standard Price *per unit* of Standard Mix × [Total Standard Quantity (units) – Total Actual Quantity (units)]
	=	$\left(\frac{₹\ 1,76,000}{1,25,000 \text{ Units}}\right)$ × (1,25,000 Units – 1,26,000 Units)
	=	₹ 1,408 (A)

Equivalent Concept – Variance Analysis

Illustration 17

GFE Associates undertake to prepare Property Tax returns. They use the weighted average method and actual costs for financial reporting purpose. However, for internal reporting, they use a standard cost system. The standards, on equivalent performance, have been established as follows:

Labour per return	*10 hrs. @ ₹30 per hour*
Overhead per return	*10 hrs. @ ₹15 per hour*

For June 2012 performance, budgeted overhead is ₹108,000 for the standard labour hours allowed.

The following additional information pertains to the month of June 2012:

June 1	*Returns in process (25% complete)*	*180 Nos.*
	Returns started in Jun	*820 Nos.*

June 30	*Returns in process (80% complete)*	*200 Nos.*
Cost Data		
June 1	*Returns in process:*	
	Labour	*₹ 16,000*
	Overheads	*8,000*
June 1 to 30	*Labour (4,000 hrs.)*	*2,00,000*
	Overheads	*1,00,000*

You are required to compute:

(a) *For each cost element, equivalent units of performance and the actual cost per equivalent unit.*

(b) *Actual cost of returns in process on June 30*

(c) *The standard cost per return, and*

(d) *The labour rate Variance, labour efficiency variance, overhead volume and overhead expenditure variance.*

Solution

(a) Statement showing cost elements equivalent units of performance and the actual cost per equivalent unit:

Detail of Returns	Detail of Input Units	Details	Equivalent Units				
			Output Units	Labour		Overheads	
				Units	%	Units	%
Returns in Process at Start	180	Returns Completed in June	800	800	100	800	100
Returns Started in June	820	Returns in Process at the end of June	200	160	80	160	80
	1,000		1,000	960		960	
Costs:				₹		₹	
From previous month				16,000		8,000	
During the month				2,00,000		1,00,000	
Total Cost				2,16,000		1,08,000	
Cost per Equivalent Unit				225.00		112.50	

Note: *Since company follows Weighted Average Method, the stages of completion of returns at the beginning of June has been ignored.*

(b) Actual cost of returns in process on June 30:

	Numbers	Stage of Completion	Rate per Return (₹)	Total (₹)
Labour	200 returns	0.80	225.00	36,000
Overhead	200 returns	0.80	112.50	18,000
				54,000

(c) Standard Cost per Return:

Labour 10 Hrs x ₹30 per hour = ₹ 300
Overhead 10 Hrs x ₹15 per hour = ₹ 150
450

(d) Computation of Variances:

Statement showing output (Jun only) element wise	Labour	Overhead
Actual performance in June in terms of equivalent units as Calculated above	960	960
Less: Returns in process at the beginning of June in terms of equivalent units i.e. 25% of returns (180)	45	45
	915	915

Variance Analysis:

Labour Rate Variance

= Actual Time x (Standard Rate – Actual Rate)

= Standard Rate x Actual Time – Actual Rate x Actual Time

= ₹ 30 x 4,000 hrs. – ₹ 2,00,000 = ₹ 80,000(A)

Labour Efficiency Variance

= Standard Rate x (Standard Time – Actual Time)

= Standard Rate x Standard Time – Standard Rate x Actual Time

= ₹ 30 x (915 units x 10 hrs.) – ₹ 30 x 4,000 hrs. = ₹ 154,500(F)

Overhead Expenditure or Budgeted Variance

= Budgeted Overhead – Actual Overhead

= ₹ 108,000 – ₹ 100,000

= ₹ 8,000(F)

Overhead Volume Variance

= Recovered/Absorbed Overhead – Budgeted Overhead

= 915 Units x 10 hrs. x ₹ 15 – ₹ 108,000 = ₹ 29,250(F)

Illustration 18

Electro-Soft Ltd. has prepared the following cost sheet based on 8,000 units of output per month

	₹
Direct Materials 1.5 kg @ ₹ 24 per kg	*36.00*
Direct Labour 3 hours @ ₹ 4 per hours	*12.00*
Factory overheads	*12.00*
Total	*60.00*

The flexible budget for factory overhead is as under:

Output (units)	*6,000*	*7,500*	*9,000*	*10,500*
Factory overhead (₹)	*81,600*	*92,400*	*1,03,200*	*1,14,000*

The actual results for the month of October, 2002 are given below:

— *Direct Materials purchased and consumed were 11,224 kg at ₹ 2,66,750.*

— *Direct Labour hours worked were 22,400 and Direct Wages paid amounted to ₹ 96,320.*

— *Factory overhead incurred amounted to ₹ 96,440 out of which the variable overhead is ₹ 2.60 per direct hour worked.*

— *Actual output is 7,620 units.*

— *Work-in-progress:*

Opening WIP:	*300 units:*
	Materials 100% complete; Labour and Overhead 60% complete
Closing WIP:	*200 units:*
	Materials 50% complete; Labour and Overheads 40% complete

You are required to analyse the variances.

Solution

Statement of Equivalent Product

	Material	Labour & Overhead
Output of Units produced	7,620	7,620
Add : Closing WIP	100	80
(200 units × 50% for Materials)		
(200 units × 40% for Labour & Overhead)	7,720	7,700
Less: Opening WIP	(300)	(180)
(300 units × 100% Complete)	7,420	7,520

Statement Showing Standard Cost & Actual Cost of Material

Standard Cost			Actual Cost		
Qty. [SQ] (Kg.)	Price [SP] (₹)	Amount [SQ × SP] (₹)	Qty. [AQ] (Kg.)	Price [AP] (₹)	Amount [AQ × AP] (₹)
11,130 (7,420 × 1.5)	24	2,67,120	11,224	23.766*	2,66,750

* Amount/Quantity

Note :

SQ = Standard Quantity = Expected Consumption for Actual Output
AQ = Actual Quantity of Material Consumed
SP = Standard Price Per Unit
AP = Actual Price Per Unit

Material Cost Variance = Standard Cost – Actual Cost
= SQ × SP – AQ × AP
= ₹ 2,67,120 – ₹ 2,66,750
= ₹ 370 (F)

Material Price Variance = Standard Cost of Actual Quantity – Actual Cost
= AQ × SP – AQ × AP
Or
= AQ × (SP – AP)
= 11,224 × (₹ 24 – ₹ 23.766)
= 2,626 (F)

Material Usage Variance = Standard Cost of Standard Quantity for Actual Output – Standard Cost for Actual Quantity
= SQ × SP – AQ × SP
Or
= SP × (SP – AQ)
= ₹ 24 × (11,130 Kg. – 11,224 Kg.)
= ₹ 2,256 (A)

Statement showing Standard Cost & Actual Cost: Labour

Standard Cost			Actual Cost		
Time SH (Hrs.)	**Rate SR (₹)**	**Amount SH × SR (₹)**	**Time AH (Hrs.)**	**Rate AR (₹)**	**Amount AH × AR (₹)**
22,560 [7,520 Units × 3 Hrs.]	4	90,240	22,400	4.30*	96,320

*Amount/Hrs.

Note :

SH = Standard Hours = Expected time (Time allowed) for Actual Output.
AH = Actual Hours paid for
SR = Standard Rate per Labour Hour
AR = Actual Rate per Labour Hour Paid

Labour Cost Variance = Standard Cost – Actual Cost
= SH × SR – AH × AR
= ₹ 90,240 – ₹ 96,320
= ₹ 6,080 (A)

Labour Rate Variance	=	Standard Cost of Actual Time – Actual Cost
	=	AH × SR – AH × AR
		Or
	=	AH × (SR – AR)
	=	22,400 × (₹ 4 – ₹ 4.30)
	=	₹ 6,720 (A)
Labour Efficiency Variance	=	Standard Cost of Standard Time for Actual Output – Standard Cost of Actual Time
	=	SH × SR – AH × SR
		Or
	=	SR × (SH – AH)
	=	₹ 4 × (22,560 hrs – 22,400 hrs)
	=	₹ 640 (F)

Statement Showing Standard/ Actual/ Budgeted Variable Overheads

Standard Variable Overheads [SH × SR]	Actual Variable Overheads [AH × AR]	Budgeted Overheads for Actual Hours [SR × AH]
[7,520 Units × 3 Hrs.] × ₹2.40 = ₹ 54,144	22,400 Hrs. × ₹ 2.60 = ₹ 58,240	22,400 Hrs. × ₹ 2.40 = ₹ 53,760

Note:

SH = Standard Hours = Expected Time for Actual Output

SR = Standard Rate per Hour

AH = Actual Hours Worked

AR = Actual Rate per Hour

Variable Overhead Cost Variance	=	Standard Variable Overheads – Actual Variable Overheads
	=	SH × SR – AH × AR
	=	₹ 54,144 – ₹ 58,240
	=	₹ 4,096 (A)
Variable Overhead Expenditure Variance	=	Budgeted Variable Overheads for Actual hours – Actual Variable Overheads
	=	SR × AH – AR × AH
	=	₹ 53,760 – ₹ 58,240 = ₹ 4,480 (A)
Variable Overhead Efficiency Variance	=	Standard Variable Overheads – Budgeted Variable Overheads for Actual Hours
	=	SH × SR – SR × AH
	=	₹ 54,144 – ₹ 53,760
	=	₹ 384 (F)

Statement showing Absorbed/Budgeted/Actual Fixed Overheads

Absorbed Fixed Overheads (SH × SR)	Budgeted Fixed Overheads (BH × SR)	Actual Fixed Overheads (AR × AH)	Budgeted Fixed Overheads for Actual Hours* (SR × AH)
(7,520 Units × 3 Hrs.) × ₹ 1. 60 = ₹ 36,096	(8,000 Units × 3 hrs.) × ₹ 1.60 = ₹ 38,400	22,400 Hrs. × ₹1.7054* = ₹ 38,200	₹ 1.60 × 22,400 hrs. = ₹ 35,840

** Amount/Hrs.*

Note:

SH = Standard Hours = Expected Time for Actual Output

SR = Standard Rate per Hour

BH = Budgeted Hours = Expected Time for Budgeted Output

AH = Actual Hours Worked

AR = Actual Rate per Hour

Workings:

Standard Variable Overhead Rate per Unit = Change in Factory Overheads/Change in Output
= (₹ 92,400 – ₹ 81,600)/(7,500 units – 6,000 units)
= ₹ 7.20 per unit

Standard Variable Overhead Rate per Hour = 2.40 per hour (7.20 per unit/3 hrs.)

Standard Fixed Overhead Rate per Unit = Total Standard Factory Overhead per Unit – Standard Variable Overhead per unit

Standard Fixed Overhead per Unit = ₹ 12 – ₹ 7.20 = ₹ 4.80

Standard Fixed Overhead Rate per hour = ₹ 4.80/3 = ₹ 1.60 per hour

Fixed Overhead Cost Variance = Absorbed Fixed Overheads –Actual Fixed Overheads
= SH × SR –AR × AH
= ₹ 36,096 – ₹ 38,200 = ₹ 2,104 (A)

Fixed Overhead Expenditure Variance = Budgeted Fixed Overheads – Actual Fixed Overheads
= BH × SR – AR × AH
= ₹ 38,400 – ₹ 38,200= ₹ 200 (F)

Fixed Overhead Volume Variance = Absorbed Fixed Overheads – Budgeted Fixed Overheads
= SH × SR – BH × SR
= ₹ 36,096 – ₹ 38,400 = ₹ 2,304 (A)

Fixed Overhead Capacity Variance = Budgeted Fixed Overheads for Actual Hours – Budgeted Fixed Overheads
= SR × AH – BH × SR
= ₹ 35,840 – ₹ 38,400 = 2,560 (A)

Fixed Overhead Efficiency Variance	= Absorbed Fixed Overheads – Budgeted Overheads for Actual hours
	= SH×SR – SR × AH
	= ₹ 36,096 – ₹ 35,840
	= ₹ 256 (F)

Preparation of Financial Profit & Loss Statement *with given* Variances

Illustration 19

The following is the Operating Statement of a company for April 2012:

				(₹)
Budgeted Profit				*2,00,000*
Variances:		*Favourable (₹)*	*Adverse (₹)*	
Sales	*Volume*		*8,000*	
	Price	*19,200*		
Direct Material	*Price*		*9,920*	
	Usage		*12,800*	
Direct Labour	*Rate*		*7,200*	
	Efficiency	*7,200*		
Fixed Overheads				
	Efficiency	*4,800*		
	Capacity		*8,000*	
	Expense	*2,800*		
		34,000	*45,920*	*11,920 (A)*
Actual Profit				*188,080*

Additional information is as under:

Budget for the year	*1,20,000 units*
Budgeted fixed overheads	*₹ 9,60,000 per annum*
Standard cost of one unit of product is:	
Direct Materials	*5 kg.@ ₹8 per kg.*
Direct Labour	*2 hours @ ₹6 per hour*
Fixed overheads are absorbed on direct labour hour basis.	
Profit	*25% on sales*

You are required to prepare the Annual Financial Profit / Loss Statement for April, 2012 in the following format:

Account	*Qty./ Hours*	*Rate / Price (₹)*	*Actual Value (₹)*
Sales *Direct Materials* *Direct Labour* *Fixed Overheads* *Total Costs* *Profit*			

Solution

Working Notes:

1. (a) Budgeted Fixed Overhead (per unit):

   = (Budgeted Fixed Overheads p.a. / Budgeted Output for the Year)

   = ₹9,60,000 / 1,20,000 units = ₹8 (per unit)

   (b) Budgeted Fixed Overhead Hour:

   = Budgeted Fixed Overhead (per unit) / Standard Labour Hours (per unit)

   = ₹8 / 2 hours = ₹4 per hour

2. Statement showing Standard Cost and Budged Selling Price

	(₹)
(a) Standard Cost (per unit):	
Direct Material	40
(5 kg. × ₹8/- per kg.)	
Direct Labour	12
(2 hours × ₹6/- per hour)	
Fixed Overhead	8
(2 hours × ₹4)	
Total Standard Cost (per unit)	60
(b) Budgeted Selling Price (per unit)	
Standard Cost (per unit)	60
Standard Profit (per unit)	20
(25% on Sales or 33-1/3% of Standard Cost)	
Budgeted Selling Price (per unit)	80

3. (a) Actual Output (units) for April, 2012:

   Fixed Overhead Volume Variance:

   = Efficiency Variance + Capacity Variance

=₹4,800 (F) + ₹8,000 (A)

= ₹3,200 (A)

Fixed Overhead Volume Variance:

= Absorbed Overheads - Budgeted Overheads

= (Standard Hours for Actual Output – Budgeted Hours) x Standard Fixed Overhead Rate

$\Rightarrow$ (2 hrs x Actual Output – 10,000 units x 2 hrs) x ₹ 4 = (-) ₹3,200

$\Rightarrow$Actual Output = 9,600 units

(b) Actual Fixed Overhead Expenses:

Fixed Overhead Expenses Variance= (Budgeted Fixed Overheads – Actual Fixed Overheads)

$\Rightarrow$ (₹80,000 – Actual Fixed Overheads) = ₹2,800 (F)

$\Rightarrow$ Actual Fixed Overheads=₹77,200

4. (a) Actual Sales Quantity (units):

Sales Margin Volume Variance

$$= \text{Budgeted Margin } \textit{per unit} \text{ x } \left(\begin{matrix}\text{Actual Sales}\\ \text{Quantity } \textit{units}\end{matrix} - \begin{matrix}\text{Budgeted}\\ \text{Quantity } \textit{units}\end{matrix}\right)$$

$\Rightarrow$ ₹20 (Actual Sales Quantity – 10,000 units) = ₹8,000 (A)

$\Rightarrow$Actual Sales Quantity = 9,600 units

(b) Actual Selling Price (per unit):

$$\text{Sales Price Variance} = \left(\begin{matrix}\text{Actual Selling}\\ \text{Price } \textit{per unit}\end{matrix} - \begin{matrix}\text{Budgeted Selling}\\ \text{Price } \textit{per unit}\end{matrix}\right) \text{x} \begin{matrix}\text{Actual}\\ \text{Sales } \textit{units}\end{matrix}$$

$\Rightarrow$ (Actual Selling Price *per unit* – ₹80) × 9,600 units= ₹19,200 (F)

$\Rightarrow$Actual Selling Price *per unit* = ₹82

5. (a) Actual Quantity of Material Consumed:

$$\text{Material Usage Variance} = \left(\begin{matrix}\text{Standard}\\ \text{Quantity}\end{matrix} - \begin{matrix}\text{Actual}\\ \text{Quantity}\end{matrix}\right) \text{x} \begin{matrix}\text{Standard Price}\\ \textit{per unit}\end{matrix}$$

$\Rightarrow$ (9,600 units × 5 kg. – Actual Quantity) x ₹8= ₹ 12,800 (A)

$\Rightarrow$ Actual Quantity = 49,600 Kg.

(b) Actual Price *per kg*:

Material Price Variance

= (Standard Price *per kg.* – Actual Price *per kg.*) x Actual Quantity of Material Consumed

⇒ (₹8 – Actual Price *per kg.*) x 49,600 Kg. =9,920 (A)

⇒Actual Price *per kg* ₹ 8.20

6. (a) Actual Direct Labour Hours Used:

Labour Efficiency Variance = (Standard Hours – Actual Hours) x Standard Rate per hour

⇒ (9,600 units × 2 hrs – Actual Hours) x ₹6 = ₹7,200 (F)

⇒Actual Direct Labour Hours =18,000 hours

(b) Actual Direct Labour Hour Rate:

$$\text{Labour Rate Variance} = \left(\begin{matrix}\text{Standard} \\ \text{Rate } \textit{per hour}\end{matrix} - \begin{matrix}\text{Actual} \\ \text{Rate} \textit{per hour}\end{matrix}\right) \times \begin{matrix}\text{Actual Direct} \\ \text{Labour Hours}\end{matrix}$$

⇒ (₹6 per hour – Actual Rate *per hour*) x 18,000 hours =₹7,200 (A)

⇒Actual Direct Labour Hour Rate= ₹6.40 per hour

Annual financial Profit /Loss Statement
(for April, 2012)

Account	Qty./ Hours	Rate/Price (₹)	Actual Value (₹)
(a)	(b)	(c)	(d)=(b)×(c)
Sales: (A) [Refer to working note 4]	9,600 units	82	7,87,200
Direct Materials: [Refer to working note 5]	49,600 kgs.	8.20 per kg.	4,06,720
Direct Labour: [Refer to working note 6]	18,000 hours	6.40 per hour	115,200
Fixed Overheads: [Refer to working note 6 (a) and 3 (b)] *(₹ 77,200/18,000 hours)* *(absorbed on direct labour hour basis)*	18,000 hours	4.288… per hour	77,200
Total Costs: (B)			5,99,120
Profit : [(A) – (B)]			188,080

Factors Contributing to Change in Profit

Illustration 20

The working results of a company for two corresponding years are shown below:

	Year 1 ₹ in lakhs	Year 2 ₹ in lakhs
Sales	*1,200*	*1,540*
Direct Material	*600*	*648*
Direct Wages and Variable Overheads	*360*	*412*
Fixed Overheads	*160*	*300*
	1,120	*1,360*
Profit	*80*	*180*

In year 2, there has been an increase in the selling price by 10%. Following are the details of material consumption and utilization of direct labour hours during the two years.

	Year 1	Year 2
Direct Material Consumption in m/t	*5,00,000*	*5,40,000*
Direct Labour Hours	*75,00,000*	*80,00,000*

You are required to:

(i) Keeping year 1 as base year, analyse the results of year 2 and work out the amount which each factor has contributed to change in profit.

(ii) Find out the break even sales for both years.

(iii) Calculate the percentage increase in selling price that would be needed over the sale value of year 2 to earn a margin of safety of 45%.

Solution

(i) Reconciliation statement showing which factor has contributed change in profit

(₹ in lacs)

	Favourable	*Adverse*
Increase in Contribution Due to Increase in Volume (*₹ 280 lacs – ₹ 240 lacs*) (*Refer to working note 3*)	40	—
Sales Price Variance (*Refer to working note 3*)	140	—
Material Usage Variance (*Refer to working note 4*)	52	—
Material Price Variance (*Refer to working note 4*)	—	—
Direct Labour Rate Variance (*Refer to working note 4*)	—	28
Direct Labour Efficiency Variance (*Refer to working note 4*)	36	—

Fixed Overhead Expenditure Variance (*Refer to working note 3*)	—	140
Change in Profit	268	168
Change in Profit (Net)		100

(ii) Break-even sales

Break-even Sales (Year 1) $= \dfrac{\text{Fixed cost}}{\text{P/V ratio}}$

(*Refer to working note 3*)

Break-even Sales (Year 2) $= \dfrac{₹\ 160 \text{ lacs}}{\left(\dfrac{₹\ 240 \text{ lacs}}{₹\ 1{,}200 \text{ lacs}}\right)} = ₹\ 800 \text{ lacs}$

(*Refer to working note 3*)

$= \dfrac{₹\ 300 \text{ lacs}}{\left(\dfrac{₹\ 480 \text{ lacs}}{₹\ 1{,}540 \text{ lacs}}\right)} = ₹\ 962.50 \text{ lacs}$

(iii) Percentage increase in selling price needed over the sales value of year 2 to earn a margin of safety of 45% in year 2

P/V Ratio = (₹ 480 lacs/₹ 1,540 lacs) × 100 = 31.169%

Break-even Sales $= \dfrac{₹\ 962.50 \text{ lacs}}{₹\ 1{,}540 \text{ lacs}} \times 100 = 62.5\%$

(*as % to sales*)

If Margin of Safety to be earned is 45% then Break-even Point should be 55%

Contribution increase required $= \dfrac{62.5 \times 31.169}{55} = 35.4193\%$

Revised Contribution = 1,540 lacs × 35.4193% = 545.45 lacs

Present Contribution = ₹ 480 lacs

Increase in Selling Price required = ₹ 65.45 lacs (₹ 545.45 lacs – ₹ 480 lacs)

Percentage increase in Selling Price

over the Sales Value of year 2 $= \dfrac{₹65.45 \text{ lacs}}{₹1{,}540 \text{ lacs}} \times 100 = 4.25\%$

Working Notes :

1. Budgeted Sales in year 2

If Actual Sales in year 2 is ₹ 110 then Budgeted Sales is ₹ 100.

If Actual Sales in year 2 is ₹ 1 then Budgeted Sales = $\dfrac{₹\ 100}{₹110}$

If Actual Sales in year 2 are ₹ 15,40,00,000 then Budgeted Sales are

$$= \frac{₹100}{₹110} \times ₹15,40,00,000 = ₹1,400 \text{ lacs}$$

2. **Budgeted figures of direct material; direct wages; and variable overhead worked out on the basis of % of sales in year 2:**

Direct Material % to Sales (in year 1) $= \dfrac{\text{Direct Wages and Variable Overhead}}{\text{Sales}}$

$$= \frac{600}{1,200} \times 100 = 50\%$$

Budgeted figure of Direct Material (in year 2) = 50% × ₹ 1,400 lacs = 700 lacs

Direct Wages and Variable Overhead $= \dfrac{\text{Direct Wages and Variable Overhead}}{\text{Sales}}$

(% to sales in year 1)

$$= \frac{360}{1,200} \times 100 = 30\%$$

Budgeted figure of Direct Wages and Variable Overhead (in year 2) = 30% × 1,400 lacs = 420 lacs

3. **Statement of figures extracted from working results of a company**

(*Figure in lacs of ₹*)

	Year 1 [Actual] (a)	Year 2 [Budgeted] (b)	Year 2 [Actual] (c)	Total [Variance] (d) = (c) – (b)
Sales : (A) (**Refer to working note 1*)	1,200	1,400*	1,540	140 (F)
Direct Material...(a) (**Refer to working note 2*)	600	700*	648	52 (F)
Direct Wages and Variable Overhead...(b) (**Refer to working note 2*)	360	420*	412	8 (F)
Total Variable Costs: (B) = (a + b)	960	1,120	1,060	60(F)
Contribution (C) = (A) – (B)	240	280	480	200 (F)
Less : Fixed Cost	160	160	300	140 (A)
Profit	80	120	180	60(F)

(4) (i) Data for Material Variances :

Standard Cost for Actual Output			Actual Cost		
Quantity of Material (m/t)	Rate per m/t (₹)	Amount (₹)	Quantity of Material (m/t)	Rate per m/t (₹)	Amount (₹)
5,83,333 $\left[\frac{₹700 \text{ lacs}}{₹120}\right]$	120*	700 lacs	5,40,000	120	648 lacs

* ₹ 600 lacs / 5 lacs m/t

Material Price Variance = (Standard Rate – Actual Rate) × Actual Quantity = Nil

Material Usage Variance = (Standard Quantity – Actual Quantity) × Standard Rate per m/t

= (5,83,333 – 5,40,000) × ₹ 120 = ₹ 52 lacs (F)

(ii) Data for labour variances overhead variances

Standard Cost for Actual Output			Actual Cost		
Labour Hours	Rate per hour (₹)	Amount (₹)	Labour Hours	Rate per hour (₹)	Amount (₹)
87,50,000 $\left[\frac{₹420 \text{ lacs}}{₹4.80}\right]$	4.80*	420 lacs	80,00,000	5.15	412 lacs

*`₹ 360 lacs / 75 lacs hours

Labour Rate Variance:

= (Standard Rate – Actual Rate) x Actual Labour Hours

= (₹ 4.80 – ₹ 5.15) x 80,00,000 = ₹ 28 lacs (A)

Labour and Variable Overhead Efficiency Variance:

= (Standard Labour Hours – Actual Labour Hours) × Standard Rate per Hour

= (87,50,000 – 80,00,000) x ₹ 4.80 = ₹ 36 lacs (F)

Selling Cost Variance

Illustration 21

Ravi, Richard, Rahim and Roop Singh are regional salesmen distributing the product of Super Perfumes Ltd. The selling price of the product is ₹ 400 per unit. The sales quota and the standard selling expenses for the year are:

Salesmen	*Sales Quota (₹)*	*Standard Selling Expenses (₹)*
Ravi	*7,50,000*	*2,25,000*
Richard	*9,00,000*	*2,47,500*
Rahim	*11,50,000*	*2,87,500*
Roop Singh	*6,00,000*	*2,25,000*

Actual data for the year were as follows: -

	Ravi	*Richard*	*Rahim*	*Roop Singh*
Days on field work	*200*	*175*	*225*	*250*
Kilometres covered	*20,000*	*18,000*	*18,000*	*30,000*
	₹	*₹*	*₹*	*₹*
Sales	*8,00,000*	*10,00,000*	*10,50,000*	*5,20,000*
Salary	*80,000*	*80,000*	*80,000*	*80,000*
Free samples	*9,000*	*7,500*	*5,375*	*8,000*
Postage and stationery	*8,000*	*9,000*	*10,000*	*6,000*
Other expenses	*9,000*	*5,000*	*4,000*	*10,000*

The salesmen are allowed conveyance allowance of ₹ 1.50 per kilometre and a daily allowance of ₹ 80 per day for the days spent on field work. Ravi gets a commission of 6 percent on sales and others are given a commission of 5 percent on sales. Corporate sales office expenses are chargeable at the rate of ₹ 30 per unit sold in the case of Ravi and Richard and ₹ 40 per unit in the case of Rahim and Roop Singh. Prepare a schedule showing the selling cost variances by salesmen.

Solution

Working Note:

		Ravi	Richard	Rahim	Roop Singh
(i)	Standard Sales Units (Sales Quota ÷ ₹ 400)	1,875	2,250	2,875	1,500
(ii)	Standard Selling Expenses per Unit (₹) (Std. Selling Expenses/Std. Sales Units)	120	110	100	150
(iii)	Actual Sales Units (Actual Sales ÷ ₹ 400)	2,000	2,500	2,625	1,300
(iv)	Actual Selling Costs	₹	₹	₹	₹
	Daily Allowance	16,000	14,000	18,000	20,000
	Conveyance Allowances	30,000	27,000	27,000	45,000
	Salaries	80,000	80,000	80,000	80,000
	Free Samples	9,000	7,500	5,375	8,000

	Postage & Stationery	8,000	9,000	10,000	6,000
	Other Expenses	9,000	5,000	4,000	10,000
	Commission on Sales	48,000	50,000	52,500	26,000
	Corporate Sales Office Expenses	60,000	75,000	1,05,000	52,000
	Total Actual Selling Cost	2,60,000	2,67,500	3,01,875	2,47,000
(v)	Standard Selling Cost (Actual Units Sold × Std. Selling Expenses per Unit)	2,40,000	2,75,000	2,62,500	1,95,000

Calculation of Variances:

Since all the selling expenses have been related to sales units, only one variance can be calculated by comparing the standard and actual selling costs as is shown in the schedule below:

Schedule showing the selling cost variances by salesman

	Ravi (₹)	Richard (₹)	Rahim (₹)	Roop Singh (₹)	Total (₹)
Standard Selling Expenses (Refer to Working Note (v))	2,40,000	2,75,000	2,62,500	1,95,000	9,72,500
Actual Selling Expenses (Refer to Working Note (iv))	2,60,000	2,67,500	3,01,875	2,47,000	10,76,375
Selling Cost Variance	20,000 (A)	7,500 (F)	39,375(A)	52,000(A)	1,03,875(A)

Illustration 22

X Manufacturing company takes over sales from the Selling Agents. In the first month of operation of direct sales, the following costs have been incurred. Prepare the actual percentage of selling cost on total sales, compare with the standard selling cost.

Compute the variances and offer your comments about the standards, which are based on actual for the previous year, and performance of the Zonal offices.

Zonal offices	*Sales Budgets (units)*	*Standard Selling Expenses*
Eastern India (E.I.)	*20,000*	*₹ 16,000*
Western India (W.I.)	*12,000*	*12,000*
Northern India (N.I.)	*6,000*	*8,000*
Southern India (S.I.)	*15,000*	*12,000*
Central India (C.I.)	*10,000*	*10,000*
Northern Western India (N.W.I.)	*5,000*	*8,000*
Selling (price per unit) – ₹ 25		

Actual:	*E.I.*	*W.I.*	*N.I.*	*S.I.*	*C.I.*	*N.W.I*
Units Sold ('000 units)	*19*	*10*	*5.9*	*17.5*	*9.5*	*5*
Salesmen's Salaries (₹ '000)	*8*	*7*	*5*	*7*	*6*	*5*
Sales Travelling (₹ '000)	*4*	*5*	*3.6*	*2.7*	*2.7*	*1.8*
Halting Charges & Bhatta (₹)	*850*	*800*	*500*	*500*	*700*	*500*
Salesmen's Commission on Selling Prices @	*1%*	*1.25%*	*1%*	*0.9%*	*1%*	*1%*

Solution

COMPARATIVE COST STATEMENT OF SELLING EXPENSES

		E.I.	W.I.	N.I.	S.I.	C.I.	N.W.I.
	Standard Data						
1.	Selling Exp. (₹)	16,000	12,000	8,000	12,000	10,000	8,000
2.	Budgeted Sales (units)	20,000	12,000	6,000	15,000	10,000	5,000
3.	Selling Cost (per unit) (₹)	0.80	1.00	1.33	0.80	1.00	1.60
4.	Actual Sales (units)	19,000	10,000	5,900	17,500	9,500	5,000
5.	Standard Selling Cost for Actual Sales (₹) [(3)×(4)]	15,200	10,000	7,847	14,000	9,500	8,000
	Actual Selling Actual Data						
	Salesmen's Salaries (₹)	8,000	7,000	5,000	7,000	6,000	5,000
	Sales Travelling (₹)	4,000	5,000	3,600	2,700	2,700	1,800
	Halting Charges etc. (₹)	850	800	500	500	700	500
	Salesmen's Commission (₹)	4,750	3,125	1,475	3,937	2,375	1,250
6.	Total Actual Selling Costs (₹)	17,600	15,925	10,575	14,137	11,775	8,550
	Analysis						
7.	Selling Costs Variance (₹)[(5) – (6)]	- 2,400	- 5,925	- 2,728	- 137	- 2,275	- 550
8.	Budgeted Sales [Budgeted Qty.× Budgeted Price] (₹)	5,00,000	3,00,000	1,50,000	3,75,000	2,50,000	1,25,000

9.	Budgeted Selling Expenses as a % of Budgeted Sales [(1)/(8)×100]	3.20%	4.00%	5.33%	3.20%	4.00%	6.40%
10.	Actual Sales (₹)	4,75,000	2,50,000	1,47,500	4,37,500	2,37,000	1,25,000
11.	Actual Selling Expenses as a % of Actual Sales	3.71%	6.37%	7.17%	3.23%	4.97%	6.84%

Comments : The above table shows that except for southern India and North – western India Zonal offices, actual sales expenses widely differ from budgeted selling expenses. However, the following points have to be noted:

(i) *The standards are based on the actual expenses for the last year. Truly speaking they are not standards and, therefore, they cannot provide realistic guidance for exercising control over the selling expenses. Variances may be there because current year's conditions might have completely changed or circumstances which were applicable last year may have ceased to become applicable now.*

(ii) *The causes of the variances cannot be correctly spelt out in the absence of details about the "Standard selling expenses." The details of actual selling expenses have been given but the details of standard selling expenses have not been given. Salesmen's salaries is a fixed charge, variance may be there on account of increase in their salaries. Sales travelling expenses are of a semi-variable nature. Less volume of sales might have resulted in less recovery of fixed sales travelling expenses such as railway freight, hotel charges.*

Finding of Missing Information, Variance Analysis

Illustration 23

Following is the standard cost card of a component:

Materials	*2 Units at ₹ 15*	*₹ 30*
Labour	*3 Hours at ₹ 20*	*₹ 60*
Total overheads	*3 Hours at ₹ 10*	*₹ 30*

During a particular month 10,000 units of the component were produced and the same was found to be at 60% capacity of the budget. In preparing the variance report for the month, the cost accountant gathered the following information:

Labour	*₹ 6,50,000*
Variable overheads	*₹ 2,00,000*
Fixed overheads	*₹ 3,00,000*
Material price variance	*₹ 70,000 (A)*
Material cost variance	*₹ 50,000 (A)*
Labour rate variance	*₹ 50,000 (F)*
Fixed overhead expenditure variance	*₹ 50,000 (A)*

You are required to prepare from the above details:

(1) Actual material cost incurred

(2) Standard cost of materials actually consumed

(3) Labour efficiency variance

(4) Variable OH efficiency variance

(5) Variance OH expenditure variance

(6) Fixed OH efficiency variance

(7) Fixed OH capacity variance

(8) Fixed OH volume variance

Solution

Computation of Requirements of the Question:

1. Actual Material Cost Incurred

Material Cost Variance =(Standard Cost of Std. Qty. for Actual Output) – (Actual Cost) Or *Actual Cost* = (Standard Cost of Std. Qty for Actual Output) – (Material Cost Variance)

= 10,000 units × 2 units × ₹ 15 + ₹ 50,000

= ₹ 3,00,000 + ₹ 50,000 = ₹ 3,50,000

2. Standard Cost of Materials Actually Consumed

Material Price Variance =(Standard Price – Actual Price) x Actual Quantity Or *Standard Cost of Actual Quantity* =(Actual Cost) + (Material Price Variance)

= ₹ 3,50,000 – ₹ 70,000 = ₹ 2,80,000

3. Labour Efficiency Variance

(Refer to working note 1)

Labour Efficiency Variance =(Standard Hours – Actual Hours) x Standard Rate per Hour

= (10,000 units × 3 hours – 35,000 hours) × ₹ 20

= (₹ 6,00,000 – ₹ 7,00,000) = ₹ 1,00,000 (A)

4. Variable Overhead Efficiency Variance

(Refer to working note 2)

Variable Overhead Efficiency Variance =(Standard Hours for Actual Output – Actual Hours) x Standard Variable Overhead Rate per Hour

= ₹ 5 x (30,000 Hours – 35,000 Hours) = ₹ 25,000 (A)

5. Variable Overhead Expenditure Variance

(Refer to working note 1)

Variable Overhead Expenditure Variance
=(Budgeted Variable Overheads for Actual Hours) – (Actual Variable Overheads)

= (₹ 5 × 35,000 Hours – ₹ 2,00,000) = ₹ 25,000 (A)

6. Fixed Overhead Efficiency Variance

(Refer to working notes 1 & 2)

Fixed Overhead Efficiency Variance
=(Standard Hours for Actual Output – Actual Hours) x Standard Fixed Overhead Rate per Hour

= ₹ 5 x (30,000 Hours – 35,000 Hours) = ₹ 25,000 (A)

7. Fixed Overhead Capacity Variance

(Refer to working notes 1 & 2)

Fixed Overhead Capacity Variance
=(Actual Hours – Budgeted Hours) x Standard Fixed Overhead Rate per Hour

= ₹ 5 x (35,000 Hours – 50,000 Hours)

= ₹ 75,000 (A)

8. Fixed Overhead Volume Variance

(Refer to working note 2)

Fixed Overhead Volume Variance
=(Actual Output – Budgeted Output) x Standard Fixed Overhead Rate per Unit

= ₹ 15 × [10,000 units – (50,000 hours / 3 hours p. u.)]

= ₹ 1,50,000 – ₹ 2,50,000 = ₹ 1,00,000 (A)

Basic Calculations:

1. Labour Rate Variance = (Standard Rate per Hour x Actual Hours) – (Actual Cost)

   Or ₹50,000 = ₹20 x Actual Hours – ₹ 6,50,000

   Or Actual Hours = 35,000

2. Standard Hours = 10,000 Units × 3 Hours = 30,000 Hours

   Budgeted Hours = $\left\{\frac{30{,}000 \text{ hours} \times 100\%}{60\%}\right\}$ = 50,000 Hours

   Budgeted Fixed Overheads= Actual Fixed Overheads + Expenditure Variance

   = ₹ 3,00,000 – ₹ 50,000 = ₹ 2,50,000

   $\left\{\begin{array}{c}\text{Standard fixed overhead} \\ \text{recovery rate per hour}\end{array}\right\} = \frac{₹\ 2{,}50{,}000}{50{,}000 \text{ hours}}$ = ₹ 5 per hour

	Total Overhead Rate per Hour	= ₹ 10
	Variable Overhead Rate per Hour (₹ 10 – ₹ 5)	= ₹ 5
3.	Standard Fixed Overhead per Unit (3 hours × ₹ 5)	= ₹ 15

Illustration 24

Mr. M provides the following information relating to 1,000 units of product 'ZED' during the month of April, 2012

Standard price per kg. of raw material – ₹ 3

Actual total direct material cost – ₹ 10,000

Standard direct labour hours – 1,600

Actual direct labour hours – 1,800

Total standard direct labour cost – ₹ 8,000

Standard variable overhead per direct labour hour – ₹ 1

Standard variable cost per unit of ZED – ₹ 1.60

Total standard variable overheads – ₹ 1,600

Actual total variable overheads – ₹ 1,620

The material usage variance is ₹ 600 (adverse) and the overall cost variance per unit of ZED is ₹ 0.07 (adverse) as compared to the total standard cost per unit of ZED of ₹ 21.

You are required to compute the following:

(a) Standard quantity of raw-material per unit of ZED.

(b) Standard direct labour rate per hour.

(c) Standard direct material cost per unit of ZED.

(d) Standard direct labour cost per unit of ZED.

(e) Standard total material cost for the output.

(f) Actual total direct labour cost for the output

(g) Material price variance.

(h) Labour rate variance.

(i) Labour efficiency variance.

(j) Variable overhead expenditure variance.

(k) Variable overheads efficiency variance.

Solution

<u>Basic Calculations:</u>

1. ***Standard Cost of Raw-Material Consumed:***	₹	₹
Total Standard Cost of ZED (1,000 units × ₹ 21)		21,000
Less: Standard Cost: Labour	8,000	
Overheads	1,600	9,600
Standard Cost of Raw Materials Used		11,400

2. ***Standard Cost of Raw-Material per Finished Unit:***

$$\frac{\text{Total Cost of Material}}{\text{Output}} = \frac{₹\,11,400}{1,000\ \text{Units}} = ₹\ 11.40$$

3. ***Standard Quantity of Raw – Material per Finished Unit and Total Quantity of Raw Material Required:***

$$\frac{\text{Standard Cost of Material per Unit}}{\text{Standard Rate per Kg.}} = \frac{₹\ 11.40}{₹\ 3.00} = 3.8\ \text{Kg. per finished unit}$$

Total Quantity – 3.8 Kg. × 1,000 units = 3,800 Kg.

4. ***Total Material Cost Variance:***

Actual Cost of Raw Material	₹ 10,000
Standard Cost of Raw Material	₹ 11,400
Total Material Cost Variance	₹ 1,400 (F)

5. ***Actual Quantity (AQ) of Raw-Material (in Kg):***

Material Usage Variance	=	Standard Cost of Standard Quantity for Actual Output – Standard Cost of Actual Quantity
	=	Standard Rate × (Standard Quantity for Actual Output – Actual Quantity)
Or ₹ 600 (A)	=	₹ 3 x (3,800 Kg. – AQ)
Or 3AQ	=	12,000 Kg. or, AQ = 4,000 Kg.

(Material usage variance is as given in the question and standard quantity is as per (3) above)

6. ***Actual Rate of Raw Material per Kg.***

$$\frac{\text{Actual Material Cost}}{\text{Actual Quantity}} = \frac{₹\ 10,000}{4,000\ \text{Kg.*}} = ₹\ 2.50\ \text{per Kg. (*As per (5) above)}$$

7. ***Standard Direct Labour Rate***

Standard Direct Labour Hours = 1,600 (given)

Standard Direct Labour Cost = ₹ 8,000 (given)

$$\text{Standard Direct Labour Hour Rate} = \frac{₹\ 8,000}{1,600\ \text{hrs.}} = ₹\ 5$$

8. ***Actual Labour Cost and Actual Labour Rate per Hour:***

Actual Total Cost of 1,000 Units		₹ 21,070
1,000 units (₹ 21 + ₹ 0.07)		
Less: Actual Cost of Material	₹ 10,000	
Actual Variable Overheads	₹ 1,620	₹ 11,620
Actual Direct Labour Cost		₹ 9,450

Actual Direct Labour Rate per Hr. = $\frac{₹\ 9,450}{1,800\text{ hrs.}}$ = ₹ 5.25

9. ***Standard Labour Hours to Produce One Unit:***

$$\frac{\text{Standard Hours}}{\text{Output in Units}} = \frac{1,600\text{ hours}}{1,000\text{ units}} = 1.6\text{ hours}$$

10. ***Standard Labour Cost per Unit:***

Standard Labour Cost per Unit = 1.6 hours × ₹ 5 = ₹ 8

11. ***Actual Hourly Rate of Variable Overheads:***

$$\frac{\text{Actual Variable Overheads}}{\text{Actual Hours}} = \frac{₹\ 1,620}{1,800\text{ hours}} = ₹\ 0.90$$

Computations of Requirements:

(a) ***Standard Quantity of Raw Material per Unit of ZED***: 3.8 kg. *(Refer to working note 3).*

(b) ***Standard Direct Labour Rate per Hour:*** ₹ 5 *(Refer to working note 7).*

(c) ***Standard Direct Material Cost per Unit of ZED:*** ₹ 11.40 *(Refer to working note 2).*

(d) ***Standard Direct Labour Cost per Unit of ZED:*** ₹ 8 *(Refer to working note 10).*

(e) ***Standard Total Material Cost for the Output:*** ₹ 11,400 *(Refer to working note 1).*

(f) ***Actual Total Direct Labour Cost for the Output***: ₹ 9,450 *(Refer to working note 8).*

(g) ***Material Price Variance:***

=Material Cost Variance – Material Usage Variance.
= ₹ 1,400 (F)* – ₹ 600 (A) *(*Refer to working note 4)*
= ₹ 2,000 (F)

Alternatively,

= Actual Quantity × (Standard Price – Actual Price)
= 4,000 units (₹ 3 – ₹ 2.50*) *(*Refer to working note 5 & 6)*
= ₹ 2,000 (F)

(h) ***Labour Rate Variance:***

= Actual Hours × (Standard Rate – Actual Rate)
= 1,800 Hours × (₹ 5 – ₹ 5.25)
= ₹ 450 (A)

(i) ***Labour Efficiency Variance:***

Standard Rate × (Standard Hours – Actual Hours)
= ₹ 5 per hour × (1,600 hours – 1,800 hours) = ₹ 1,000 (A)

(j) *Variable Overhead Expenditure Variance:*

= Actual Hours × (Standard Rate per Hour – Actual Rate per Hour)
= 1,800 Hours × (₹ 1 – Re. 0.90*) = ₹ 180 (F) *(*Refer to working note 11)*

(k) *Variable Overhead Efficiency Variance:*

= Standard Variable Overhead Rate per Hour × (Standard Hours for Actual Output – Actual Hours)
= ₹ 1 per hour × (1,600 hours – 1,800 hours) = ₹ 200 (A)

Reconciliation between Budgeted Profit and Actual Profit through given Variances

Illustration 25

The budgeted output of a single product manufacturing company for the year ending 31st March was 5,000 units. The financial results in respect of the actual output of 4,800 units achieved during the year were as under:

	₹
Direct material	*29,700*
Direct wages	*44,700*
Variable overheads	*72,750*
Fixed overheads	*39,000*
Profit	*36,600*
Sales	*2,22,750*

The standard wage rate is ₹ 4.50 per hour and the standard variable overhead rate is ₹ 7.50 per hour.

The cost accounts recorded the following variances for the year:

Variances	***Favourable*** ₹	***Adverse*** ₹
Material price		*300*
Material usage	–	*600*
Wage rate	*750*	–
Labour efficiency	–	*2,250*
Variable overhead expenses	*3,000*	–
Variable overhead efficiency	–	*3,750*
Fixed overhead expense	–	*1,500*
Selling price	*6,750*	–

Required:

(i) Prepare a statement showing the original budget.

(ii) Prepare the standard product cost sheet per unit.

(iii) Prepare a statement showing the reconciliation of originally budgeted profit and the actual profit.

Solution

Basic Calculations:

		₹
(a)	Actual Sales	2,22,750
	Less : Price Variance (Favourable)	6,750
	Standard Sales	2,16,000
	Units Sold	4,800

Budgeted Price per Unit: $= \frac{₹\,2,16,000}{4,800 \text{ units}} = ₹\ 45$

		₹	₹
(b)	Material Used		29,700
	Less: Price Variance (Adverse)	300	
	Usage Variance (Adverse)	600	900
	Standard Cost		28,800

Standard Material Cost per Unit $= \frac{₹\ 28,800}{4,800 \text{ units}} = ₹\ 6$

		₹
(c)	Direct Wages Spent	44,700
	Add: Wage Rate Variance (Favourable)	750
		45,450
	Less: Efficiency Variance (Adverse)	2,250
	Standard Wages	43,200

Standard Wage Rate per Unit $= \frac{₹\ 43,200}{4,800 \text{ units}} = ₹\ 9$

(d) Standard Direct Wage Rate is ₹ 4.50 per Hour

Hence Standard Time per Unit: ₹ 9 ÷ 4.50 Hour = 2 Hours

(e) Variable Overheads :

Standard Rate ₹ 7.50 per Hour

Variable Overhead per Unit: 2 hrs. × ₹ 7.50 = ₹ 15

(*Note*: Alternatively, this may be calculated by adjusting variances as in other cases)

		₹
(f)	Fixed Overhead Spent	39,000
	Less : Fixed Overhead Expense Variance (Adverse)	1,500
	Budgeted Overheads	37,500

Std. Fixed Overhead Rate per Unit : $\frac{₹\ 37,500}{5,000 \text{ units}} = ₹\ 7.50$

(g) Fixed Overhead Recovered: 4,800 Units × ₹ 7.50 = ₹ 36,000 (Absorbed)

(h) Fixed Overhead Volume Variance

(Absorbed Overheads – Budgeted Overheads) = ₹ 36,000 – ₹ 37,500

= ₹ 1,500 (Adverse)

(i) Budgeted Sales: 5,000 units × ₹ 45 = ₹ 2,25,000

(j) Actual Sales = ₹ 2,22,750

(k) Sale Volume Variance ₹45 × (4,800 Units – 5,000 Units)

[Budgeted Price × (Actual Qty. – Budgeted Qty.)] = ₹9,000 (A)

(i) Statement Showing the Original Budget:

		₹
Budgeted Sales	(5,000 units × ₹ 45)	2,25,000
Less: Budgeted Costs:		
Direct Material	(5,000 units × ₹ 6)	30,000
Direct Wages	(5,000 units × ₹ 9)	45,000
Variable Overheads	(5,000 units × ₹ 15)	75,000
Fixed Overheads	(5,000 units × ₹ 7.50)	37,500
Profit :		37,500

(ii) Statement Showing Standard Product Cost Sheet per Unit:

	₹
Direct Materials	6.00
Direct Wages	9.00
Prime Cost	15.00
Variable Overheads	15.00
Fixed Overheads	7.50
Total Cost	37.50
Profit	7.50
Selling Price	45.00

(iii) Statement Showing Reconciliation of the Original Budgeted Profit and the Actual Profit

Particulars	₹	₹	₹
Budgeted Profit:			37,500.00
(Budgeted Quantity x Budgeted Margin)			
Sales Margin Variances:			
Sales Margin Volume Variance*			(1,500.00)
Standard Profit			36,000.00
Effect of Other Variances			
Material Cost Variances:			
Material Price Variance	(300.00)		
Material Usage Variance	(600.00)	(900.00)	
Labour Cost Variances:			
Labour Rate Variance	750.00		
Labour Efficiency Variance	(2,250.00)	(1,500.00)	
Variable Overhead Cost Variances:			
Variable Overhead Expenditure Variance	3,000.00		
Variable Overhead Efficiency Variance	(3,750.00)	(750.00)	
Fixed Overhead Cost Variances:			
Fixed Overhead Expenditure Variance	(1,500.00)		
Fixed Overhead Volume Variance	(1,500.00)	(3,000.00)	
Sales Margin Variances:			
Sales Margin Price Variance		6,750.00	600.00
Actual Profit for the Month			36,600.00

Adverse Shown by (-) Symbol

*Sales Margin Volume Variance	= Sales Volume Variance x Budgeted Net Profit Ratio
	= ₹ 9,000 (A) x [₹ 7.50/ ₹ 45 x100] % = 1,500 (A)

Reconciliation between Standard Profit and Actual Profit, Analysis of Variances

Illustration 26

The following information is available in respect of Y Ltd. for a week:

(a) 400 kg of raw material were actually used in producing product 'EXE'. The purchase cost thereof being ₹ 24,800. The standard price per kg of raw material is ₹ 60. The expected output is 12 units of product 'EXE' from each kg of raw material. Raw material price variance and usage variance as computed by cost accountant are ₹ 800 (adverse) and ₹

600 (adverse) respectively.

(b) The week is of 40 hours. The standard time to produce one unit of 'EXE' is 30 minutes. The standard wage rate is ₹ 5 per labour hour. The company employs 60 workers who have been paid hourly wage rate as under :

Number of workers	*6*	*8*	*46*
Hourly wage rate (₹)	*4.80*	*5.20*	*5.00*

(c) Budgeted overheads for a four-weekely period is ₹ 81,600. The actual fixed overheads spent during the said week are ₹ 19,800.

(d) Entire output of 'EXE' has been sold at its standard selling price of ₹ 15 per unit.

You are required to :

(i) Compute the variances relating to labour and overheads.

(ii) Prepare a statement showing total standard costs, standard profit and actual profit for the week.

Solution

Basic Calculations:

1. Standard Quantity and Cost of Raw Material required for Actual Output:

Material Cost Variance = Standard Material Cost – Actual Material Cost

⇒ ₹ 800(A) + ₹ 600(A) = Standard Material Cost – ₹ 24,800

⇒ Standard Material Cost = ₹ 23,400

⇒ Standard Price per Kg. × Standard Qty. for Actual Output = ₹ 23,400

⇒ Standard Qty. for Actual Output = ₹ 23,400/60 = 390 Kg.

⇒ Standard Qty. per Unit Output × Actual Output in Units = 390 Kg.

⇒ Actual Output in Units = 390 Kg. × 12 = 4,680 Units

2. Basic data for the computation of Labour Variances:

Standard Labour Cost for Actual Output				Actual Cost		
Standard Hours	**Rate Per Hour**	**Amount**	**Standard Cost for Actual Hours**	**Actual Hours**	**Rate Per Hour**	**Amount**
2,340 (4,680 units × 1/2 hr.)	₹5	₹11,700	₹12,000 (2,400 hrs × ₹ 5)	240 320 1,840	₹4.80 ₹5.20 ₹5.00	₹1,152 ₹1,664 ₹ 9,200
2,340 hrs.		₹11,700	₹12,000	2,400 hrs.		₹12,016

3. Basic data for the computation of Fixed Overhead Variances:

Budgeted Std. Data		Actual Data	
Budgeted Fixed Overhead (₹) *(for 1 week)*	20,400	Actual Fixed Overhead (₹)	19,800
Budgeted Hours *(60 workers × 40 hrs. per week)*	2,400	Actual Labour Hours	2,400
		Actual Output (units)	4,680
Budgeted Output (units)	4,800		
Std. Rate p.h. (₹)	8.50		
Std. Rate p.u. (₹)	4.25		

Computation of Variances:

(i) Computation of labour and overhead (variances):

Labour Cost Variance: (Refer to Working note 2)

= (Std. Cost– Actual Cost)

= ₹ 11,700 – ₹ 12,016 = ₹ 316 (A)

Labour Rate Variance:

= Standard Cost of Actual Time – Actual Cost

= ₹ 12,000 – ₹ 12,016

= ₹ 16 (A)

Labour Efficiency Variance:

= Standard Cost of Standard Time for Actual Output – Standard Cost of Actual Time

= (₹ 11,700 – ₹ 12,000) = ₹ 300 (A)

Fixed Overhead Cost Variance:

= Fixed Overheads Absorbed – Actual Fixed Overheads

= 4,680 Units × ₹ 4.25 – ₹ 19,800

= ₹ 19,890 – ₹ 19,800 = ₹ 90 (F)

Fixed Overhead Volume Variance:

= Std. Fixed Overhead Rate per Unit x (Actual Output – Budgeted Output)

= ₹ 4.25 x (4,680 units – 4,800 units)

= ₹ 510 (A)

Fixed Overhead Expenditure Variance:

= Budgeted Fixed Overheads – Actual Fixed Overheads

= ₹ 20,400 – ₹ 19,800

= ₹ 600 (F)

(ii) Statement showing Total Standard Cost, Standard Profit and Actual Profit for the week

	₹	₹	₹
Sales			70,200
(4,680 units × ₹ 15)			
Less : Standard Costs of :			
Direct Material		23,400	
Direct Labour		11,700	
Overheads		19,890	(54,990)
(4,680 × ₹ 4.25)			
(*Refer to working notes 1 to 3*)			
Standard Profit			15,210
Less : Adjustment for Variance :			
Raw Material :		1,400 (A)	
Price Variance :	800 (A)		
Usage Variance :	600 (A)		
Labour :		316 (A)	
Rate Variance :	16 (A)		
Efficiency Variance	300 (A)		
Overhead :		90 (F)	
Expenditure variance :	600 (F)		
Volume variance :	510 (A)		(1,626)
Actual profit			13,584

5.6 Behavioural Aspects of Standard Costing

1. *Projection of fixed overheads and estimated selling price in a Standard Cost Sheet is a circular exercise with no added value.*

   In an award winning article," COST / MANAGEMENT ACCOUNTING: THE 21ST CENTURY PARADIGM", published in Management Accounting (USA), December 1995, William L Ferrara argues that while preparing a Standard Cost Sheet, one of the objectives of which is to assist management in pricing products, a professional cannot project fixed overheads until and unless he is aware of the production quantum to be effected. The forecast of future production can only be made if a tentative selling price of the product is known because, in a competitive market, it is the selling price which decides the sale quantity and therefore the production volume. The authors contend that in case the selling price is known at the time of projecting fixed overheads then the re computation of the same is a valueless exercise.

2. *Traditional costing tools like standard costing induce a static behaviour in the employees.*

   During the past decade and a half, various writers such as Johnson and Kaplan, Ferrara and Monden etc have questioned the productivity and use of traditional systems such as standard costing and variance analysis. They argue that the use of standard costing renders employees static and curbs innovation and that companies following traditional standard costing find it difficult to improve upon standards because of severe resistance from employees who are convinced that the established best practise cannot be improved further.

3. *Fear of adverse variances forces managers to give undue importance to material price, labour rate and efficiency and capacity utilisation. These concepts are detrimental to the modern day world class manufacturing environment characterised by concepts of JIT and TQM.*

   In a World Class Manufacturing environment, characterised by Just in Time policies, the focus of the management is to produce only as much as is required. This requires purchase of small quantities of raw material, increase in the number of set ups and minimal importance to capacity utilisation. Policies like this result in increased adverse variances related to raw material prices, labour efficiency and production volume. Critics argue that the fear of such adverse variances affects goal congruence and forces managers to behave against their company's policies.

4. *Traditional costing does not provide the management with what is the allowable cost; rather it emphasises on the standard or actual costs.*

   This is looked upon as one of the major reasons for lack of innovation especially in the global era where competition amongst companies is unprecedented. It is argued that techniques like Target costing are much more motivating when compared to Traditional costing since the former encourage the use of concepts like value engineering and value analysis.

6

Costing of Service Sector

LEARNING OBJECTIVES

After finishing this chapter, you will be able to understand and appreciate the peculiarities involved in the costing of service sector. Through this chapter you will gain an understanding as to the various methods applied by service sector entities to determine their unit costs.

6.1 Introduction

Service sector companies provide their customers with services or intangible products. The activities of service sector may be used for both: (i) Provision of services to outside customers (ii) Provision of services internally (i.e. captive consumption). The types of services that may be provided, by service sector are of diverse nature and have their own peculiarities and requirements in respect of the cost accounting treatment, however, the general principles of costing discussed in earlier chapters relating to manufacturing sector also apply to service sector.

6.2 Main Characteristics of Service Sector

(i) Activities are labour intensive: Due to their inherent nature, the activities of service sector are generally labour intensive. The direct material cost is either small or non-existent. The labour cost (i.e. salary and wages) constitutes a significant portion of the total operating costs of a service sector entity. For example, cost of stationery used by a professional consultant for expressing an opinion in black and white, for a client will be small or even non existent in case he gives verbal opinion. In the preceding example direct labour cost (i.e. the salary of the consultant) constitutes a significant portion of the total operating cost as compared to the material cost which constitutes a negligible portion.

(ii) Cost-unit is usually difficult to define: The selection of cost unit for service sector is relatively difficult to ascertain as compared to the selection of cost unit for manufacturing sector.

The following table provides some examples of the cost units for service sector.

(A)	**To External Customers**	**Cost Unit**
(i)	Hotel	Bed nights available, Bed night occupied

(ii)	School	Student hours, Full time students
(iii)	Hospital	Patient per day, Room per day
(iv)	Accounting firm	Charged out client hours
(v)	Transport	Passenger km., quintal km.
(B)	**Internal services**	**Cost Unit**
(i)	Staff canteen	Meals provided, No. of staff
(ii)	Machine maintenance	Maintenance hours provided to user department
(iii)	Computer department	Computer time provided to user department

(iii) Product costs in service sector: Costs are classified as product or period costs in manufacturing sector for various reasons viz.

(i) To determine the unit manufacturing costs so that inventories can be valued and the selling prices be determined & verified.

(ii) To report production costs on income statement.

(iii) To analyse costs for control purposes.

The only difference between manufacturing and service sector is that in service sector there is no physical product that can be stored, assembled and valued. Services are rendered and cannot be stored or placed in a vault. In service sector the cost of material is insignificant. Rendering a loan service, representing someone in court of law or selling an insurance policy are typical services performed by professionals. For computing unit cost of these services the most important cost would be professional's labour cost. The direct labour cost is traceable to service rendered. In addition to labour cost the service sector like manufacturing sector incurs various overhead cost. In service sector those overhead costs which are incurred for offering a service are classified as service overhead (like factory overhead in manufacturing sector). In order to arrive at the correct cost incurred for rendering services, it is imperative that such overheads be allocated/apportioned over the cost units.

6.3 Collection of Costing Data In Service Sector

Costs are accumulated under various heads for control purposes and for decision making. Costs thus, collected are usually grouped under fixed costs and variable costs. The format in which costs data is presented depends upon the nature of industry and the need of the management. *In the Cost Accounting book of PE-II level we have discussed in the chapter of operating costing the costing methods appropriate for different types of service sector i.e. Transport Sector, Hospital, Hotel, College etc*. Often composite cost units such as passenger km, bed, nights etc. are used by these organisation for ascertaining the cost per unit in respect of these services respectively. For preparing a cost sheet under operating costing, costs are usually accumulated for a specified period viz. a month, a quarter or year etc. Since there is no direct relationship between the costs and level of services there is tendency in service sector to view all the costs as over- heads. If such a view is taken the cost control will be achieved by controlling the inputs rather than the outputs i.e. control will be relatively ineffective. This gives an impetus for the application of Activity Based Costing in service sector, with a desire to secure better understanding of costs, as an aid to decision making and cost control. Although the Activity Based Costing was initially developed for manufacturing sector, it can be applied with equal effectiveness to service sector for understanding cost behaviour of service sector entities as well.

6.4 Costing Methods Used in Service Sector

Like manufacturing sector companies in the service sector, generally adopt one out of the following two basic costing methods to assign costs to services. These are:

1. **Job costing method:** In job costing method the cost of a particular service is obtained by assigning costs to a distinct identifiable service. In service sector like Accounting firm, Advertising campaigns etc. job costing method is used. For assigning indirect costs (overheads) models such as Activity Based Costing may be used.
2. **Process costing method:** In process costing system the cost of a service is obtained by assigning costs to masses of similar unit and then computing unit cost on an average basis. Retail banking, Postal delivery, Credit card etc. uses process costing method.
3. **Hybrid costing method:** Many companies uses a method of costing which is neither job costing nor process costing method. They in fact uses a hybrid costing method which combines elements of both job costing and process costing methods.

   The job costing and process costing for manufacturing sector have been discussed in the Cost Accounting book of PE-II level. The same cost principles as discussed for manufacturing sector will apply to service sector.

6.4.1 Job costing method in service sector: The two significant costs which are incurred in service sectors are:

(i) Direct labour

(ii) Service overheads

For ascertaining the price of a service provided by service sector if job costing method is followed, the costs for each job are to be monitored continuously. There are two main uses of this job cost information:

1. To guide decisions on job pricing
2. To assist in enhancing profitability by cost planning and cost control

The five steps which are generally adopted for assigning costs to individual jobs are as follows:

Cost Accumulation

(i) **Identify the job that is chosen as cost object:** For instance, litigation work for Motorola India Ltd. by Dua & Associates by assuming that work requires 100 budgeted hours of professional labour.

(ii) **Identify the direct cost categories for the job:** In the above example the professional hours required for doing litigation work is a direct cost.

(iii) **Identify indirect costs (overheads) associated with the job:** This step requires identification of indirect costs incurred for providing services. These costs may include the costs of support labour, computer time, travel, telephone/fax machine, photocopying etc.

Cost Allocation

(iv) **Select the cost allocation base to be used in assigning each indirect cost to the job:** This step requires the selection of cost allocation base that has a *cause and effect* relationship between changes in it and changes in the level of indirect costs. The allocation base suitable for allocating indirect cost of law firm is professional labour hours.

(v) **Identify the rate per unit of the cost allocation base used to allocate cost to the job:** The budgeted indirect cost allocation rate is computed by using following formula

$$\text{Budgeted indirect cost rate} = \frac{\text{Budgeted total indirect cost}}{\text{Budgeted total quantity of cost allocation base}}$$

The indirect cost of job is computed as = Budgeted indirect cost rate × Professional labour hours reqd. for job

Example

Dua and Associates is a law firm specialising in carrying out litigation work for clients. It has 25 professionals who work for clients (5 partners and 20 associates). The average budgeted total compensation per professional for 1999 is ₹ 1,04,000. Each professional is budgeted to have 1,600 billable hours to clients in 1999. Dua and Associates is a highly respected firm and all professionals work for clients to their maximum 1,600 billable hours. All professional labour costs are traceable to jobs on a per hour basis. The budgeted indirect cost (legal support) in 1999 is ₹ 22,00,000. The indirect costs are allocated to the jobs using professional labour hours as the allocation base. In this example for ascertaining the cost of job company consider single direct cost rate (i.e. Professional labour) and single indirect cost rate.

The job costing method example given above uses single direct cost and single indirect cost for computing the cost of the job. The use of single direct cost and single indirect cost is likely to overstate job with more professional hours and understate the jobs with less professional hours. This requires refinement of costing system. The refinement in costing system means changes in existing costing system that results in a better measure of the way the jobs uses the resources of the organisation. Many organisations in service sector for refining their costing system are using activity based costing (ABC). Activity based costing focusses on activities as the fundamental cost objects. It uses the cost of these activities as the basis for assigning costs of services to jobs. For understanding the implementation of ABC in service sector consider the above example.

In the above example Dua and Associates for refining its costing method reviewed its work activities and collected information about how each litigation work vary in their use of resources. They identified that the direct costs are of the following five categories. These are:

1. Professional Partner labour
2. Professional Associate labour
3. Office support labour.
4. Phone/fax/photocopying: Traced on as identified basis or per monthly billing from third parties.
5. Travel: Traced on as identified basis, per monthly billing from third parties.

Similarly, indirect costs are identified into two categories. These are:

1. *General support:* The allocation base is budgeted professional labour hours.
2. *Out of town support:* The allocation base in law firm is lawyer days at non-local site.

With refined costing system service organisation has more accurate cost information for services. This helps them to adjust their pricing policies for services

6.4.2 Process costing method in service sector: In this method the cost of service is obtained by assigning costs to masses of units and then computing unit costs on an average basis. Let us explain this with an illustration.

Illustration 1

The loan department of a Canara Bank performs several functions in addition to home loan application processing task. It is estimated that 25% of the overhead costs of loan department are applicable to the processing of home-loan application. The following information is given concerning the processing of a loan application:

Direct professional labour:

	(₹)
Loan processor monthly salary :	*80,000*
(4 employees @ ₹ 20,000 each)	
Loan department overhead costs (monthly)	
Chief loan officer's salary	*5,000*

Telephone expenses	*750*
Depreciation Building	*2,800*
Legal advice	*2,400*
Advertising	*400*
Miscellaneous	*650*
Total overhead costs	*12,000*

You are required to compute the cost of processing home loan application on the assumption that one hundred home loan applications are processed each month.

Solution:

Statement showing computation of the cost of processing a typical home loan application

	(₹)
Direct professional labour cost	80,000
(4 employees @ ₹ 20,000 each)	
Service overhead cost (25% of ₹ 12,000)	3,000
Total processing cost per month	83,000
No. of applications processed per month	100
Total processing cost per home loan application	830

6.4.3 Customer Costing in service sector: The customer costing is a new approach to management. The central theme of this approach is customer satisfaction. In some service industries, such as public relations, the specific output of industry may be difficult to identify and even more difficult to quantify. Further, where there are multiple customers, identifying support activities i.e. common costs with particular customer may be more problematic. In such cases, it is important to cost customers. An analysis based on Activity Based Costing of customers profitability provides valuable information to help management in pricing customer. Consider a banking sector. A bank's activities for customer will include the following types of activities. These are:

(i) Stopping a cheque

(ii) Withdrawal of cash

(iii) Updation of pass book

(iv) Issue of duplicate pass book

(v) Returning a cheque because of insufficient funds

(vi) Clearing of a customer cheque.

Different customers or categories of customers use different amount of these activities and so customer profiles can be build up and customer can be charged according to the cost to serve them.

For example (i) In Computer Institute the cost of providing a course for enrolled students may be determined by a variety of factors, such as type of course (Oracle or Java) and the level of course (introduction or advanced). (ii) A hotel may have activities that are provided for specific types of customers such as well laid gardens, swimming pool and a bar. Older guest may appreciate and use the garden, families the swimming pool and business guests the bar. If the activities are allocated to relevant guest a correct cost per bed occupied can be calculated for each type of category.

For customer costing purpose, the costs are divided into following categories. These are:

(i) Customer Specific costs: These are the direct and indirect cost of providing service to each customer plus customer related cost assigned to them. For example cost of ex- press courier service to a client/customer who requests overnight delivery of some agreement.

(ii) Customer-line categories: These are the costs which are broken into the broad segments of customers and not individual customer.

(iii) Company costs: These are those costs which are not allocated to either customer line or individual customers but charge to company. The example is the cost of advertisement to promote sale of service.

6.5 Pricing of Service Sector

In the chapter of pricing strategies and transfer pricing in this book we have discussed the methods followed for pricing by manufacturing sector. The service sector follows a different approach for pricing their service. Although a service has no physical existence it must be priced and billed to customers. Most service organizations use a form of time and material pricing to arrive at the price of a service. Service companies such as appliance repair shops, automobile repair business arrive at prices by using two computations, one for labour and other for materials and parts. As with a cost based approach a mark up percentage is used to add the cost of overhead to the direct cost of labour, materials and parts. If materials and parts are not part of service being performed, then only direct labour costs are used as basis for determining price. For professionals such as accountants and consultants a factor representing all overhead costs is applied to the base labour costs to establish a price for the services.

Illustration 2

Motorcraft has just completed repair work on Car No. DL 6CB 2051 of Mr. X. The parts used to repair the vehicle cost ₹ 500. The company's 40% mark up rate on parts covers parts–related overhead costs. Labour involved 9 hours of time from a Motorcraft service engineer whose *wages are ₹ 50 per hour. The current overhead work up rate on labour is 80%. You are required to compute how much Mr. X will be billed for his car repairs.*

Solution:

Computation of the amount Mr. X should be billed for his car repairs. Parts charges:

	(₹)	(₹)
Repairs parts used	500	
Overhead charges		
(40% of ₹ 500)	200	
Total parts charges	700	
Labour charges:		
9 hours @ ₹ 50 per hour	450	
Overhead charges		
(80% of ₹ 450)	360	
Total labour charges		810
Total billing amount		1510

7

Transfer Pricing

LEARNING OBJECTIVES
After studying this chapter students will understand. • Purpose of transfer pricing • Responsibility of a division as responsibility centre • Conflicts between the divisions • Setting of transfer price where the profit of the organisation can be higher.

7.1 Introduction

The whole organisation can be divided into a numbers of divisions, the performance of each division can be measured in terms of both the income earned and the costs which are incurred. In profit centred divisional approach the manager of each division is responsible for cost, income and profit of his division. Further he is given freedom to make all decisions affecting his division. In such a decentralised organisation there may be transfer of goods from one division to another division. The price charged for transfer of goods of one division to another division is the cost to receiving division and income of supplying division. It means that the transfer price fix will affect the profitability of both divisions.

7.1.2. Definition: Transfer price can be defined as the price charged for products exchanged in internal transactions between sellers (or transferors) and buyers (or transferees) who belong to the same organisation usually a decentralised organisation.

7.2 Objectives of Transfer Pricing System

The main-objectives of intra-company transfer pricing are as below:

(i) This motivates manager of a division to maximize profit of the division and inturn the profit of the company as a whole.

(ii) To utilise capacity of the plant and other resources as maximum as possible.

(iii) To optimise allocation of financial resources.

7.3 Methods of Transfer Pricing

The methods of pricing usually employed in industry when goods or services are transferred from one unit to the other can be broadly classified under the following three categories:

i) At cost or variants of cost e.g. actual manufacturing cost; standard cost; full cost and full cost plus mark up.

ii) At market price.

iii) At bargained or negotiated prices.

A brief discussion of these methods is given below.

7.3.1 Pricing at Cost

(a) *Actual manufacturing cost:*

In this method goods or services are transferred at their actual cost of production. It is useful for those units where the responsibility of profit performance is centralised. Under this method, it is difficult to measure the performance of each profit centre.

(b) Standard cost:

Under this method all transfers of goods and services are made at their standard cost. Any difference between actual and standard cost viz., variances are usually absorbed by the supplying unit. In some cases, variances are transferred to the user unit as well. This will result in the inventory being carried at identical standard cost by both the supplying and receiving units. Here also the profit performance responsibility is centralised and thus it cannot be measured for individual units involved.

(c) Full cost:

Full cost means cost of production plus expenses like selling and distribution, administration, research and development cost etc. In this method, the supplying unit is not allowed to make any profit on transfers to other units. But it is free to earn profit on outside sale. One good thing about this method is that the supplying unit is allowed to recover the full cost of the goods/services transferred.

(d) Full cost plus mark up:

Under this method the supplying unit transfers goods and services at full cost plus some mark-up. The mark-up added to full cost is either expressed as a percentage of full cost or of capital employed. Selling expenses here are recovered by the supplying unit without incurring them, especially when the goods/services are transferred internally. Due to this defect the use of full cost plus method is not appreciated by the internal receiving units. To overcome this defect either the use of standard cost plus or actual cost plus are preferred. Use of either of the preceding method facilitates the task of measuring profit performance and efficiency of the units involved.

7.3.2. Pricing at market price: Under this method, the transfer prices of goods/services transferred to other units/divisions are based on market prices. In a competitive market goods/services cannot be transferred to its users at a higher price. Such a competitive market provides an incentive to efficient production. Since market prices will, by and large be determined by demand and supply in the long run, it is claimed that profits which results under this method, will provide a good indicator of the overall efficiency of the various units.

Competitive market prices provide reliable measures of divisional income because these prices are established independently rather than by individuals who have an interest in the results. The main limitations of this method are:

(i) Difficulty in obtaining market prices.

(ii) Difficulty in determining the elements of selling and distribution expenses such as commission, discounts, advertisement and sales promotion etc., so that necessary adjustment may be made in the market price to provide benefit of these expenses, to the profit centre, receiving the goods.

Illustration 1

SV Ltd. manufactures a product which is obtained basically from a series of mixing operations. The finished product is packaged in the company-made glass bottles and packed in attractive cartons. The company is organised into two independent divisions viz. one for the manufacture of the end-product and the other for the manufacture of glass bottles. The product manufacturing division can buy all the bottle requirements from the bottle manufacturing division. The General Manager of the bottle manufacturing division has obtained the following quotations from the outside manufacturers for the supply of empty bottles.

No. of empty bottles	***Total purchase value (₹)***
8,00,000	*14,00,000*
12,00,000	*20,00,000*

A cost analysis of the bottle manufacturing division for the manufacture of empty bottles reveals the following production costs:

No. of empty bottles	***Total cost (₹)***
8,00,000	*10,40,000*
12,00,000	*14,40,000*

The production cost and sales value of the end product marketed by the product manufacturing division are as under:

Volume (Bottles of end product)	***Total cost of end product (excluding cost of empty bottles)***	***Sales value (Packed in bottles)***
	(₹)	*(₹)*
8,00,000	*64,80,000*	*91,20,000*
12,00,000	*96,80,000*	*1,27,80,000*

There has been considerable discussion at the corporate level as to the use of proper price for transfer of empty bottles from the bottle manufacturing division to product manufacturing division. This interest is heightened because a significant portion of the Divisional General Manager's salary is in incentive bonus based on profit centre results.

As the corporate management accountant responsible for defining the proper transfer prices for the supply of empty bottles by the bottle manufacturing division to the product *manufacturing division, you are required to show for the two levels of volumes of 8,00,000 and 12,00,000 bottles, the profitability by using (i) market price and (ii) shared profit relative to the costs involved basis for the determination of transfer prices. The profitability position should be furnished separately for the two divisions and the company as a whole under each method. Discuss also the effect of these methods on the profitability of the two divisions.*

Solution

Statement showing profitability of two divisions at two different levels of output using different transfer prices

No. of bottles	***8,00,000***	***12,00,000***
	(₹)	(₹)
Sales value (Packed Product) : (A)	91,20,000	1,27,80,000
Less : Costs		
Product Manufacturing Division	64,80,000	96,80,000
Bottle Manufacturing Division	10,40,000	14,40,000
Total costs : (B)	75,20,000	1,11,20,000
Profit :{(A) – (B)}	16,00,000	16,60,000
Profit prorated to Bottle Mfg. Division and Product Mfg. Division.		
Share of Bottle Manufacturing Division:		
16,00,000 × 10,40,000/75,20,000	2,21,276	
16,60,000 × 14,40,000/1,11,20,000		2,14,964
Balance profit relates to Product Mfg. Division	13,78,724	14,45,036
	16,00,000	16,60,000
	(₹)	(₹)
Transfer prices of bottles		
Costs	10,40,000	14,40,000
Profit as computed above	2,21,276	2,14,964
Total price	12,61,276	16,54,964
Transfer price per bottle	₹ 1.577	₹ 1.379

From the above computations, it is observed that shared profit relative to the cost involved is ₹ 2,21,276 (₹ 0.2766 per bottle) at 8,00,000 production level and ₹ 2,14,964 (₹ 0.179 per bottle) at 12,00,000 production level. The profit of Product Mfg. Division is ₹13,78,724 (₹1.723per bottle) at 8,00,000 production level and ₹ 14,45,036 (₹ 1.2042 per bottle) at 12,00,000 production level.

Profitability based on market price

No. of bottles	8,00,000	12,00,000
Bottle Mfg. Division	(₹)	(₹)
Market price	14,00,000	20,00,000
Less: Cost	10,40,000	14,40,000
Profit (i)	3,60,000	5,60,000
Product Mfg. Division		
Sales value	91,20,000	1,27,80,000
Less: Bottle cost (at Market Price)	14,00,000	20,00,000
Product cost	64,80,000	96,80,000
Profit (ii)	12,40,000	11,00,000
Total profit : (i) + (ii)	16,00,000	16,60,000

	Profit based on cost (₹)		Profit based on Market price (₹)	
Production level	Bottle Mfg. Div.	Product Mfg. Div.	Bottle Mfg. Div.	Product Mfg. Div.
8,00,000 bottles	2,21,276	13,78,724	3,60,000	12,40,000
12,00,000 bottles	2,14,964	14,45,036	5,60,000	11,00,000

Observations:

1. Market price methods give a better profitability to Bottle Mfg. Division at both the production levels.
2. Market price method gives a lower profitability to Product Mfg. Division as compared to Bottle Mfg. Division.
3. Under Cost-based method, there is a better profit at lower level of production in Bottle Mfg. Division. However in Product Mfg. Division 12,00,000 production level gives a higher profit. But in Market price method, the position is quite reverse.

7.3.3. Bargained or Negotiated prices: Under this method each decentralised unit is considered as an independent unit and such units decide the transfer price by negotiations or bargaining. Divisional managers have full freedom to purchase their requirement from outside if the prices quoted by their sister unit are lower. A system of negotiated prices develops business like attitude amongst divisions of the company. In order to avoid any reduction in overall profits of the company, the top management may impose restriction on the external purchase/sale of goods. In order to have an effective system of intra-company transfer pricing; the following points should be kept in view:

1. Prices of all transfers in and out of a profit centre should be determined by negotiation between the buyer and the seller (i.e. transferee and transferor)
2. Negotiations should have access to full data on alternative sources and markets and to public and private information about market prices.
3. Buyers and sellers should be completely free to deal outside the company.

Illustration 2

Fasteners Ltd. is having production shops reckoned as cost centres. Each shop charges other shops for material supplied and services rendered. The shops are motivated through goal congruence, autonomy and management efforts. Fasteners Limited is having a welding shop and a painting shop. The welding shop welds annually 75,000 purchased items with other 1,50,000 shop made parts into 12,000 assemblies. The assemblies are having variable cost of ₹ 9.50 each and are sold in market at ₹ 12 per assembly. Out of the total production, 80% is diverted to painting shop at same price ruling in the market. Welding shop incurs a fixed cost of ₹ 25,000 per annum. The painting shop is having fixed cost of ₹ 30,000 and its cost of painting including transfer price from welding shop comes to ₹ 20 per unit. This shop sells all units transferred to it by welding shop at ₹ 25 per assembly.

You are required to:

(a) Find out profit of individual cost centres and overall profitability of the concern.

(b) Recommend course of action if painting shop wishes to purchase its full requirement (at market price which is ₹ 10 per assembly) either from open market or from welding shop at market price of ₹ 10 per assembly.

Give reasons for your recommendations.

Solution

Fasteners Limited

(a) Present profitability of individual shops and overall profitability

Particulars	Welding shop			Painting shop		
	Qty. Unit	Rate (₹)	Value (₹)	Qty Unit	Rate (₹)	Value (₹)
Sale in open market	2,400	12.00	28,800	9,600	25.00	2,40,000
Transfer to painting shop	9,600	12.00	1,15,200			
Total sales : (A)	12,000		1,44,000	9,600		2,40,000
Less: Variable Cost: (B)	12,000	9.50	1,14,000	9,600	20.00	1,92,000
Contribution : {(A) – (B)}			30,000			48,000
Less: Fixed cost			25,000			30,000
Profit			5,000			18,000

Overall profit for the company (₹ 5,000 + ₹ 18,000) = ₹ 23,000

(b) (i) When Painting shop purchases all its requirement from open market at a price of ₹10 per unit

	Welding shop			Painting shop		
	Qty. Unit	Rate (₹)	Value (₹)	Qty Unit	Rate (₹)	Value (₹)
Sale	2,400	12.00	28,800	9,600	25.00	2,40,000
Less: Variable cost	2,400	9.50	22,800	9,600	18.00*	1,72,800
Contribution			6,000			67,200
Less: Fixed cost			25,000			30,000
Profit/(Loss)			(19,000)			37,200

Overall profit for the company

₹ 37,200 – ₹ 19,000 = ₹ 18,200

*It is given in the question that cost of painting including transfer price from welding shop is ₹ 20 per unit. The transfer price from welding shop is ₹ 12 per unit. Therefore, the variable cost of ₹ 8 (₹ 20 – ₹ 12) is incurred by painting shop exclusively. The painting shop will be purchasing its requirement from open market at ₹ 10 per unit. Therefore, the variable cost per unit in painting shop will be ₹ 18 (₹ 10 + ₹ 8). This point should be noted carefully.

(b)(ii) When all the requirements of painting shop is met by transfer from welding shop at a transfer price of ₹ 10 per unit

	Welding shop			Painting shop		
	Qty. Unit	Rate (₹)	Value (₹)	Qty Unit	Rate (₹)	Value (₹)
Sale in the open market	2,400	12.00	28,800	9,600	25.00	2,40,000
Transfer to painting shop	9,600	10.00	96,000			
Total sales	12,000		1,24,800			
Less: Variable cost	12,000	9.50	1,14,000	9,600	18.00	1,72,800
Contribution			10,800			67,200
Less: Fixed cost			25,000			30,000
Profit/(Loss)			(14,200)			37,200

Overall profit of the company = ₹ 37,200 – ₹ 14,200 = ₹ 23,000

For the purpose of comparison, the results of the three alternatives are summarised below:

	Welding shop (₹)	Painting Shop (₹)	Overall Profit (₹)
Profit under (i)	5,000	18,000	23,000
Profit/(Loss) under (b)(i)	(19,000)	37,200	18,200
Profit/(Loss) under (b)(ii)	(14,200)	37,200	23,000

Alternative (b)(ii) should be accepted due to the following reasons:

(a) It gives a maximum overall profit of ₹ 23,000. The discussion is confined to either b(i) or b(ii).

(b) Each shop is treated as a separate cost centre and not a profit centre.

(c) The policy of overall goal congruence of the company is followed.

Illustration 3

Division A of Better Margins Ltd. has been given a budgeted target of selling 2,00,000 components COM 21, it manufactures at a price which would fetch a return of 25% on the average assets employed by it. The following figures are relevant:

Fixed overhead	*₹ 4,00,000*
Variable cost	*₹ 1 per unit*
Average assets :	
Sales debtors	*2,00,000*
Stocks	*6,00,000*
Plant and other assets	*4,00,000*

However, the marketing department of the company finds out by a survey that the maximum number of COM 21, the market can take, at the proposed price is only 1,40,000 units.

Fortunately Division B is willing to purchase the balance 60,000 units. The Manager, Division A is willing to sell to Division B at a concessional price of ₹ 4 per unit. But the Manager, Division B is ready to pay ₹ 2.25 only per unit, as he feels he can himself make COM 21 in his Division at that price.

Rather than sell to Division B at ₹ 2.25, the Manager, Division A feels he will restrict the activity of his Division to the manufacture and sale of 1,40,000 components only. By this, he could reduce ₹ 80,000 in stocks, ₹ 1,20,000 of plant and other assets and ₹ 40,000 in selling and administration expenses.

As a Cost Accountant, you are asked to work out the various computations and show that selling 60,000 COM 21 to Division B at ₹ 2.25 per unit would be in the interest of the organisation.

Solution

Neither selling price nor total sales is given. Division A of Better Margins Ltd. expects a return of 25% on average assets employed i.e., ₹ 12,00,000.

Total sales will be:	(₹)
(a) Profit (25% of 12,00,000)	3,00,000
(b) Fixed overhead	4,00,000
(c) Variable cost (2,00,000 × ₹ 1)	2,00,000
Total sales	9,00,000
Sales per unit (₹ 9,00,000 ÷ 2,00,000 units)	₹ 4.50

	Transfer to Division B and sale to outside parties	Sale to outside parties only
Sales (units)	2,00,000	1,40,000
Sales value (1,40,000 units @ ₹ 4.50)	6,30,000	6,30,000
(60,000 units @ ₹ 2.25)	1,35,000	Nil
(A)	7,65,000	6,30,000
Less: Variable cost @ ₹ 1 per unit (B)	2,00,000	1,40,000
Contribution (A – B)	5,65,000	4,90,000
Less: Fixed overhead	4,00,000	3,60,000
Net profit	1,65,000	1,30,000
Average assets employed	12,00,000	10,00,000
Return on investment	13.75%	13.00%

If the component is transferred to Division B as well as sold to outside parties, it is more profitable as the contribution, net profit and return on investment is more than the existing proposal. Therefore selling the components to Division B at ₹ 2.25 per unit is in the overall interest of the company.

*Reduction in selling and administration expenses (fixed in nature) by ₹ 40,000.

It is assumed that when Better Margins Ltd. is selling 1,40,000 units to the market only, selling price remain constant due to continuous demand of the product in the market. Market is ready to consume the product at a selling price of ₹ 4.50 per unit. Though, average assets employed will change but effect of that in selling price per unit is ignored.

Illustration 4

A boatyard is divided into three profit centres whose managers are rewarded according to results. Transactions between these profit centres are frequent.

Sales centre (S) *buys and sells new boats.*

If it needs to take part-exchange from a customer in order to sell a new boat, it transfers the part-exchanged boat to B at an agreed price.

Brokerage (B) *buys and sells second-hand boats:*

(i) in part-exchange from S (B names the price at which it can buy a comparable boat that is in a suitable condition for resale to an end-user customer, but deducts the likely cost of repairs) and

(ii) from other sources, on a normal trading basis.

Repairs (R) *does repairs for*

(i) B (to put boats into saleable condition) and

(ii) other customers.

The following situation arises:

S can sell to a customer for ₹ 35,000 a new boat which would cost ₹ 29,000. To do so, it needs to offer ₹ 16,000 in part-Exchange for the customer's old boat. However, the customer's boat is estimated by R to need repairs that will cost:

Materials	*₹ 300*
Labour	*60 hours at ₹ 15 per hour*

B can buy for ₹ 15,000 a boat comparable to the one being offered by the customer in part-exchange but which needs no repair. B could then sell that boat for ₹ 19,000.

Other data:

R's labour rate per hour is made up as follows:

	₹	
Variable cost	*6.00*	
Fixed cost	*4.50*	*(based on 20,000 budgeted hours p.a.)*
Profit	*4.50*	
	15.00	

– *45% of R's time is reserved for work from B*

– *Annual fixed cost is budgeted at:*

S ₹ 70,000

B ₹ 80,000

You are required, in relation to the above situation, to set out the contribution to profit for each profit centre that would result:

(i) Assuming that all estimates and budgets materialised as expected,

(ii) Assuming that all estimates and budgets materialised as in (i), except that the repairs undertaken by R took an extra 10 hours and ₹ 100 of materials due to a problem not noticed by B or R.

Solution (a)(i)

Statement showing the contribution to profit for each assuming that all estimates and budgets materialised as expected.

Sales Centre (S)	(₹)	(₹)	(₹)
New Boat Sold			
– Selling price			35,000
– Purchase price			29,000
Gross margin			6,000
Less: Second hand boat			
Part-exchange of old boat		16,000	

Broker's Price	15,000		
Less: Repairs	1,200	(13,800)	(2,200)
Contribution			3,800
Brokerage Centre (B)			
Second-hand boat sold			19,000
Less: Paid to Centre S		13,800	
Paid to Centre R		1,200	15,000
Contribution			4,000
Repair Centre (R)			
Sales to Centre B			1,200
Less: Materials		300	
Direct labour variable cost		360	660
Contribution			540

(ii) Assuming Additional Costs

It is noticed that all estimates and budgets are materialised except that repairs undertaken by R took an extra 10 hours and ₹ 100 of materials due to a problem not noticed by B or R.

R is responsible for giving correct repair costs and, therefore, he has to bear the additional cost:

	(₹)	(₹)
Repair Centre (R)'s contribution		540
Less: Extra cost of materials	100	
Extra D.L. variable cost (10 hrs × ₹ 6)	60	160
Revised contribution		380

However, full details are not given in the question. 'B' is a middleman passing on R's costs to S and as such should not bear additional costs. Had the item been noticed originally then S would have paid the cost and perhaps it should be passed back. This would be particularly so if R had insufficient opportunity for a complete inspection. In that case extra cost should be:

	(₹)
Material	100
Labour (10 hrs. × ₹ 15)	150
	250

Reduced contribution of S = ₹ 3,800 – ₹ 250 = ₹ 3,550

	(₹)
Original contribution of R	540
Add.: Saving in variable cost	
[10 hrs × (₹ 15 – ₹ 6)]	90
Increased contribution of R	630

Note: Other solutions are equally acceptable if well argued and logically justified.

Illustration 5

Division A is a profit centre which produces three products X, Y and Z. Each product has an external market.

Products	*X (₹)*	*Y (₹)*	*Z (₹)*
External market price per unit	*48*	*46*	*40*
Variable cost of production in division A	*33*	*24*	*28*
Labour hours required per unit in division A	*3*	*4*	*2*

Product Y can be transferred to Division B, but the maximum quantity that might be required for transfer is 300 units of Y.

The maximum external sales are:

X 800 units Y 500 units Z 300 units

Instead of receiving transfers of product Y from Division A, Division B could buy similar product in the open market at a slightly cheaper price of ₹ 45 per unit.

What should the transfer price be for each unit for 300 units of Y, if the total labour hours available in Division A are:

(a) 3,800 hours

(b) 5,600 hours

Solution

Working Notes:

(i) Hours required to meet maximum demand:

External sales Product	(i)	Hours reqd. per unit (ii)	Total Hrs. (iii) = (i) × (ii)
X	800 units	3	2,400
Y	500 units	4	2,000
Z	300 units	2	600
		Total	5,000

(ii) Contribution per unit:

Product	X (₹)	Y (₹)	Z (₹)
Selling price	48	46	40
Less : Variable cost	33	24	28
Contribution per unit : (A)	15	22	12
Labour hours required per unit : (B)	3	4	2
Contribution per hour (₹): (A) / (B)	5	5.5	6
Ranking	III	II	I

(a) **If only 3,800 hours are available in Division A.**

300 units of Z (maximum), which will take*	600 hrs.
500 units of Y (maximum), which will take	2,000 hrs.
400 units of X to use remaining hrs.	1,200 hrs.
	3,800 hrs.

***Note :** Labour hours required per unit are given in the question. If 300 units of Y are to be transferred to 'B' division, then 1,200 hours will have to be used for production of Y instead of X. It means Division A will sacrifice production of 400 units of X, which are yielding ₹ 5 per hour. Given above is the optimum mix for Division A for 3,800 hrs. If 300 units of Y are to be transferred to 'B' division with time constraint of 3,800 hours, then additional 300 units of Y will have to be produced sacrificing the production of 400 units of X which is yielding contribution.

Transfer price	(₹)
(i) Variable cost of Y	24.00
Opportunity cost	
(ii) Contribution relating to 'X' forgone for producing additional units of Y (4 hrs × ₹ 5*)	20.00
	44.00

*Y takes 4 hours and in each hour production of X would have generated contribution of ₹ 5.

(b) **If 5,600 hours are available**

Maximum time required to meet external sales *(Refer to Working note 1)*	5,000 hrs.
Hours now available	5,600 hrs.

(i) It means 600 hrs can be easily used for the production of Y and transfer price will be variable cost only

i.e. (600 hrs. ÷ 4 hrs) × ₹ 24 ₹ 3,600

Note: Y takes 4 hours per unit

(ii) For producing additional 150 units, production of X will be disturbed.

Variable costs		
(i) 150 units of Y @ ₹ 24	₹ 3,600	
Opportunity cost		
(ii) Contribution of 'X' units for gone (600 hrs. × ₹ 5)	₹ 3,000*	₹ 6,600
Total price for 300 units		₹10,200

∴ Average transfer price should be ₹ 34 per unit

*Contribution per hour of X forgone.

Illustration 6

Your company fixes the inter-divisional transfer prices for its products on the basis of cost, plus a return on investment in the division. The Budget for Division A for 2012-13 appears as under:

Investment in Division A

	(₹)
Fixed Assets	5,00,000
Current Assets	3,00,000
Debtors	2,00,000
Annual fixed cost of the division	8,00,000
Variable cost per unit of product	10
Budgeted volume	4,00,000 units per year
Desired ROI	28%

Determine the transfer price for Division A.

Solution

The desired rate of return is 28% on investments. Investments include:

(i) Fixed assets after depreciation

(ii) Net working capital.

In the question, current assets and debtors are given but current liabilities and creditors are not indicated. Therefore, these are assumed to have nil value.

Investments

	(₹)	(₹)
Fixed assets		5,00,000
Net working capital		
Current assets	3,00,000	
Debtors	2,00,000	5,00,000
Total investments		10,00,000

The desired rate of return is 28%

∴ The profit margin will be

$\left(\frac{28}{100} \times ₹\ 10,00,000\right)$ ₹ 2,80,000

Budgeted volume 4,00,000 units

	(₹)
Profit margin per unit (₹ 2,80,000 ÷ 4,00,000 units)	0.70
Fixed cost per unit (₹80,000 ÷ 4,00,000 units)	2.00
Variable cost per unit	10.00
Transfer price per unit	12.70

Illustration 7

A company is organised on decentralised lines, with each manufacturing division operating as a separate profit centre. Each division manager has full authority to decide on sale of the division's output to outsiders and to other divisions.

Division C has always purchased its requirements of a component from Division A. But when informed that Division A was increasing its selling price to ₹ 150, the manager of Division C decided to look at outside suppliers.

Division C can buy the component from an outside supplier for ₹ 135. But Division A refuses to lower its price in view of its need to maintain its return on the investment.

The top management has the following information :

C's annual purchase of the component	*1,000 units*
A's variable costs per unit	*₹ 120*
A's fixed cost per unit	*₹ 20*

Required:

(i) Will the company as a whole benefit, if Division C bought the component at ₹ 135 from an outside supplier ?

(ii) If A did not produce the material for C, it could use the facilities for other activities resulting in a cash operating savings of ₹ 18,000. Should C then purchase from outside sources?

(iii) Suppose there is no alternative use of A's facilities and the market price per unit for the component drops by ₹ 20. Should C now buy from outside?

Solution

(i) The company as a whole will not be benefited if Division C bought the component from an outside supplier at ₹ 135/- per unit

	(₹)
Purchase cost from outside supplier (1,000 units × ₹ 135 per unit)	1,35,000
Less : Saving in variable cost of division A by reducing Divisions' output (1,000 units × ₹ 120 per unit)	1,20,000
Net cost (benefit) to the company as a whole	15,000

The company as a whole will not be benefited, as it will be required to incur an additional cost of ₹ 15,000 if Division C bought the company from outside supplier.

(ii) The company will be benefitted if C purchase the component from an out-side supplier and Division A uses the facilities for other activities.

	(₹)	(₹)
Purchase cost from outside supplier		
(1,000 units × ₹ 135)		1,35,000
Less : Savings in variable cost of Division A for the units		
purchased by Division C from outside		
(1,000 units × ₹ 120 per unit)	1,20,000	
Cash operating saving of Division A for the use of facilities		
for other activities	18,000	1,38,000
Net cost (benefit) to the company as a whole		(3,000)

It is advisable that Division C should purchase the component from outside sources as this decision will benefit the company by ₹ 3,000.

(iii) The company will be benefitted if C purchase the component from an out-side supplier and there is no alternative use of Division A's facilities.

Purchase cost from outside supplier

	(₹)
(1,000 units × ₹ 115)	1,15,000
Less : Saving in variable cost of Division A by reducing division's output	1,20,000
(1,000 units × ₹ 120)	
Net cost (benefit) to the company	(5,000)

It is advisable that the Division C should buy the component from outside as this decision will benefit the company by ₹ 5,000.

Illustration 8

Division Z is a profit centre, which produces four products –A, B, C and D. Each product is sold in the external market also. Data for the period is as follows:

	A	B	C	D
Market price per unit	*₹ 150*	*₹ 146*	*₹ 140*	*₹ 130*
Variable cost of production per unit	*₹ 130*	*₹ 100*	*₹ 90*	*₹ 85*
Labour hours required per unit	*3*	*4*	*2*	*3*

Product D can be transferred to division Y, but the maximum quantity that might be required for transfer is 2,500 units of D.

The maximum sales in the external market are:

A *2,800 units*

B	*2,500 units*
C	*2,300 units*
D	*1,600 units*

Division Y can purchase the same product at a slightly cheaper price of ₹125 per unit instead of receiving transfers of product D from division Z.

What should be transfer price for each unit for 2,500 units of D, if the total labour hours available in division Z are:

(i) 20,000 hours? *(ii) 30,000 hours?*

Solution

Working note:

Ranking of products when availability of time is the key factor

Product	(₹)	(₹)	(₹)	(₹)
Market price per unit	150	146	140	130
Less: Variable cost of production per unit	130	100	90	85
Contribution per unit	20	46	50	45
Contribution per hour	6.66	11.50	25	15
	(₹20/3 hrs.)	*(₹46/4 hrs.)*	*(₹50/2 hrs.)*	*(₹45/3 hrs.)*
Ranking	IV	III	I	II

(i) Statement of product mix (when total available hours in division Z are 20,000)

Product (Refer to W.N.)	Maximum demand (units)	Hours per unit	Units produced	Hours used	Balance hours
(a)	**(b)**	**(c)**	**(d)**	**(e)=(b)×(c)**	**(f)**
C	2,300	2	2,300	4,600	15,400 (20,000 – 4,600)
D	1,600	3	1,600	4,800	10,600 *(15,400 – 4,800)*
B	2,500	4	2,500	10,000	600 *(10,600 – 10,000)*
A	2,800	3	200	600	Nil *(600 – 600)*

Note: Time required to meet the demand of 2,500 units of product D for division Y is 7,500 hours. This requirement of time viz., 7,500 hours for providing 2,500 unit of product D for division Y can be met by sacrificing the production of 1,725 units of product B (1,725 units × 4 hours = 6,900 hurs) and 200 units of product B (200 units × 3 hours = 600 hours)

Statement of Transfer Price for each unit for 2,500 unit of D.

Transfer price	2,500 units of product D	Per unit of product D
Variable cost (2,500 units × ₹ 85)	2,12,500	85.00
Opportunity cost of the contribution foregone by not producing 200 units of A. (200 units × ₹ 20)	4,000	1.60
Opportunity cost of the contribution foregone by not producing 1,725 units of B (1,725 units × ₹ 46)	79,350	31.74
Transfer price	2,95,850	118.34

(ii) Statement of product mix (when total available hours in division Z are 30,000)

Product (Refer to W.N.)	Maximum demand (units)	Hours per unit	Units produced	Hours used	Balance hours
(a)	(b)	(c)	(d)	(e)=(b)×(c)	(f)
C	2,300	2	2,300	4,600	25,400 *(30,000 – 4,600)*
D	1,600	3	1,600	4,800	20,600 *(25,400 – 4,800)*
B	2,500	4	2,500	10,000	10,600 *(20,600 – 10,000)*
A	2,800	3	2,800	8,400	2,200 *(10,600 – 8,400)*

Note: The required time for producing 2,500 units of product D for Divison Y is 7,500 hours. This requirement can be met to the extent of 2,200 hours out of the balance hours (as shown in the last column of the above table). The remaining re-quirement of 5,300 hours can be met by sacrificing the output of 1,766.66 units of product A.

Statement of Transfer Price for each unit for 2,500 units of D

	2,500 units of product D	Per unit of product D
Variable cost *(2,500 units × ₹ 85)*	2,12,500	85.00
Opportunity cost of the contribution foregone by not producing 1,766.66 units of product A. *(1,766.66 units × ₹ 20)*	35,333.20	14.13

Transfer price	2,47,833.20	99.13

Illustration 9

AB Ltd. manufactures Foam, Carpets and Upholstery in the three divisions. Its operating statement for 2011-12 showing the performance of these divisions drawn for the use of management is reproduced below:

(Rupees in '000)

Particulars	***Manufacturing Divisions***			***Total***
	Foam	***Carpets***	***Upholstery***	
Sales revenue (A)	*1,600*	*1,200*	*1,200*	*4,000*
Manufacturing Costs :				
Variable	*1,200*	*700*	*680*	*2,580*
Fixed (Traceable)	—	*100*	*20*	*120*
(B)	*1,200*	*800*	*700*	*2,700*
Gross Profit (A – B) = (C)	*400*	*400*	*500*	*1,300*
Expenses :				
Administration	*134*	*116*	*172*	*422*
Selling	*202*	*210*	*232*	*644*
(D)	*336*	*326*	*404*	*1,066 (B)*
Net Income (C – D)	*64*	*74*	*96*	*234*
Division's Ranking	*3rd*	*2nd*	*1st*	

(A) *Sales include Foam transferred to the Upholstery division at its manufacturing cost of ₹ 2,00,000.*

(B) *Common expenses of ₹ 1,30,000 and ₹ 1,00,000 on account of administration and selling respectively stand apportioned to these divisions at 10% of Gross Profit in case of administration and 2.5% of Sales in case of selling expenses. Rest of ₹ 8,36,000 of the expenses are traceable to respective divisions.*

The manager of the Foam division is not satisfied with the above approach of presenting operating performance. In his opinion his division is best among all the divisions. He requests the management for preparation of revised operating statement using contribution approach and showing internal transfer at market price.

You are required to :

(a) *Draw the revised operating statement using contribution approach and pricing the internal transfer at market price.*

(b) *Compute relevant ratios to show comparative profitability of these divisions and rank them in the light of your answer at (a) above. Further, offer your comments on the contention of the manager of Foam division.*

(c) *State why the contribution approach and pricing of internal transfers at market price are*

more appropriate in realistic assessment of the performance of various divisions.

Solution

(a) Revised Operating Statement using Contribution approach

(₹ '000)

Divisions :	Foam	Carpets	Upholstery	Total
Sales revenue	1,680	1,200	1,200	4,080
(*Refer to working note 1*)				
Less : Variable manufacturing costs :	1,200	700	760	2,660
(*Refer to working note 2*)				
Contribution : (A)	480	500	440	1,420
Traceable Costs :				
Fixed manufacturing costs	—	100	20	120
Administration expenses	94	76	122	292
(*Refer to working note 3*)				
Selling expenses	162	180	202	544
(*Refer to working note 4*)				
Total : (B)	256	356	344	956
Operating income : [(A) – (B)]	224	144	96	464
Less : Common expenses				
(₹ 130 + ₹ 100)				230
Net income of the company				234

Working Notes:

1. **Computation of sales revenue of Foam division**

	(₹ '000)
Sales of foam division to outside customers	1,400 (₹ 1,600 – ₹ 200)
Less: Variable manufacturing costs (₹ 1200 – ₹ 200)	1,000
Mark-up on outside sale	400
Percentage of mark-up 40% (₹ 400/₹ 1000) × 100	
Transfer price of foam to upholstery division	280
Sales of foam division to outside customers	1,400
Total	1,680

2. **Computation of variable manufacturing costs of**

Upholstery Division in (₹ '000)

(₹ 680 – ₹ 200 + ₹ 280) = ₹ 760

3. **Computation of Traceable Administration Expenses**

(₹ '000)

Divisions	Foam	Carpets	Upholstery	Total
Given Administration expenses	134	116	172	422
Less : Common expenses (10% of Gross Profit)	40	40	50	130
Traceable Administration expenses	94	76	122	292

4. **Computation of Traceable Selling Expenses**

(₹ '000)

Divisions	Foam	Carpets	Upholstery	Total
Given Selling expenses	202	210	232	644
Less : Common expenses (2.5% of Sales)	40	30	30	100
Traceable selling expenses	162	180	202	544

(b) **Comparative Profitability & Ranking Statement (based on contribution approach relevant ratios calculated by using figures of (a) part)**

(₹ '000)

Divisions	Foam	Carpets	Upholstery
Contribution margin ratio (in %)	28.57 (₹ 480/1680) × 100	41.67 (₹ 500/1200) × 100	36.67 (₹ 440/1200) × 100
Ranking	III	I	II
Operating Income Ratio (in %)	13.33 (₹ 224/1680) × 100	12 (₹ 144/1200) × 100	8 (₹ 96/1200) × 100
Or			
Ranking	I	II	III

Comment: The Manager of Foam Division, appears to be correct in raising objection over the approach used for presenting operating performance of three divisions for the year 2011-12. His division is the best among all, on the basis of operating income/sales ratio which is the highest inspite of its contribution margin ratio to be the lowest.

(c) The use of contribution approach for reporting is more realistic for assessing the performance of various divisions as it considers variable and traceable costs only and avoids common costs while finding out profitability. This approach enables the management to rightly interpret the information. Further pricing of internal transfers at market price will give due credit to specific profit centre i.e. transferor.

Illustration 10

City Instrument Company (CIC) consists of the Semi-conductor Division and the Mini-computer Division each of which operates as an independent profit centre. Semi-conductor Division employs craftsmen, who produce two different electronic components, the new-high performance Super-chip and an older product called Okay-chip. These two products have the following cost characteristics:

	Super-chip		***Okay chip***	
Material	*Parts*	*₹ 20*	*Parts*	*₹ 10*
Labour	*2 hours × ₹ 140*	*₹ 280*	*1/2 hour × ₹ 140*	*₹ 70*

Annual Overhead in Semi-conductor Division is ₹ 40,00,000 all fixed. Owing to high skill level necessary for the craftsmen, the Semi-conductor Division's capacity is set at 50,000 hours per year.

To date, only one customer has developed a product utilising super-chip, and this customer orders a maximum of 15,000 super-chips per year at a price of ₹ 600 per chip. If CIC cannot meet his entire demand, the customer curtails his own production. The rest of the semi-conductor's capacity is devoted to Okay-chips, for which there is unlimited demand at ₹ 120 per chip.

The Mini-computer Division produces only one product, a process control unit, which requires a complex circuit board imported at a price of ₹ 600. The control unit's costs are

	Control Unit	
Material	*Circuit board*	*₹ 600*
	Other parts	*80*
Labour	*5 hours @ ₹ 100*	*500*

The Mini-Computer Division is composed of only a small assembly plant and all overhead is fixed at a total of ₹ 8,00,000 per year. The current market price for the control unit is ₹ 1,400 per unit.

A joint research project has just revealed that with minor modifications, a single super-chip could be substituted for the circuit board currently used by the Mini-computer division. The modification would require an extra one hour of labour by Mini-computer's staff, for a total of 6 hours per control unit. Mini-computer has therefore asked Semi-conductor division to declare a transfer price at which Semi-conductor division would sell super-chip internally.

Required:

(i) *Mini-computer expects to sell 5,000 control units this year. From the overall view point of CIC, how many super-chips should be transferred to Mini-computer Division to replace circuit boards?*

(ii) If the demand for the control unit is sure to be 5,000 units, but its price is uncertain, what should be the transfer price of super-chip to ensure proper decisions ? (All other data unchanged)

(iii) If demand for the control unit rises to 12,000 units at a price of ₹ 1,400 per unit, how many of 12,000 units should be built using Super-chip ? (All other data unchanged.)

Solution:

Working Notes:

1. **Contribution per hour of Super-chips and Okay-chips:**

	Super-chips	Okay-chips
Selling price per unit (₹)	600	120
Less : Variable cost per unit (₹)	300	80
Contribution per unit (₹)	300	40
Hours required per unit	2	0.5
Contribution per hour	150	80
	(₹ 300/2 hrs.)	(₹ 40/0.5 hrs.)

2. **Details of hours utilised in meeting the demand of 15,000 units of Super-chips and utilising the remaining hours for Okay-chips out of available hours of 50,000 per annum:**

Hours utilised for manufacturing 15,000 units of Super-chips (15,000 units × 2 hours)	30,000
Hours utilised for manufacturing 40,000 units of Okay-chips (40,000 units × 0.5 hours)	20,000
	50,000

3. **Contribution of a process control unit (using an imported complex circuit board):**

	(₹)
Selling price per unit : (A)	1,400
Variable costs :	
Circuit board (Imported)	600
Other parts	80
Labour cost (5 hours × ₹ 100)	500
Total variable cost : (B)	1,180
Contribution per unit (₹) {(A) – (B)}	220

4. Contribution of a process control unit (using a Super chip):

	(₹)
Selling price per unit : (A)	1,400
Variable costs :	
Super chip	300
(Material + Labour costs)	
Other parts	80
Labour cost	600
(6 hours × ₹ 100)	
Total variable cost : (B)	980
Contribution per unit : {(A) – (B)}	420

5. Incremental contribution per unit of a process control unit, when instead of using imported complex circuit board Super-chip is used:

Incremental contribution per unit (₹) : 200

{*₹ 420 – ₹ 220*} {*Refer to working notes 3 & 4*}

(i) Super-chip to be transferred to Mini Computer Division to replace Circuit Boards :

Out of 50,000 available hours 30,000 hours are utilised for meeting the demand of 15,000 units of Super-chips, the rest 20,000 hours may be used for manufacturing 40,000 Okay-chips, which yields a contribution of ₹ 40 per unit for ₹ 80/- per hour (*Refer to Working note 1*) or a contribution of ₹ 160 per two-equivalent hours.

In case the company decides to forego the manufacturing of 20,000 units of Okay- chips in favour of 5,000 additional units of Super-chips to be used by Mini-Computer

Division (instead of complex imported Circuit Board) for manufacturing process control units. This decision would increase the existing contribution of Mini-Computer Division by ₹ 200/- per two-equivalent hours (*Refer to Working note 5*).

After taking into account the profit foregone of Okay-chips, the existing contribution of Mini-Computer Division of CIC would increase by ₹ 40 per two equivalent hours.

Hence the entire requirement of 5,000 units of Super-chips be produced and transferred to Mini-Computer Division.

(ii) Minimum transfer price of Super-chip to Mini Computer Division :

= Variable cost of a Super-chip + Opportunity cost of foregoing the production of an Okay-chip and using the craftsman time for Super-chip

= ₹ 300 + 2 hours × ₹ 80

= ₹ 460

(iii) Super-chips to be produced for the production of 12,000 units of process control units:

After meeting out the order of 15,000 Super-chips per year, the concern is left out with 20,000 hours. Use of Super-chips for control units production would increase the existing contribution of Mini-Computer Division by ₹ 200/- per unit. Out of the remaining 20,000 craftmen hours, 10,000 units of Super-chips can be made, which may be used for the production of 10,000 process control units.

7.4 Conflict between a Division and the Company

Usually a conflict between a division of the company and the company as a whole is faced by the management of decentralised units when products or services are exchanged among different divisions of the company. Such a conflict becomes more significant in the case of those concerns where profitability is used as a criterion for evaluating the performance of each division.

The essence of decentralisation is reflected in the freedom to make decisions. Under such a set up it is expected that the top management should not interfere with the decision making process of its subordinates heading different units. In other words, management of decentralised units is given autonomy with regard to decision making. The management of such companies also expects that each division should not only achieve its own objective necessary for evaluating the performance but should also achieve the objective of goal congruence.

In case of failure of a division to achieve the objective of 'goal congruence' the management of the company may dictate their 'transfer price'. Such interference of management of the company is usually the main basis of conflict between a division and the company as a whole.

Further this conflict is aggravated if the management advocates the transfer of goods and services at cost. As such the transfer price will not reflect a good picture about the performance of the transferring division. The profitability of the transferring division will not be known by the use of such a transfer price.

Each division appreciates the transfer of its goods/services at usual selling price/market price so as to arrive at the correct return/profitability figure, used for measuring the performance. There is no incentive to the transferring division if goods and services are transferred at variable cost.

To resolve the conflicts between the divisions, following methods may be suggested

(i) Dual Rate transfer pricing system

(ii) Two Part transfer pricing system.

(i) Dual Rate transfer pricing system: In a decentralised business environment where each division is treated as a responsibility centre, management may resolve conflicts by introducing Dual Rate transfer pricing. In this system two separate transfer prices are used to price an interdivisional transaction. For example supplying division is allowed to record a transaction at full cost plus mark-up. On the other hand receiving division may be charged only marginal cost for such transaction. In this system, accounts of the both divisions show profit and accordingly performance of the division is evaluated.

Drawbacks of Dual Rate transfer pricing system are as follows:

(a) Since, both division i.e., supplying and receiving division report profit in their respective accounts, therefore, it creates confusion. Profit of the company as a whole can not be calculated by simply adding profits of the all divisions.

(b) Profit shown by the divisions are artificial and should be used only for performance evaluation.

(c) Divisions are not competing effectively.

(ii) Two part transfer pricing system: This transfer pricing system is also known as Marginal cost plus a fixed lump-sum fee. In this system supplying division transfer its products to receiving division at marginal cost and a fixed fee for a particular period say per annum. The advantage of this system is that supplying division recovers fixed overheads and profit margin through fixed fee and receiving division get intermediate products at marginal cost. Both supplying and receiving division shall also be able to report profit from inter-divisional transactions.

7.5 Multinational Transfer Pricing

In the discussions above, we have focused on how transfer pricing policies affect the motivation of managers. However, in multinational companies, other factors may dominate. Multinational companies use transfer prices to minimize worldwide income taxes, import duties, and tariffs. For example, Nike might prefer to make its profits in a country, with its maximum corporate tax rate of 28%, rather than in some other place where the rate is 35%.

Suppose a division in a high-income-tax-rate country produces a sub-assembly for another division in a low-income-tax-rate country. By setting a low transfer price, the company can recognize most of the profit from the production in the low-income-tax-rate country, thereby minimizing taxes. Likewise, items produced by divisions in a low-income-tax-rate country and transferred to a division in a high-income-tax-rate country should have a high transfer price to minimize taxes.

Sometimes import duties offset income tax effects. Most countries base import duties on the price paid for an item, whether bought from an outside company or transferred from another division. Therefore, low transfer prices generally lead to low import duties.

How transfer price can be used as a tax saving tool in a multinational transactions with an associate or business establishment, can be illustrated with the help of the following example:

Example : ABC Ltd. In India manufactures a product-A, where tax rate assumed as 35% and sales product-A, where tax rate assumed as 35% and sales product-A to its associates in Mexico, where the tax rate is assumed as 45%.

Data for the year 2011-12 is as follows:

Units sold = 5,000 units

	Product-A Manufactured in India (₹)	Product-A Sold in Mexico (₹)	ABC Ltd. as a whole (₹)
Sale price per unit	—	2,000	2,000
Transfer price p.u	800	(800)	
Variable costs p.u	(200)	(400)	(600)
Contribution margin p.u	600	800	1,400
Total contribution	30,00,000	40,00,000	70,00,000
Fixed costs	(10,00,000)	(20,00,000)	(30,00,000)
Profit before Tax	20,00,000	20,00,000	40,00,000
Tax rate	35%	45%	
Taxes	(7,00,000)	(9,00,000)	(16,00,000)
Profit after tax	13,00,000	11,00,000	24,00,000

If product-A was sold to associate in Mexico at ₹ 1,200 p.u., then the Profit of the company as a whole could have been increased.

If Transfer Price set at ₹ 1,200 per unit

	Product-A Manufactured in India (₹)	Product-A Sold in Mexico (₹)	ABC Ltd. as a whole (₹)
Sale price per unit	—	2,000	2,000
Transfer price p.u	1,200	(1,200)	
Variable cost p.u	(200)	(400)	(600)
Contribution margin p.u	1000	400	1,400
Total contribution	50,00,000	20,00,000	70,00,000
Fixed costs	(10,00,000)	(20,00,000)	(30,00,000)
Profit before tax	40,00,000	—	40,00,000
Tax rate	35%	45%	
Taxes	(14,00,000)	—	(14,00,000)
Profit after taxes	26,00,000	—	26,00,000

From the above example one can see how multinational transfer pricing can be used as a tax avoidance tool.

To curb this type of practice, in India, organisations having transactions with their cross border divisions has to follow Arms – Length price. The General Anti Avoidance Rule, popularly known as GAAR has been proposed (in Finance Budget 2012) to curb this type of tax avoidance mechanism.

In summary, transfer pricing is more complex in a multinational company than it is in a domestic company. Multinational companies try to achieve more objectives through transfer-pricing policies, and some of the objectives may be conflicting.

8

Uniform Costing and Inter Firm Comparison

8.1 Uniform Costing

Meaning

Uniform Costing is not a distinct method of costing. In fact, when several undertakings start using the same costing principles and/or practices they are said to be following uniform costing. The basic idea behind uniform costing is that the different concerns in an industry should adopt a common method of costing and apply uniformly the same principles and techniques for better cost comparison and common good. The principles and methods of compilation, analysis, apportionment and absorption of overheads differ from one concern to the other in the same industry; but if a common or uniform pattern is adopted by all, it helps mutually in cost control and cost reduction. Therefore, it is necessary that a uniform method of costing should be adopted by the member unit of an industry.

8.1.1 Objectives of Uniform Costing: The main objectives of Uniform Costing are as follows:—

1. *Facilitates Comparison:* To facilitate the comparison of costs and performances of different units in the same industry; it provides objective basis.
2. *Eliminates Unhealthy Competition:* To eliminate unhealthy competition among the different units of an industry.
3. *Improves Efficiency:* To improve production capacity level and labour efficiency by comparing the production costs of different units with each other.
4. *Provides Relevant Data:* To provide relevant cost information/data to the Government for fixing and regulating prices of the products.
5. *Ensures Standardisation:* To bring standardisation and uniformity in the operation of participating units.
6. *Reduces Cost:* To reduce production, administration, selling and distribution costs, and to exercise control on fixed costs.

8.1.2 Essential requisites for the installation of Uniform Costing System : A successful system of uniform costing has the following requirements :—

1. The firms in the industry should be willing to share/furnish relevant data/information.

2. A spirit of cooperation and mutual trust should prevail among the participating firms.

3. Mutual exchange of ideas, methods used, special achievements made, research and know how etc. should be frequent.

4. Bigger firms should take the lead towards sharing their experience and know-how with the smaller firms to enable the latter to improve their performance.

5. Uniformity must be established with regard to several points before the introduction of uniform costing in an industry. In fact, uniformity should be with regard to following points:

   (*a*) Size of the various units covered by uniform costing.

   (*b*) Production methods.

   (*c*) Accounting methods, principles and procedures used.

8.1.3 Advantages of Uniform Costing: The advantages accruing from the use of uniform costing system are as follows:-

1. The management of each firm will be saved from the exercise of developing and introducing a costing system of its own.

2. A costing system devised by mutual consultation and after considering the difficulties and circumstances prevailing in different firms is readily adopted and successfully implemented.

3. It facilitates comparison of cost figures of various firms to enable the firms to identify their weak and strong points besides controlling costs.

4. Optimum achievement of efficiency is attempted by all the firms by utilising the experience of other concerns in the industry.

5. Standing in the industry of each firm will be known by making a comparison of its cost data with others.

6. Services of cost consultants or experts may be available jointly to each firm in the industry by sharing their experiences and expenses.

7. Research and development benefits of bigger firms may be made available to smaller firms.

8. It helps in the reduction of labour turnover, as a uniform wage system is the pre-condition of a uniform costing system.

9. It helps Trade Associations in negotiating with the Government for any assistance or concession in the matters of taxation, exports, subsidies, duties and prices determination etc.

10. Unhealthy competition is avoided among the firms in the same industry in framing pricing policies and submitting tenders.
11. Prices fixed on the basis of uniform costing are representative of the whole industry and thus are reliable.
12. Uniform costing provides a basis for the comparative assessment of the performance of two firms in the same industry but in different sectors.
13. It helps the Government in regulating the prices of essential commodities such as bread, sugar, cement, steel etc.

8.1.4 Limitations of Uniform Costing:

1. Sometimes it is not possible to adopt uniform standards, methods and procedures of costing in different firms due to differing circumstances in which they operate. Hence, the adoption of uniform costing becomes difficult in such firms.
2. Disclosure of cost information and other data is an essential requirement of a uniform costing system. Many firms do not wish to share such information with their competitors in the same industry.
3. Small firms in an industry believe that uniform costing system is only meant for big and medium size firms, because they cannot afford it.
4. It induces monopolistic trend in the business, due to which prices may be increased artificially and supplies withheld.

8.2 Inter-Firm Comparison

Meaning

It is technique of evaluating the performance, efficiency, costs and profits of firms in an industry. It consists of voluntary exchange of information/data concerning costs, prices, profits, productivity and overall efficiency among firms engaged in similar type of operations for the purpose of bringing improvement in efficiency and indicating the weaknesses. Such a comparison will be possible where uniform costing is in operation.

An inter-firm comparison indicates the efficiency of production and selling, adequacy of profits, weak spots in the organisation, etc. and thus demands from the firm’s management an immediate suitable action. Inter-firm comparison may enable the management to challenge the standards which it has set for itself and to improve upon them in the light of the current information gathered from more efficient units. Such a comparison may be carried out in electrical industry, printing firms, cotton spinning firms, pharmaceuticals, cycle manufacturing, etc.

8.2.1 Requisites of inter-firm comparison system: The following requisites should be considered while installing a system of inter-firm comparison:–

1. *Centre for Inter-Comparison:* For collection and analysing data received from member units, for doing a comparative study and for dissemination of the results of study a Central body is necessary. The functions of such a body may be :–

   (*a*) Collection of data and information from its members;

   (*b*) Dissemination of results to its members;

   (*c*) Undertaking research and development for common and individual benefit of its members;

   (*d*) Organising training programmes and publishing magazines.

2. *Membership*: Another requirement for the success of inter-firm comparison is that the firms of different sizes should become members of the Centre entrusted with the task of carrying out inter-firm comparison.

3. *Nature of information to be collected*: Although there is no limit to information, yet the following information useful to the management is in general collected by the Centre for inter-firm comparison.

   (*a*) Information regarding costs and cost structures.

   (*b*) Raw material consumption.

   (*c*) Stock of raw material, wastage of materials, etc.

   (*d*) Labour efficiency and labour utilisation.

   (*e*) Machine utilisation and machine efficiency.

   (*f*) Capital employed and Return on capital.

   (*g*) Liquidity of the organisation.

   (*h*) Reserve and appropriation of profit.

   (*i*) Creditors and debtors.

   (*j*) Methods of production and technical aspects.

4. *Method of Collection and presentation of information*: The Centre collects information at fixed intervals in a prescribed form from its members. Sometimes a questionnaire is sent to each member; the replies of the questionnaire received by the Centre constitute the information/data. The information supplied by firms is generally in the form of ratios and not in absolute figures. The information collected as above is stored and presented to its members in the form of a report. Such reports are not made available to non-members.

8.2.2 Advantages of Inter-firm comparison: The main advantages of inter-firm comparison are:–

1. Such a comparison gives an overall view of the industry as a whole to its members– the present position of the industry, progress made during the past and the future of the industry.
2. It helps a concern in knowing its strengths or weaknesses in relation to others so that remedial measures may be taken.
3. It ensures an unbiased specialized reporting on particular problems of the concern.
4. It develops cost consciousness among members of the industry.
5. It helps Government in effecting price regulation.
6. It helps to improve the quality of products manufactured and to reduce the cost of production. It is thus advantageous to the industry as well as to the society.

8.2.3 Limitations of inter-firm comparison

The following are the limitations in the implementation of a scheme of inter-firm comparison :

1. Top management feels that secrecy will be lost.
2. Middle management is usually not convinced with the utility of such a comparison.
3. In the absence of a suitable Cost Accounting System, the figures supplied may not be reliable for the purpose of comparison.
4. Suitable basis for comparison may not be available.

Summary

- When several undertakings start using the same costing principles and/or practices they are said to be following uniform costing.
- The main objectives of Uniform Costing are as follows :
  - ✓ *Facilitates Comparison*
  - ✓ *Eliminates Unhealthy Competition*
  - ✓ *Improves Efficiency*
  - ✓ *Provides Relevant Data*
  - ✓ *Ensures Standardisation*
  - ✓ *Reduces Cost*
- Inter-Firm comparison is technique of evaluating the performance, efficiency, costs and profits of firms in an industry

It indicates the efficiency of production and selling, adequacy of proof- its, weak spots in the organisation, etc. and thus demands from the firm's management an immediate suitable action

- Requisites of inter-firm comparison system are as follows:
  - ✓ *Centre for Inter-Comparison*
  - ✓ *Membership*
  - ✓ *Nature of information to be collected*
  - ✓ *Method of Collection and presentation of information*

9

ProfitabilityAnalysis-Product Wise/Segment Wise/ Customer Wise

LEARNING OBJECTIVES

After studying this chapter you will be able to understand:

- The various components of operating income and its impact on the profitability,
- Effect of growth component, price recovery component and productivity components in the change of operating income,
- Reasons for the difference in the operatingprofit of two periods and its reconciliation,
- Product wise profitability analysis- Direct Product Profitability (DPP)
- Segment wise profitability analysis and Customer profitability analysis.
- Four perspectives of the Balanced scorecard.

9.1 Profitability Analysis

An organisation which operates in a competitive environment has to adopt various strategies to survive profitably into the market where it operates. Porter in its generic strategy theory has suggested that a firm can survive profitably in the long term if it chooses its generic strategy according to the environment in which it operates and which conformsto the overall corporate objectives. A firm would be profitable if it is either a cost leader i.e. it can produce its product at a lower cost than its competitor and enjoy maximum market share or if it produces its products with some peculiar features which make it different from others. Whichever, approacha firm may choose it has to be very careful on the part of actual performance and any deviation from the set performance target.To achieve its objectives it has to put some performance measurement mechanism into place so that any deviation can be measured and corrective action can be taken.

ProfitabilityAnalysis can be useful to measure the performance of a firm against the acceptable standards. In this chapter we will learn toanalyseoperating profit of an organization from various angles. Profitability can be analysedas per the requirement of the management, to assist them to identify the critical success factors and to take appropriate decisions.

9.1.1 Operating profit analysis: Operating profit of a firm is affected by various components which are responsible for changes in the revenue and costs. A change in the profit may be due to revenue or costs orboththe factors. For the purpose of analysing operating income, we spread our analysis into three main areas or components which are (a) Growth component (b) Price recovery component and (c) Productivity component.Analysis will cover both revenue and cost effect,wherever applicable,on these components separately.

(a) Growth component measures the change in the quantity of output sold. The growth component of the change in the operating income measures the increase/ decrease in revenue and in costs due to selling more/ less quantity units from the previous period.

Revenue effect of growth component:

Revenue effect of growth can be measured with the help of the following formula:

$$\text{Revenue effect of growth} = \begin{pmatrix} \text{Actual unit of output} & - & \text{Actual units of output} \\ \text{sold in current year} & & \text{sold in last year} \end{pmatrix} \times \text{Selling price in last year}$$

The revenue effect of growth measures the increase/ decrease in revenue solely due to change in number of units sold.

Cost effect of growth component:

Cost effect of growth measures the effect of variable cost and fixed cost separately.

For variable cost components:

$$\text{Cost effect of growth} = \begin{pmatrix} \text{Unit of input required to} & & \text{Actual units of input} \\ \text{produce current year output} & - & \text{used to produce last year's} \\ \text{in last year} & & \text{output} \end{pmatrix} \times \text{Input price in last year}$$

For fixed cost components:

$$\text{Cost effect of growth} = \begin{pmatrix} \text{Actual units of capacity in} & & \\ \text{last year if adequate to} & - & \text{Actual units of Capacity} \\ \text{produce current year's} & & \text{in last year} \\ \text{production in last year} & & \end{pmatrix} \times \text{Rate in last year}$$

(b) Price recovery component of change in operating income measures the changes in the revenue and costs solely due to changes in prices.

Revenue effect of price recovery component:

Revenue effect of price recovery can be measured with the help of the following formula:

$$\text{Revenue effect of Price recovery} = \begin{pmatrix} \text{Selling price in} & - & \text{Selling price in} \\ \text{current year} & & \text{last year} \end{pmatrix} \times \begin{matrix} \text{Actual units of output} \\ \text{sold in current year} \end{matrix}$$

The revenue effect of price recovery measures the increase/ decrease in revenue solely due to change in selling prices.

Cost effect of price recovery component:

Cost effect of price recovery measures the effect of variable cost and fixed cost separately.

For variable cost components:

$$\text{Cost effect of Price recovery} = \begin{pmatrix}\text{Input price in} & - & \text{Input price in} \\ \text{current year} & & \text{last year}\end{pmatrix} \times \begin{matrix}\text{Units of input required to produce} \\ \text{current year's output in last year}\end{matrix}$$

For fixed cost components:

$$\begin{matrix}\text{Cost effect of} \\ \text{Price recovery}\end{matrix} = \begin{pmatrix}\text{Rate per unit} & - & \text{Rate per unit} \\ \text{in current year} & & \text{in last year}\end{pmatrix} \times \begin{matrix}\text{Actual units of capacity in last year,} \\ \text{if adequate to produce current} \\ \text{year's output in last year}\end{matrix}$$

(c) Productivity component measures the change in the operating income due to changes in the product mix and/ or yield of inputs as compared with the last year. This component uses current year's prices of input to measure the changes in costs only.

For variable cost components:

$$\begin{matrix}\text{Cost effect of} \\ \text{productivity}\end{matrix} = \begin{pmatrix}\text{Actual units of input} & & \text{Units of input required} \\ \text{used to produce current} & - & \text{to produce current year's} \\ \text{year's output} & & \text{output in last year}\end{pmatrix} \times \text{Input price in current year}$$

For fixed cost components:

$$\begin{matrix}\text{Cost effect of} \\ \text{productivity}\end{matrix} = \begin{pmatrix} & & \text{Actual units of capacity in} \\ \text{Actual units of capacity} & - & \text{last year if adequate to} \\ \text{in current year} & & \text{produce current year's} \\ & & \text{production in last year}\end{pmatrix} \times \text{Rate in current year}$$

Reconciliation of Operating Profit

	Costs	**Revenue**
Operating profit in last year	xxx	
Add/(Less): Revenue and Cost effect of Growth component	(+) F (-) A	(+) F (-) A
Add/(Less): Revenue and Cost effect of Price recovery component	(+) F (-) A	(+) F (-) A
Add/ (Less): Cost effect of Productivity component	(+) F (-) A	-
	xxx	xxx
Operating Income in current year (Revenue – Costs)	xxx	

F= Favourable, A= Adverse

Illustration 1

Y Limited is a manufacturer of Cardboard boxes. An analysis of its operating income between 2012and 2013 shows the following:

	Income Statement (amount in 2012)	***Revenue & Cost effect of Growth component in 2013***	***Revenue & Cost effect of Price recovery component in 2013***	***Cost effect of productivity component in 2013***	***Income Statement (amount in 2013)***
Revenue (₹)	*40,00,000*	*2,00,000(F)*	*4,20,000(F)*	*-*	*46,20,000*
Cost (₹)	*29,20,000*	*60,000 (A)*	*2,56,000(A)*	*58,000(F)*	*31,78,000*
Operating Income (₹)	*10,80,000*	*1,40,000(F)*	*1,64,000(F)*	*58,000(F)*	*14,42,000*

Y limited sold 4,00,000 boxes and 4,20,000 boxes in 2012 and 2013 respectively. During 2013 the market for cardboard boxes grew 3% in terms of number of units and all other changes are due to company's differentiation strategy and productivity. Compute how much of the change in operating income from 2012 to 2013 is due to the industry market size factor, productivity and product differentiation and also reconcile the profit of both years due to these factors.

Solution:

Reconciliation of Operating Income

Particulars	*Amount (₹)*
Operating Income in 2012	10,80,000
Add: Change Due to Industry Market Size Factor (W.N.-1)	84,000
Changes Due to Productivity (W.N.-2)	58,000
Changes Due to Product Differentiation (W.N.-3)	2,20,000
Operating Income in 2013	14,42,000

Workings:

Total Increase in Sale of Cardboard Boxes 20,000 Boxes (4,20,000 Boxes – 4,00,000 Boxes). Out of this increase in Sales of 20,000 Boxes,12,000 Boxes (3% of 4,00,000) is due to *growth in market size*, and the remaining 8,000 Boxes (20,000 Boxes – 12,000 Boxes) are due to an increase in *market share* .

W.N.1 Effect of the Industry Market Size Factor on operating income:

$$=\text{Revenue and Cost Effect of Growth Component in 2013} \times \frac{\text{Increase in Sales Unit Due to Market Growth}}{\text{Total Growth in Sales Unit (from 2012 to 2013)}}$$

$$= ₹1,40,000 \times \frac{12,000\,\text{Boxes}}{20,000\,\text{Boxes}}$$

$$= ₹84,000\ (F)$$

W.N.2. Effect of Productivity *on operating income*:

= Cost Effect of Productivity Component in 2013

= ₹58,000 (F)

W.N.3 Effect of Product Differentiation *on operating income*:

Particulars	*Amount (₹)*
Increase in the Selling Price (Revenue Effect of the Price Recovery Component)	4,20,000 (F)
Increase in Prices of Inputs (Cost Effect of the Price Recovery Component)	2,56,000 (A)
Growth in Market Share Due to Product Differentiation* $\left(₹\ 1,40,000 \times \frac{8,000\ \text{Boxes}}{20,000\ \text{Boxes}}\right)$	56,000 (F)
Total	2,20,000 (F)

* Revenue and Cost Effect of Growth Component in 2013 ×

$$\frac{\text{Increase in Sales Unit Due to Product Differentiation}}{\text{Total Growth in Sales Unit (from 2012 to 2013)}}$$

9.2 Profitability Analysis-Product wise

For a profit making organisation, profit earned from an operation is a key performance indicator which assures and controls the direction towards the organisation’s objectives. In today’s competitive business era most of the firms are having a portfolio of various ranges of products either for the same consumer market or for different consumer markets. A firm which has a portfolio ofprofitable products enjoys high profitability. However, it is very important to know the relative profitability of an individual product so that management can concentrate on the profitable products and weed out the loss making products from the products’ portfolio. Direct product profitability is one among the various analytical methods which analyse the profitability for each product or segment of products separately.

9.2.1 Direct Product Profitability (DPP):DPP is used to measure the profitability of an individual product and assist management to know the true profitability to make appropriate decisions.As opposed to the traditional absorption costing, where normally labour hours or machine hours are used as a basis for absorption of indirect costs, DPP uses variety of measures like space used for transportation and storing of goods, refrigeration time etc. DPP is generally used in the retail tradeto determine profitability from an individual product.

CIMA describe DPP*“used primarily within the retail sector, DPP involves the attribution of both the purchase price and other indirect costs (for example distribution, warehousing and retailing) to each product line. Thus a net profit, as opposed to a gross profit, can be identified for each product. The cost attribution process utilizes a variety of measures (for example warehousing space and transport time) to reflect the resource consumption of individual products.”*

In recent years DPP has developed considerably in parallel with activity-based costing. DPP hasbecome much more sophisticated and is now very similar to activity-based costing. Infact it may be regarded as the application or extension of activity based costing.

Benefits of DPP

- Better cost analysis
- Better pricing decisions
- Better management of stores and warehouse space
- The rationalisation of product ranges.

Direct product profitability statement: Retail organisations traditionally deducted the boughtincost of goods from the selling price to give a gross margin. The gross margin is uselessmeasure for controlling the costs of the organisation itself or making decisions about the profitabilityof the different products. This is because none of the costs generated by the retailorganisation itself are included in its calculation. For example, it does not include the storagecosts of the different goods and these costs vary considerably from one goods to another. Amethod was needed which relates the indirect costs to the goods according to the way the goodsuses or creates these costs.

Indirect costs, for DPP may be analysed into basic cost categories as follows:

(i) *Overhead cost:* This is incurred through an activity that is not directly linked to a particularproduct.

(ii) *Volume related cost:* The cost is incurred in relation to the space occupied by products.This includes storage and transport costs.

(iii) *Product batch cost:* This cost is often a time based cost. If product items (that is a numberof identical products which are handled together as a batch) are stocked on shelves alabour time cost is incurred.

(iv) *Inventory financing costs:* This is the cost of tying up money in stock and is the cost ofthe product multiplied by interest rate per day or per week.

Direct product profit can be derived as shown below:

Direct Product Profit	
Sales	xx
*Less:*Cost of Goods Sold	xx
Gross Margin	xx
$\pm$ Adjustments	xx
Adjusted Gross Margin	xx
*Less:*Direct Product Costs (Warehouse, Transportation, Store etc.)	xx
Direct Product Profit	xx

Table 1, given below shows the DPP for product A. Directly attributable costs have been groupedinto three categories and are deducted from the gross margin to determine the good's DPP.

Table- 1
Direct product profit for product A

	(₹)	*(₹)*
Selling price p.u.		150.00
Less : Bought-in price		80.00
Gross margin		70.00
Less : Direct product costs :		
Warehouse costs	16.00	
Transport costs	18.00	
Store costs	22.00	56.00
Direct product profit p.u.		14.00

Warehouse and store costs will include items such as labour, space and insurance costs, while,transport costs will include labour, fuel and vehicle maintenance costs. The usual way to spreadthese costs across the different goods sold is in relation to volume or area occupied, as mostcosts increase in direct proportion to the volume of the good or the space it occupies. However,there are some exceptions to this; for example, insurance costs may be better spread on value oron a risk index. Risk is greater with refrigerated or perishable goods. Refrigeration costs mustonly be related to those products that need to be stored in the refrigerator.

The result of this type of DPP cost analysis may give information such as that given in thefollowing table:

Table-2

Profit	*Gross margin (%)*	*DPP (%)*
Ice-cream	20.40	4.60
Baby food	11.00	5.50
Tooth paste	31.20	18.80
Wine	45.30	17.20
Paper tissues	15.70	0.00

Above table-2 shows that for ice-cream there is a considerable gap between the gross marginand the DPP because its refrigerated storage is expensive. It also shows that paper tissues,which had quite a healthy gross margin, are just breaking even with DPP; this is because theyare very bulky relative to their price.

While the supermarket or other retailer does not have theluxury of stopping selling paper tissues, because obviously it would lose considerable trade if itdid not stock a complete range of goods, it does have other choices. The choices are merchandisingones, such as where to display the stock and in what position on the shelves. Stocks at eyelevelsells more quickly than the above or below eye level. The brand with the greatest marginshould be placed at eye level. Goods at the front of the store tend to sell faster than goods at theback. This explains why tissues are rarely found close to the entrance or the cash till.

With manufactured products cost per unit for the different products is often calculated and theproducts ranked. For a retail organisation DPP per unit may not be the best measure to use.

DPPper unit of time adds another dimension to the measurement and DPP per unit of time permeasure of space adds a third. This is automatically built in when overheads are spread if a costeach product uses this rate multiplied by the volume and the number of days or weeks in thesystem. In the example in Table 1, the store costs would be based on a rate per cubic centimeter or metre per day and the product cost can be calculated according to its size and the time it takes to flowthrough the system. For example, if the store cost per cubic cm is ₹ 0.0073 per day and goodA is 10 cubic cms and the average stay in the store is three days, the store cost per item is ₹0.0073 × 10 cms. × 3 days = ₹ 0.22.

According to Doherty et al (1993) the single most valuable aspect of DPP lies in its diagnosticcapabilities, allowing managers to ask questions, such as why did a product group over or underperform. Table-3 shows that product group A-2 accounts for 3.07 per cent of sales but for 3.31per cent of DPP. Upon investigation it emerged that stock-turn was managed particularly well inthis group. Product B-2 has 3.05 per cent of sales but accounts for only 2.85 per cent of DPP. Aninvestigation of the warehouse costs might explain this or the opportunity of offering multipacksmight reduce costs.

Table–3
DPP values by Product Group. From Doherty et al 1993.

Product Group	*Sales as a % of total sales*	*DPP as a % of total DPP*
A-1	3.20	3.40
A-2	3.07	3.31
B-1	2.84	2.89
B-2	3.05	2.85
C-1	2.75	2.66
A-3	2.26	2.48

Illustration 2

Jigyasa India Ltd. (JIL) has 30 retail stores of uniform sizes ‘Fruity &Sweety Retails’ across the country. Mainly three products namely ‘Butter Jelly’, ‘Fruits & Nuts’ and ‘Icy Cool’ are sold through these retail stores. JIL maintains stocks for all retail stores in a centralised warehouse. Goods are released from the warehouse to the retail stores as per requisition raised by the stores. Goods are transported to the stores through two types of vans i.e. normal and refrigerated. These vans are to be hired by the JIL.

Costs per month of JIL are as follows:

	(₹)
Warehouse Costs:	
Labour& Staff Costs	*27,000*
Refrigeration Costs	*1,52,000*
Material Handling Costs	*28,000*
Total	*2,07,000*
Head Office Cost:	
Salary & Wages to Head Office Staff	*50,000*
Office Administration Costs	*1,27,000*
Total	*1,77,000*
Retail Stores Costs:	
Labour Related Costs	*33,000*
Refrigeration Costs	*1,09,000*
Other Costs	*47,000*
Total	*1,89,000*

Average transportation cost of JIL per trip to any retail stores are as follows:

Normal Van	*₹3,200*
Refrigerated Van	*₹4,900*

The Chief Financial Manager asked his Finance managers to calculate profitability based on three products sold through Fruity &Sweety retail stores rather than traditional method of calculating profitability.

The following information regarding retail stores are gathered:

	Butter Jelly	Fruits & Nuts	Icy Cool
No. of Cartons per cubic metre (m^3)	*42*	*28*	*40*
No. of Items per cartons (units)	*300*	*144*	*72*
Sales per month (units)	*18,000*	*4,608*	*1,152*
Time in Warehouse (in months)	*1*	*1.5*	*0.5*
Time in Retail Stores (in months)	*1*	*2*	*1*
Selling Price per unit (₹)	*84*	*42*	*26*
Purchase Price per unit (₹)	*76*	*34*	*22*

Butter Jelly and Icy-Cool are required to be kept under refrigerated conditions.

Additional information:

Total Volume of All Goods Sold per month	*40,000 m^3*
Total Volume of Refrigerated Goods Sold per month	*25,000 m^3*
Carrying Volume of each van	*64 m^3*

Required:

Calculate the Profit per unit using Direct Product Profitability (DPP) method.

Solution:

Direct Product Profitability (DPP) Statement

(Amount in ₹)

	Butter Jelly	***Fruits & Nuts***	***Icy Cool***
Selling Price *per unit*	84.00	42.00	26.00
Less: Purchase Price *per unit*	76.00	34.00	22.00
Gross Profit ...(A)	8.00	8.00	4.00
Direct Product Costs:			
Warehouse Costs *per* m^3 [W.N.-1]	7.46	2.07	3.73
Retail Stores Costs *per* m^3 [W.N.-2]	6.36	4.00	6.36
Transportation Costs [W.N.-3]	76.56	50.00	76.56
Total DPP costs *per* m^3	90.38	56.07	86.65
Items *per* m^3[W.N.-4]	12,600	4,032	2,880
Cost *per item* ...(B)	0.007	0.014	0.030
Direct Product Profit ...(A) - (B)	7.993	7.986	3.97

Working Notes

(1) Warehouse Related Costs

	General Costs (₹)	***Cost Related with Refrigerated Goods (₹)***
Labour& Staff Costs	27,000	---
Refrigeration Costs	---	1,52,000
Material Handling Costs	28,000	---
Total	55,000	1,52,000
Volume of Goods Sold	40,000 m^3	25,000 m^3
Cost *per* m^3 *per month*	1.38	6.08

Products	***Time in Warehouse***	***Cost per*** m^3 ***per month (₹)***	***Total Cost (₹)***
Butter Jelly	1 Month	7.46 (1.38 + 6.08)	7.46
Fruits & Nuts	1.5 Months	1.38	2.07
Icy-cool	0.5 Months	7.46 (1.38 + 6.08)	3.73

(2) Retail Stores Related Costs

	General Costs (₹)	Cost Related with Refrigerated Goods (₹)
LabourRelated Costs	33,000	---
Refrigeration Costs	---	1,09,000
Other Costs	47,000	---
Total	80,000	1,09,000
Volume of Goods Sold	40,000 m^3	25,000 m^3
Cost *per* m^3 *per month*	2.00	4.36

Products	Time in Retail Stores	Cost *per* m^3 *per month*	Total Cost
Butter Jelly	1 Month	₹6.36 (₹2.00 + ₹4.36)	₹6.36
Fruits & Nuts	2 Months	₹2.00	₹4.00
Icy-Cool	1 Month	₹6.36 (₹2.00 + ₹4.36)	₹6.36

(3) Transportation Costs

	Normal Van Costs	Refrigerated Van Costs
Cost *per trip*	₹3,200	₹4,900
Volume of Van	64 m^3	64 m^3
Cost *per* m^3 *per trip*	₹50.00	₹76.56

(4) No. of Items *per* m^3

Products	No. of Cartons (m^3)	No. of Items *per Cartons* (units)	No. of Items *per* m^3
Butter Jelly	42	300	12,600 (42 × 300)
Fruits & Nuts	28	144	4,032 (28 × 144)
Icy- Cool	40	72	2,880 (40 × 72)

9.3 Profitability Analysis- Segment wise

As a part of performance evaluation, a firm can segment its operations in many different ways. The basis of segmentation can be product, price, geographical boundaries, demographics, consumers or production processes etc.Unlike financial reporting, there is no need to follow any prescribed criteria as set by any standards.In management accounting the purpose of segment wise profitability analysis is to assist management to make better decisions, hence, it is customisable.

Costs, in segment analysis can be classified into direct costs and indirect costs. All expenses which are directly related with the existence of a segment are called direct costs. Direct costs can be variable or fixed in nature. On the other hand indirect costs are common costs which cannot be attributed totally to a particular segment. Generally, there are two approaches followed to calculate segmental profit which are (i) Full cost approach and (ii) Contribution approach.

(i) Full cost approach: Under this approach profit for a segment is calculated taking all direct andindirect costs into account. The indirect costs are allocated to the segment based on appropriate allocation base. Under this approach, since full costs are taken into account while calculating segmental profit so sum total of profits of each segment equals to total organisational profit. Mathematically it can be written as:

Segmental profit = Segmental revenue – Direct costs – Allocated indirect costs.

And

Organisational profit = ∑Segmental profits

(ii) Contribution approach: Under this approach instead of calculating segment wise net income contribution is calculated. To calculate segmental contribution only direct costs are deducted from the segmental revenue. All indirect expenses are deducted from the overall organisational contribution. While following this approach one may find that the segment which was seemed to be unprofitable is contributing towards the indirect expenses. Segmental contribution can be simply calculated as:

Segmental contribution = Segmental revenue – Direct costs

And

Organisational profit = ∑Segmental contribution – Total indirect costs

9.4 Profitability Analysis- Customer wise

9.4.1Customer Profitability Analysis: In many organisations it is just as important to costcustomers as it is to cost products. Different customers or groups of customers differ in theirprofitability. This is a relatively new technique that ABC makes possible because it creates costpools for activities. Customers use some activities but not all, and different groups of customershave different 'activity profiles'.

Service organisations, such as a bank or a hotel, in particular need to cost customers. A bank'sactivities for a customer will include the following types of activities:

- Withdrawal of cash
- Unauthorised overdraft
- Request for a statement
- Stopping a cheque
- Returning a cheque because of insufficient funds.

Different customers or categories of customers will each use different amounts of these activitiesand so customer profitability profiles can be built up, and customers can be charged accordingto the cost to serve them. A hotel may have activities that are provided for specific types ofcustomers, such as well laid-out gardens, a swimming pool and a bar. Older guests may appreciateand use the garden, families use the swimming pool and business guests use the bar. If theactivities are charged to the relevant guests a correct cost per bed occupied can be calculated forthis type of category. This will show the relative profitability and lead to strategies for encouragingthe more profitable guests.

Even a manufacturing organisation can benefit from costing its customers. Not all customerscost the same to serve even if they require the same products. Some customers may be locateda long way from the factory and transport may cost more. Other customers may be disruptiveand place rush orders that interrupt production scheduling and require immediatespecial transport.Some customers need after sales service and help with technical matters, etc.

Illustration3

A manufacturing organisation has four different customers A, B, C and D. A single product issold to them at different prices because of trade discount offered. Data is given for cost per unit of business activity. You are required to prepare customer profitability statement.

Information on four customers

Customers	*A*	*B*	*C*	*D*
No. of units sold	*60,000*	*80,000*	*1,00,000*	*70,000*
Selling price net of discount	*25p*	*23p*	*21p*	*22p*
No. of sales visits	*2*	*4*	*6*	*3*
No. of purchase orders	*30*	*20*	*40*	*20*
No. of deliveries	*10*	*15*	*25*	*14*
Kilometres per journey	*20*	*30*	*10*	*50*
No. of rush deliveries	—	—	*1*	*2*

Cost of each activity

Sales visit	*₹ 210 per visit*
Order placing	*₹ 60 per order*
Product handling	*₹ 0.10 per item*
Normal delivery cost	*₹ 2 per kilometer*
Rushed delivery cost	*₹ 200 per delivery*

Solution:

Customer profitability Statement

Customers	A (₹)	B (₹)	C (₹)	D (₹)
Revenue net of discount	15,000 (60,000 × ₹ 0.25)	18,400 (80,000 × ₹0.23)	21,000 (1,00,000 × ₹0.21)	15,400 (70,000× ₹0.22)
Costs : Sales visits	420	840	1,260	630
Ordering processing	1,800	1,200	2,400	1,200
Product handling	6,000	8,000	10,000	7,000
Normal Delivery	400	900	500	1,400
Rush deliveries	—	—	200	400
	8,620	10,940	14,360	10,630
Operating profit	6,380	7,460	6,640	4,770
Percentage profitability	43%	41%	32%	31%

It is apparent from above solution that all four customers are profitable, but customers C and Dnot particularly so when compared with customers A and B. There are several reasons for thisrange in profitability one reason is the negotiation of favourable terms such as higher tradediscount as compared to other customers.

Benefits of Customer Profitability Analysis:

1. It helps the supplier to identify which customers are eroding overall profitability andwhich customers are contributing to it.

2. It can help to provide a basis for constructive dialogue between buyer and seller to improve margins.

9.5 The Balanced Scorecard

In today's business environment information becomes a vital element and to gain competitive advantage over the peers, it cannot be denied. In this era of information age competition, a company cannot survive just by injecting huge capital investment in new technology for physical assets only or by excellent management of financial assets and liabilities. In this information age both manufacturing and service organisation needs new capabilities for competitive success. Merely investing in and managing physical, tangible assets is not enough but an organisation must be able to mobilise and exploit its intangible or invisible assets which in turn becomes a decisive factor.

Intangible assets enable an organisation to:

- Maintain and further development in customer relationships to retain loyalty of existing customers and to serve new market/ customer segments effectively and efficiently.

- Introduce products and services as per the desire of targeted customer and market segments.
- Produce customised high-quality products and services economically with short gestation periods.
- Mobilise employee skills and motivation for better and consistent deliberation in process capabilities, quality, and response times.
- Deploy information technology, data bases and effective management information systems.

The balanced scorecard is a method which displays organisation's performance into four dimensions namely financial, customer, internal and innovation. The four dimensions acknowledge the interest of shareholders, customers and employees taking into account of both long-term and short-term goals.

Kaplan and Norton classified performance measures into four business 'perspectives':

(i) The financial perspective

(ii) The customer perspective

(iii) The internal business perspective

(iv) The learning and growth perspective

(i) Financial Perspective: "How Do We Look To Shareholders?" In this step manager of a division or a unit, links its business objectives to the corporate strategy of the company as a whole.Financial performance measures indicate whether the company's strategy implementation and execution are contributing to its revenue and earnings. To identify key performance measures in this perspective, managers, during strategic planning ask "How do we look to shareholders?"

Corporate strategy and strategic initiatives are examined from the financial perspective to see feasibility of these initiatives of being met. The financial objectives chosen at the onset of the balanced scorecard implementation should serve two purposes:

1. To provide definite performance that was expected at the time of strategies selection.
2. To provide a focus for objectives and appropriate measures in each of the other three perspectives.

(ii) Customer Perspective: "How Do Customer View Us?" In this stage, companies identify customers and market segments in which they compete and also the means by which they provide value to these customers and markets. Managers identify the lead indicators which make a particular business unit or product different from that of others. Lead indicator may vary from customer to customer or market segment. If for example, a customer values on-time delivery then on-time delivery becomes a lead indicator. Examples of lead indicators may include any number of customer considerations, including:

- On-time delivery
- On-site service

- After sales support
- Defects per order
- Cost of the product
- Free shipments etc.

By delivering quality as per the customer demand and need, business units can improve outcome measures such as customer satisfaction, retention, acquisition and loyalty.

(iii) Internal Business Perspective: "At What Must We Excel?" In this stage companies identify processes and activities which are necessary to achieve the objectives as identified at financial perspectives and customer perspective stage. These objectives may be achieved by reassessing the value chain and making necessary changes to the existing operating activities. If maintaining net earnings is the financial objective of a company and after sales service can increase customer retention, then internal business perspective needs to improve after sales services to satisfy customer requirements to maintainnet earnings. This objective may be achieved by providing for example toll free customer help lines, setting up service centresin all major cities.

(iv) Learning and Growth Perspective: "How Do We Continue To Improve And Create Value?" In the learning and growth perspective, Companies determine the activities and infrastructure that the company must build to create long term growth, which are necessary to achieve the objectives set in the previous three perspectives. Organisational learning and growth comes from three principle sources:

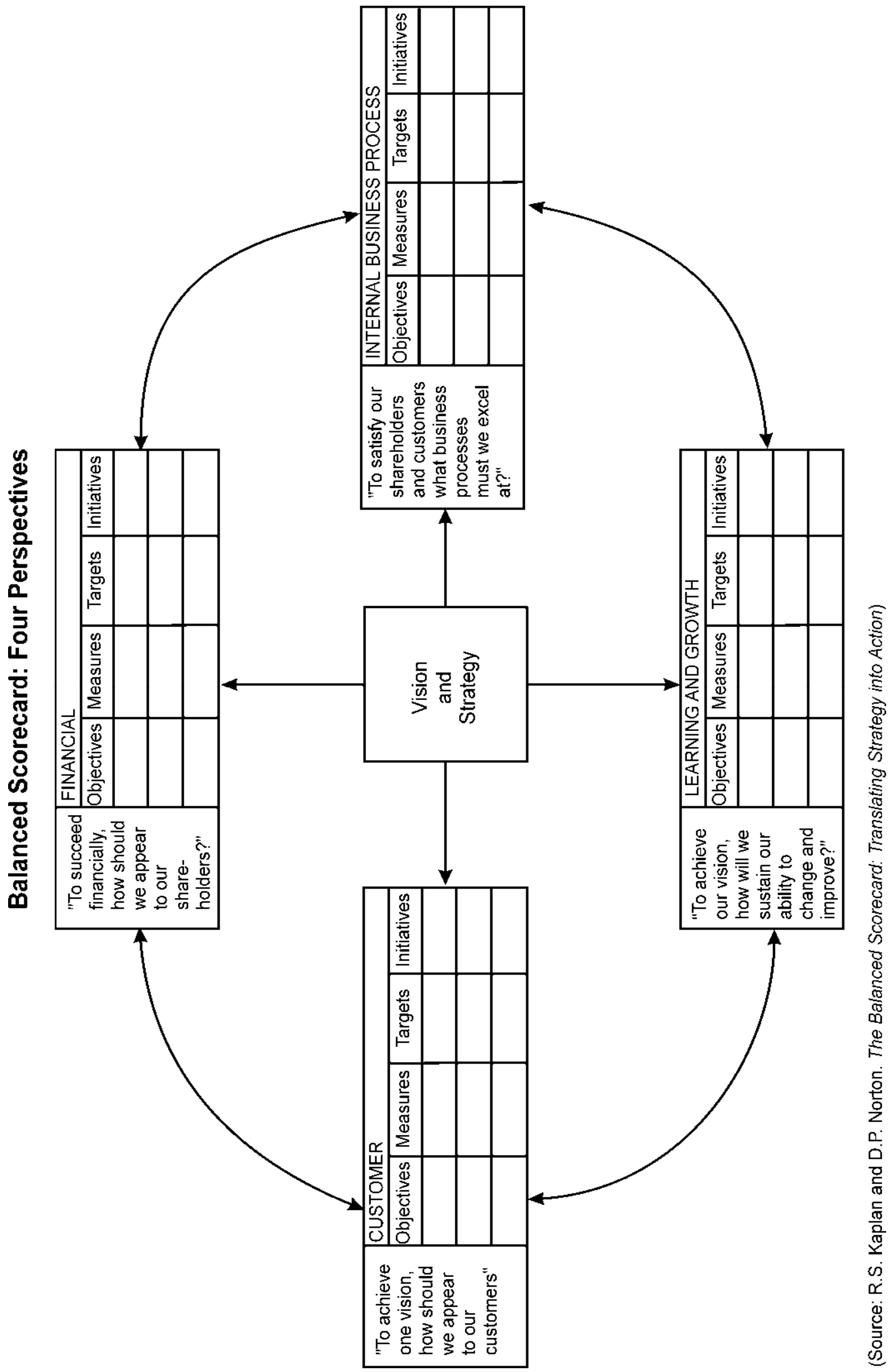
- People i.e. employee capabilities
- Systems i.e. information system capabilities and
- Organisational procedures i.e. motivation, empowerment and alignment.

Since, the balanced scorecard is intended to improve long-term performance, managers may invest in resources needed in the short-run but this should not affect business unit's performance.

The ultimate result of using the Balanced Scorecard approach should be an improved longtermfinancial performance. Since the scorecard gives equal importance to the relevant non –financial measures, it should discourage the short termism that leads to cuts in spending on newproduct development, human resource development etc which are ultimately detrimental forthe future prospects of the company.

The responsibility to devise and implement a Balanced Scorecard should be that of themanagersworking with the business. Since every company is different, it shall need to work out for itselfthe various financial and non – financial measures, which need to be focused upon for its owndevelopment. Since the Balanced Scorecard is recommended as a management tool usedboth for internal and external reporting purposes, it is again the

The following figure summarises the ideas of a Balanced Scorecard:



manager's responsibility todecide as to what information needs to be disclosed and how any problems of confidentialitycan best be overcome.

The following are some reasons why Balanced Scorecards sometimes fail to provide for thedesired results;

- Managers mistakenly think that since they already use non – financial measures, theyalready have a Balanced Scorecard.
- Senior executives misguidedly delegate the responsibility of the Scorecard implementationto middle level managers.
- Company's try to copy measures and strategies used by the best companies rather thandeveloping their own measures suited for the environment under which they function.
- There are times when Balanced Scorecards are thought to be meant for reporting purposesonly. This notion does not allow a Business to use the Scorecard to manage Business ina new and more effective way.

It may be noted that the above-mentioned difficulties refer to the internal use of the Scorecard,unless it is used internally successfully, it should not be used as a basis for external reporting.

FINAL COURSE STUDY MATERIAL

PAPER 5

ADVANCED MANAGEMENT ACCOUNTING

MODULE – 3

BOARD OF STUDIES
THE INSTITUTE OF CHARTERED ACCOUNTANTS OF INDIA

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CHAPTER 16 – LEARNING CURVE THEORY

APPENDIX

10

Linear Programming

LEARNING OBJECTIVES

After studying this unit you will be able to:

- Formulate the Linear Programming Problem
- Solve the Linear Programming Problem with two variables using graphical method
- Solve the Linear Programming Problem using simplex method
- Recognize problems that Linear Programming can handle

10.1 Introduction

Linear programming is a mathematical technique for determining the optimal allocation of resources and achieving the specified objective when there are alternative uses of the resources like money, manpower, materials, machines and other facilities. The objective in resource allocation may be either cost minimization or profit maximization. The technique of linear programming is applicable to all problems in which the total effectiveness about the use of resources can be expressed as a linear function of individual allocations, and the limitations on resources give rise to *linear* equalities or inequalities of individual allocations. The adjective linear, is to be particularly noted here. It implies that all the limitations or constraints and the objective must be expressed as linear functions.

Although the technique of linear programming is applicable to all such problems, there are some more easy methods for solving specific categories of problems. Special algorithms have been evolved to ease up computational task. The algorithms have acquired special names of their own. This has led to the following categories of the linear programming problems:

(i) General Linear Programming Problems.

(ii) Transportation Problems.

(iii) Assignment Problems.

We know that there is an opportunity cost for scarce resources that should be included in the relevant cost calculation for decision making and variance calculations. In practice several resources may be scarce. The opportunity cost of these scarce resources can be determined by the use of linear programming techniques. Our objective in this chapter is to examine linear programming techniques and to consider how they can be applied to some specific types of decisions that a firm may have to make.

General Linear Programming Problems: An examination of the following simple example should illustrate the basic concepts of linear programming problem (abbreviated as LPP).

Illustration 1

A small scale industry unit manufactures two products, X_1 and X_2 which are processed in the machine shop and the assembly shop. The time (in hours) required for each product in the shops are given in the matrix below. Profit per unit is also given along.

	Machine shop	***Assembly shop***	***Profit per unit***
Product X_1	*2 hours*	*4 hours*	*₹ 3*
Product X_2	*3 hours*	*2 hours*	*₹ 4*
Total time available (hours) (in a day)	*16*	*16*	

Assuring that there is unlimited demand for both products how many units of each should be produced every day to maximize total profit?

Solution

Let x_1 and x_2 be the number of units of X_1 and X_2 respectively that maximize the profit.

To begin solving the problem, let us restate the information in mathematical form. To do so, we introduce a term "*objective function*". This refers to the expression which shows the relationship of output to profit.

$3x_1$ = total daily profit from the sale of x_1 units product X_1

$4x_2$ = total daily profit from the sale of x_2 units product X_2

Objective function Max Profit = $3x_1 + 4x_2$

Department time constraints

Time used in making x_1 and x_2 units of two products must not exceed the total daily time available in two shops. In other words, (the hours required to make 1 unit of product X_1) multiplied by (the number of units of product X_1) plus (the hours required to make 1 unit of product X_2) multiplied by (the number of units of product X_2) must be less than or equal to the daily time available in each shop. Mathematically it is stated as

$2x_1 + 3x_2 \leq 16$ Machine shop constraint

$4x_1 + 2x_2 \leq 16$ Assembles shop constraint

In order to obtain meaningful answers, the values calculated for x_1 and x_2 cannot be negatives; they must represent real product X_1 and product X_2. Thus, all elements of the solution to the formulated linear programming problem must be either greater than or equal to 0 ($x_1 \geq 0$, $x_2 \geq 0$). The machine and assembly shops constraints are called structural constraints.

The problem can be summarized in a mathematical form as below:

Maximize:

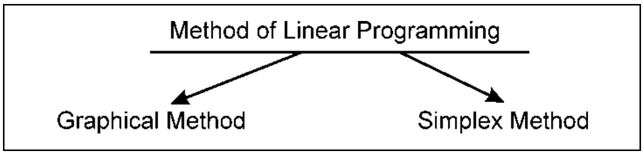
$$\text{Profit} = 3x_1 + 4x_2$$

Subject to the constraints

$$2x_1 + 3x_2 \leq 16$$

$$4x_1 + 2x_2 \leq 16$$

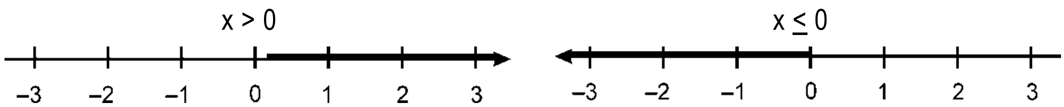
$$x_1 \geq 0, x_2 \geq 0.$$



10.2 Graphical Method

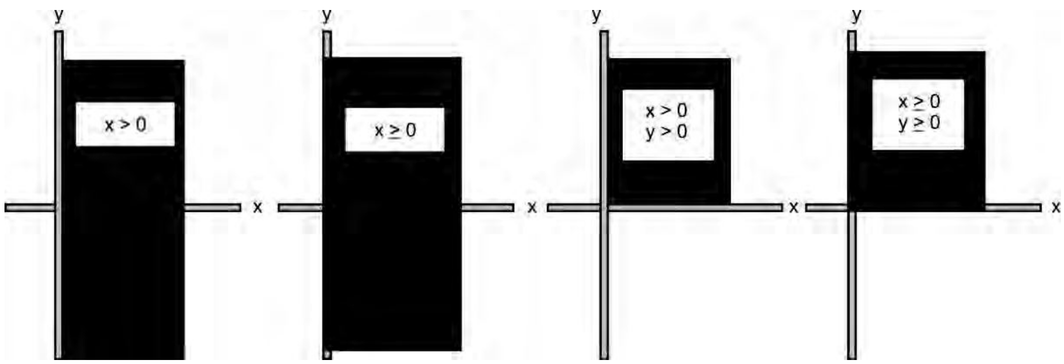
The graphical method discussed here provides necessary grounding in understanding the computational steps of the simplex algorithms, it itself being of little use in handling practical problems. It is possible to solve conveniently linear programming problems graphically as long as the number of variables (products, for example) is not more than two. Before we proceed to explain the graphical method, let us first know how linear inequalities are handled graphically.

Linear inequalities in one variable and the solution space: Any relation or function of degree one that involves an inequality sign is a *linear inequality*. It may be of one variable, or, may be of more than one variable. Simple examples of linear inequalities are those of one variable only; viz. $x > 0$, $x \leq 0$, etc.



The values of the variables that satisfy an inequality are called the solution space, and is abbreviated as S.S. The solution spaces for (i) $x > 0$, (ii) $x \leq 0$ are shown by the dark portion in the above diagrams.

Linear inequalities in two variables: Now we turn to linear inequalities in two variables *x,* and y, and shade a few S.S.

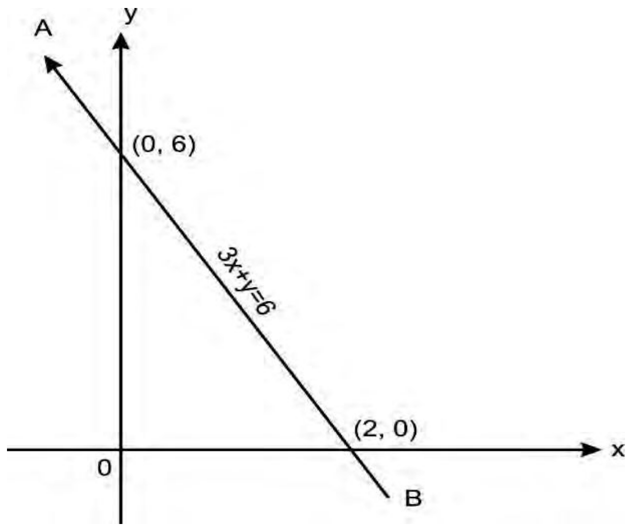


Let us now consider a linear inequality in two variables given by $3x + y < 6$.

The inequality mentioned above is true for certain pairs of numbers, (x, y) that satisfy $3x + y < 6$. By trial, we may arbitrarily find such a pair to be (1,1) because $3 \times 1 + 1 = 4$, and $4 < 6$.

Linear inequalities in two variables may be solved easily by extending our knowledge of straight lines.

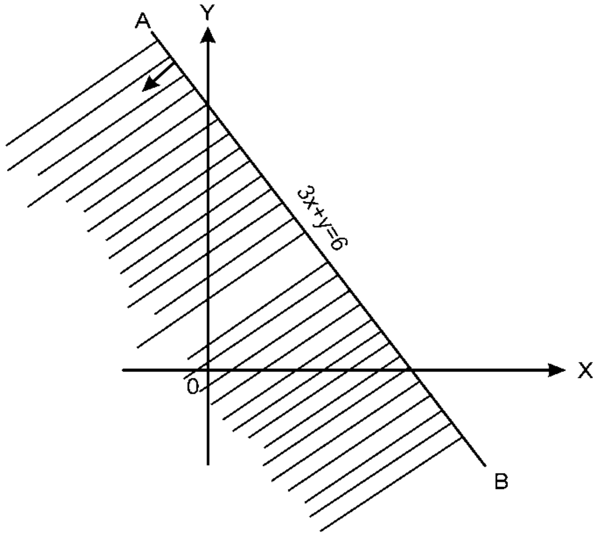
For this purpose, we replace the inequality by an equality and seek the pairs of number that satisfy $3x + y = 6$. We may write $3x + y = 6$ as $y = 6 - 3x$, and draw the graph of this linear function.



Let $x = 0$ so that $y = 6$. Let $y = 0$, so that $x = 2$

Any pair of numbers (x,y) that satisfies the equation $y = 6 - 3x$ falls on the line AB.

Therefore, if y is to be less than $6 - 3x$ for the same value of x, it must assume a value that is less than the ordinate of length $6 - 3x$.



All such points (x,y) for which the ordinate is less than $6 - 3x$ lie below the line AB.

The region where these points fall is indicated by an arrow and is shaded too in the diagram above.

Now we consider two inequalities

$3x + y < 6$ and $x - y < -2$

being satisfied simultaneously by x and y.

The pairs of numbers (x, y) that satisfy both the inequalities may be found by drawing the graphs of the two lines $y = 6 - 3x$ and $y = 2 + x$, and determining the region where both the inequalities hold. It is convenient to express each equality with y on the left-side and the remaining terms in the right-side.

The first inequality $3x + y < 6$ is equivalent to

$$y < 6 - 3x$$

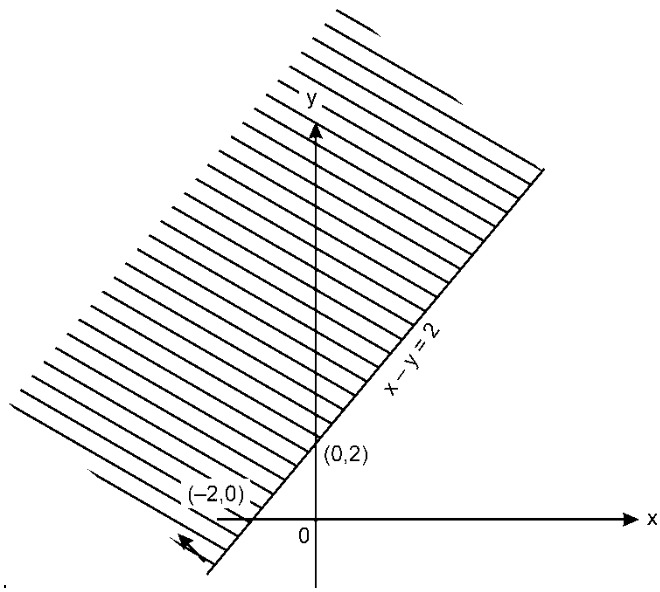
and requires the value of y for each x to be less than that of $y = 6 - 3x$. This inequality is therefore satisfied by all points lying below the line $y = 6 - 3x$. The region where these points fall has been shaded in the previous diagram.

We consider the second inequality $x - y < -2$, and note that this is equivalent to $y > 2 + x$.

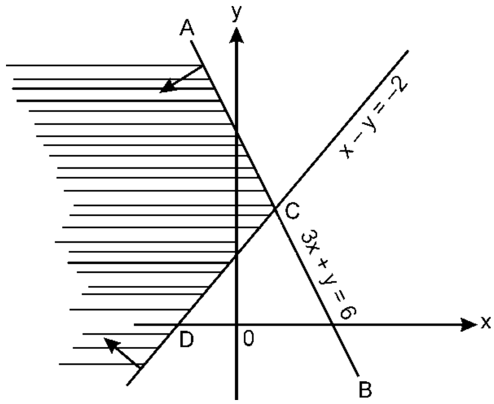
It requires the value of y for each x to be larger than that of $y = 2 + x$. This inequality is, therefore, satisfied by all points lying above the line $y = 2 + x$.

The region is indicated by an arrow and is shaded too in the diagram given below:

Let x = 0 so that y = 2; Let y = 0 so that x = –2.



By superimposing the above two graphs we determine the common region ACD in which the pairs *(x,y)* satisfy both the inequalities.



Note: [1] The inequalities $3x + y \leq 6$ and $x - y \leq 2$ differ from the preceding ones in that these also include equality signs. It means that the points lying on the corresponding lines are also included in the region.

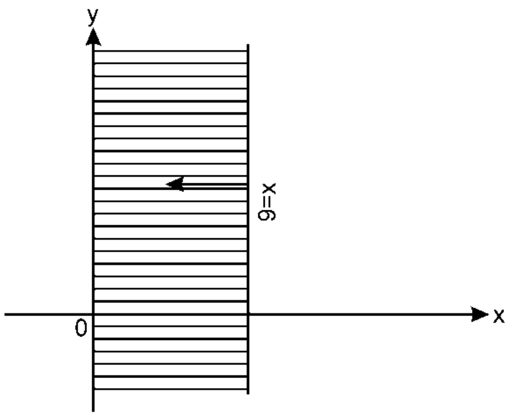
[2] The procedure may be extended to any number of inequalities.

Graphical solution for linear programming problem: We consider the problem of maximizing a linear function Z = $x + 2y$ subject to the restrictions

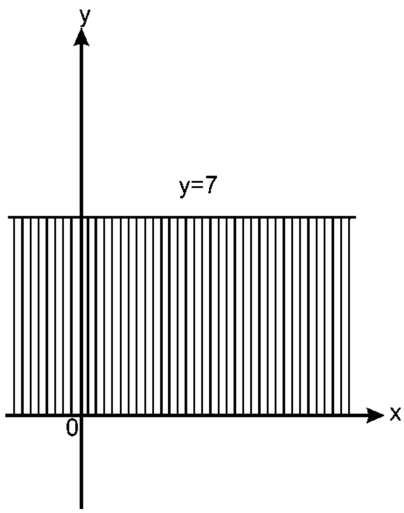
$x \geq 0, y \geq 0, x \leq 6, y \leq 7, x + y \leq 12$

We note that the above inequalities may be grouped as follows:

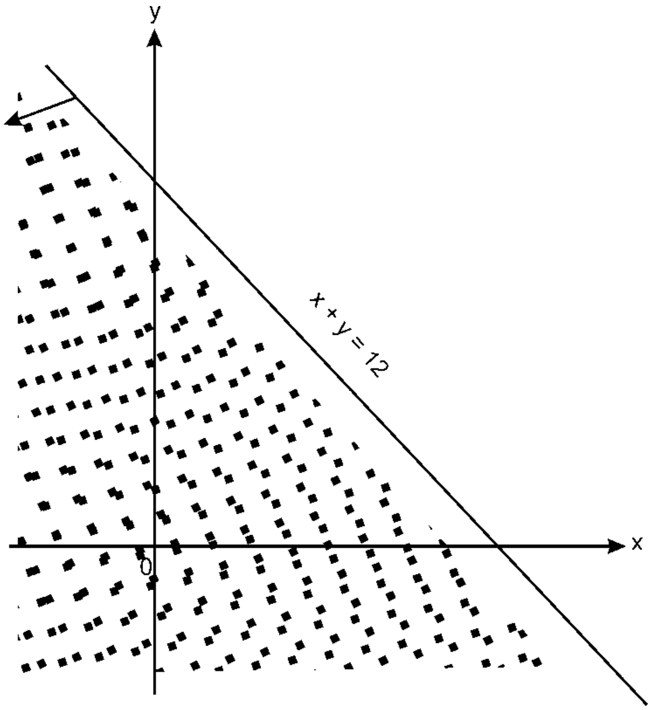
$\begin{Bmatrix} x \geq 0 \\ x \leq 6 \end{Bmatrix}$ $\begin{Bmatrix} y \geq 0 \\ y \leq 7 \end{Bmatrix}$ $\{x + y \leq 12\}$



$x \geq 0, x \leq 6$

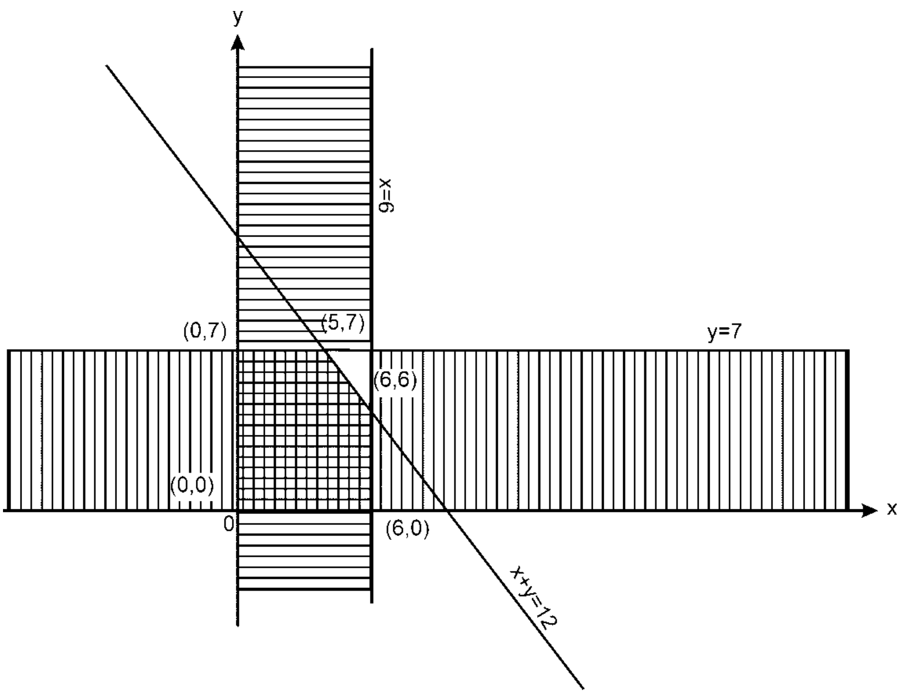


$y \geq 0, y \leq 7$



x + y ≤ 12

By superimposing the above three diagrams, we determine the common region in the *xy* plane where all the five inequalities are simultaneously satisfied.



This common region is known as feasible region or the solution set (or the polygonal convex sets).

A region is said to be *bounded* if it can be totally included within a (very large) circle. The shaded region enclosed by deep lines in the previous diagram is bounded, since it can be included within a circle.

The objective function attains a maximum or a minimum value at one of the corner points of the feasible solution known as extreme points of the solution set. Once these extreme points (the points of intersection of lines bounding the region) are known, a compact matrix representation of these points is possible. We shall denote the matrix of the extreme points by E.

The coefficients of the objective function may also be represented by a column vector. We shall represent this column vector by C.

The elements in the product matrix EC shows different values, which the objective function attains at the various extreme points. The largest and the smallest elements in matrix EC are respectively the maximum and the minimum values of the objective function. The row in matrix EC in which this happens is noted and the elements in that row indicate the appropriate pairing and are known as the *optimal solution.*

In the context of the problem under consideration.

$$E = \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & 7 \\ 5 & 7 \\ 6 & 0 \\ 6 & 6 \end{bmatrix} \end{array}, C = \begin{array}{c} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{array} \begin{array}{c} x \\ y \end{array}$$

$$EC = \begin{bmatrix} 0 & 0 \\ 0 & 7 \\ 5 & 7 \\ 6 & 0 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0\times1+0\times2 \\ 0\times1+7\times2 \\ 5\times1+7\times2 \\ 6\times1+0\times2 \\ 6\times1+6\times2 \end{bmatrix} = \begin{bmatrix} 0 \\ 14 \\ 19 \\ 6 \\ 18 \end{bmatrix}$$

The given objective function viz. Z = x + 2y is maximum at the points (5, 7) present in the third row of the matrix E. Thus the optimal solution is $x = 5$, $y = 7$, and the maximum value of the objective function is 19.

We now list the steps to be followed under graphical solution to a linear programming problem.

Step 1. Determine the region that satisfies the set of given inequalities.

Step 2. Ensure that the region is bounded*. If the region is not bounded, either there are

* It is inconceivable for a practical problem to have an unbounded solution.

additional hidden conditions which can be used to bound the region or there is no solution to the problem.

Step 3. Construct the matrix E of the extreme points, and the column vector C of the objective function.

Step 4. Find the matrix product EC. For maximization, determine the row in EC where the largest element appears; while for minimization, determine the row in EC where the smallest element appears.

Step 5. The objective function is optimized corresponding to the same row elements of the extreme point matrix E.

Note: If the slope of the objective function be same as that of one side of feasible region, there are multiple solutions to the problem. However, the optimized value of the objective function remains the same.

Illustration 2

A manufacturer produces two products A and B, and has his machines in operation for 24 hours a day. Production of A requires 2 hours of processing in machine M_1 and 6 hours in machine M_2. Production of B requires 6 hours of processing in machine M_1 and 2 hours in machine M_2. The manufacturer earns a profit of ₹ 5 on each unit of A and ₹ 2 on each unit of B. How many units of each product should be produced in a day in order to achieve maximum profit?

Solution

Let x_1 be the number of units of type A product to be produced, and x_2 is that of type B product to be produced. The formulation of the L.P.P. in this case is as below:

Maximize $Z = 5x_1 + 2x_2$

subject to the constraints.

$2x_1 + 6x_2 < 24$

$6x_1 + 2x_2 < 24$

$x_1 \geq 0,\ x_2 \geq 0$

For the line $2x_1 + 6x_2 = 24$

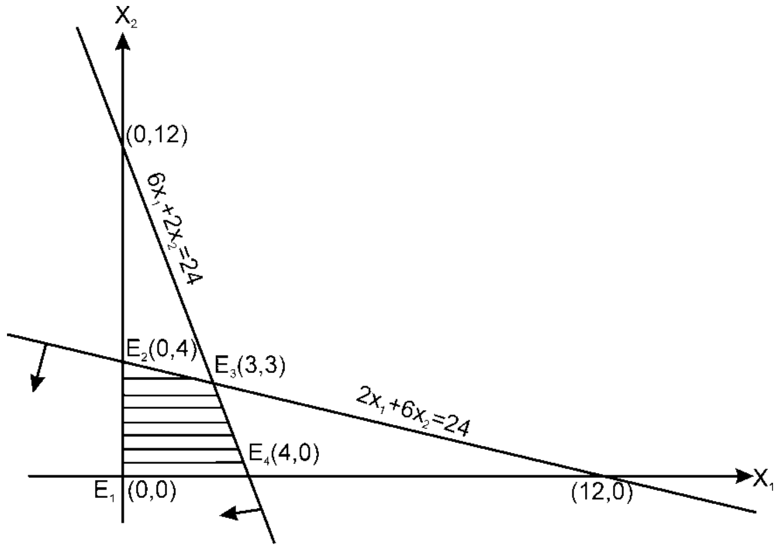
let $x_1 = 0$, so that $x_2 = 4$

let $x_2 = 0$, so that $x_1 = 12$

For the line $6x_1 + 2x_2 = 24$

let $x_1 = 0$, so that $x_2 = 12$

let $x_2 = 0$, so that $x_1 = 4$



The shaded portion in the diagram is the feasible region and the matrix of the extreme points E_1, E_2, E_3 and E_4 is

$$E = \begin{array}{c c} x_1 & x_2 \end{array}$$
$$E = \begin{bmatrix} 0 & 0 \\ 0 & 4 \\ 3 & 3 \\ 4 & 0 \end{bmatrix} \begin{array}{l} E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}$$

The column vector for the objective function is C = $\begin{bmatrix} 5 \\ 2 \end{bmatrix} \begin{array}{l} x_1 \\ x_2 \end{array}$

The column vector the values of the objective function is given by

$$EC = \begin{bmatrix} 0 & 0 \\ 0 & 4 \\ 3 & 3 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0\times5+0\times2 \\ 0\times5+4\times2 \\ 3\times5+3\times2 \\ 4\times5+0\times2 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 21 \\ 20 \end{bmatrix} \begin{array}{l} E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}$$

Since 21 is the largest element in matrix EC, therefore the maximum value is reached at the extreme point E3 whose coordinates are (3,3).

Thus, to achieve maximum profit the manufacturer should produce 3 units each of both the products A and B.

Summary of Graphical Method

It involves:

(i) Formulating the linear programming problem, i.e. expressing the objective function and constraints in the standardised format.

(ii) Plotting the capacity constraints on the graph paper. For this purpose normally two terminal points are required. This is done by presuming simultaneously that one of the constraints is zero. When constraints concerns only one factor, then line will have only one origin point and it will run parallel to the other axis.

(iii) Identifying feasible region and coordinates of corner points. Mostly it is done by breading the graph, but a point can be identified by solving simultaneous equation relating to two lines which intersect to form a point on graph.

(iv) Testing the corner point which gives maximum profit. For this purpose the coordinates relating to the corner point should put in objectives function and the optimal point should be ascertained.

(v) For decision – making purpose, sometimes, it is required to know whether optimal point leaves some resources unutilized. For this purpose value of coordinates at the optimal point should be put with constraint to find out which constraints are not fully utilized.

10.3 Trial & Error Method of solving Linear Programming Problem

Method of Solving LPP: Trial & Error Method (or Algebraic Approach): Graphical method cannot be used when there are more than 2 variables in a LPP., in such a case we use the simplex method which is highly efficient and versatile as also amenable to further mathematical treatment and offers interesting economic interpretations. However, its underlying concepts are rather lengthy to discuss and the student should patiently go through the following material on the trial and error method to gain a good grasp over the simplex technique.

Slack Variable

Consider the following example:

Example: Maximize $Z = 3x_1 + 4x_2$

Subject to $2x_1 + 3x_2 \leq 16$

$4x_1 + 2x_2 \leq 16$

$x_1, x_2 \geq 0$

The $\leq$ type inequalities can be transformed into equalities by the addition of non negative variables, say s_1 and s_2 (known as slack variables) as below. The slack variables represent idle resources, therefore, they are to be positive or non negative. Further the contribution per unit of a slack variable is always taken as zero in the objective function of a LPP. This is so because profits are not made on unused resources.

$$\begin{Bmatrix} 2x_1 + 3x_2 + s_1 = 16 \\ 4x_1 + 2x_2 + s_2 = 16 \end{Bmatrix} \qquad \text{...(A)}$$

And the objective function may be rewritten as below.

Maximize $Z = 3x_1 + 4x_2 + 0\ s_1 + 0\ s_2$

Surplus Variable

In ≥ type inequalities, we subtract a variable (called the **surplus variable**) to make it an equality. The value of this variable can be interpreted as the excess amount of the resources utilized over and above the given level. The contribution per unit of a surplus variable is also taken as zero in the objective function.

The Linear Programming Theorems: The trial and error and simplex methods are based on the concept of slack variables and theorems described below:

Extreme Point Theorem: It states that an optimal solution to a LPP occurs at one of the vertices of the feasible region. This should be obvious from the discussion on the graphical method.

Now the vertices are defined by the intersection of equations. The first step of the method is, therefore, to convert the inequalities into equalities by the addition (or subtraction) of the slack (or surplus variables) depending on the direction of the inequality.

It is to be noted that the system of equations (A) above has more variables than the number of equations. Such a system of equations has an infinite number of solutions; yet it has finite and few vertices, the co-ordinates of which can be determined by applying the Basis theorem.

Basis Theorem: It states that for a system of m equations in n variables (where $n > m$) has a solution in which at least $(n\text{-}m)$ of the variables have value of zero as a vertex. This solution is called a basic solution.

Extreme point theorem can be extended to state that the objective function is optimal at least at one of the basic solutions. Some of the vertices may be infeasible in that they have -ve co-ordinates and have to be dropped in view of the non-negativity condition on all variables including the slack and surplus variables.

Consider the following LPP for elucidating the basis theorem.

Maximize $Z = 3x_1 + 4x_2$

Subject to $2x_1 + 3x_2 \leq 16$

$4x_1 + 2x_2 \leq 16$

$x_1, x_2 \geq 0.$

Introducing slack variables s_1 and s_2, we have

Maximize $Z = 3x_1 + 4x_2 + 0\ s_1 + 0\ s_2$

Subject to $2x_1 + 3x_2 + 1\ s_1 + 0\ s_2 = 16$

$4x_1 + 2x_2 + 0\ s_1 + 1\ s_2 = 16$ ... (B)

$x_1, x_2, s_1, s_2 \geq 0$

Here n (number of variables) = 4 and m (number of equations) = 2. Thus $n - m$ = 2. According to the Basis theorem we set 2 (= $n - m$) variables in (B) equal to zero at a time, solve the resulting system of equations and obtain a basic solution. Thus if we zeroise x_1 and x_2, the resulting system of equations would be

$$\begin{Bmatrix} 1s_1 + 0s_2 = 16 \\ 0s_1 + 1s_2 = 16 \end{Bmatrix} \quad \text{......(C)} \quad \text{Set 1 } (x_1 = x_2 = 0)$$

These equations directly yield s_1 = 16 and s_2 = 16 as the basic solution i.e. the co-ordinate of a vertex.

The other sets of equations, upon zeroising two variables at a time in (B), would be as follows:

$$\begin{Bmatrix} 2x_1 + 3x_2 = 16 \\ 4x_1 + 2x_2 = 16 \end{Bmatrix} \quad \text{Set 2 } (s_1 = s_2 = 0)$$

$$\begin{Bmatrix} 2x_1 + 0s_2 = 16 \\ 4x_1 + 1s_2 = 16 \end{Bmatrix} \quad \text{Set 3 } (x_2 = s_1 = 0)$$

$$\begin{Bmatrix} 2x_1 + 1s_1 = 16 \\ 4x_1 + 0s_1 = 16 \end{Bmatrix} \quad \text{Set 4 } (x_2 = s_2 = 0)$$

$$\begin{Bmatrix} 3x_2 + 1s_1 = 16 \\ 2x_2 + 0s_1 = 16 \end{Bmatrix} \quad \text{Set 5 } (x_1 = s_2 = 0)$$

$$\begin{Bmatrix} 3x_2 + 0s_2 = 16 \\ 2x_2 + 1s_2 = 16 \end{Bmatrix} \quad \text{Set 6 } (x_1 = s_1 = 0)$$

It is a simple matter to solve these six sets of simultaneous equations and obtain the six basic solutions *i.e.* co-ordinates of the six vertices of the feasible region. The solutions are given below. The student may verify these by solving each of the sets of simultaneous equations.

Set	Solution
1	s_1 =16, s_2 = 16
2	x_1 = 2, x_2 = 4
3	x_1 = 8, s_2 = –16
4	x_1 = 4, s_1 = 8
5	x_2 = 8, s_1 = –8
6	x_2 = 16/3, s_2 = 16/3

Since solutions of set 3 and 5 yield a negative co-ordinate each, contradicting thereby the non-negativity of constraints, thus these are infeasible and are to be dropped from consideration.

Now according to the basis theorem the optimal solution lies at one of these vertices. By substituting these co-ordinates the values of objective function are derived below:

Set	Solution	Z (Profit)
1	$s_1 = 16$, $s_2 = 16$	0
2	$x_1 = 2$, $x_2 = 4$	22
3	Infeasible	
4	$x_1 = 4$, $s_1 = 8$	12
5	Infeasible	
6	$x_2 = 16/3$, $s_2 = 16/3$	$21\frac{1}{3}$

Thus solution of set 2 is optimal with a profit of 22. In this way we can solve a LPP simply by employing the theorems stated above; but the simplex method is a further improvement over the trial and error method. Following three are the inefficiencies in the trial and error method.

Inefficiencies of Trial and Error Method:

(a) In a LPP where *m* and *n* are larger, solving of numerous sets of simultaneous equations would be extremely cumbersome and time-consuming.

(b) Scanning the profit table we notice that we jump from profit 0 to 22 to $21^1/_3$ *i.e.* there are ups and downs. The simplex method ensures that successive solutions yield progressively higher profits, culminating into the optimal one.

(c) Some of the sets yield infeasible solutions. There should be means to detect such sets and not to solve them at all.

10.4 The Simplex Method for Maximization Problems

The simplex method is a computational procedure - *an algorithm* - for solving linear programming problems. It is an iterative optimizing technique. In the *simplex process, we must first find an initial basis solution (extreme point). We then proceed to an adjacent extreme point. We continue moving from point to point until we reach an optimal solution.* For a maximization problem, the simplex method always moves in the direction of steepest ascent, thus ensuring that the *value of the objective function improves with each solution.*

Simplex Algoritham

Maximize

$Z = c_1 x_1 + c_2 x_2 + \ldots\ldots\ldots\ldots\ldots\ldots + c_n x_n$

Subject to the constraints

$k_{11}x_1 + k_{12}x_2 + \ldots\ldots\ldots. + k_{1n}x_n \leq b_1$

$k_{21}x_1 + k_{22}x_2 + \ldots\ldots\ldots. + k_{2n}x_n \leq b_2$

$k_{m1}x_1 + k_{m2}x_2 + \ldots\ldots\ldots. + k_{mn}x_n \leq b_m$

$x_1, x_2, \ldots x_n \geq 0$

- x_1, x_2……………x_n are the **'Decision Variables'**.
- c_j (j = 1, 2,……., n) in the objective function are called the **'Profit or Cost coefficients'**.
- b_i (i = 1, 2 …….m) are called **'Resources'**.
- Constants k_{ij} (i = 1,2,……,m; j = 1, 2,…..n) are called **'Structural Coefficients'**.

- **An inequality of the "≤" type is changed into an equality:**

  By the addition of a non-negative slack variable. By adding a suitable positive quantity s_i to the left hand side, the inequality constraint can be written as:

  $k_{i1}x_1 + k_{i2}x_2 + \ldots\ldots + k_{in}x_n + s_i = b_i$ (i = 1, 2, …..m)

- **An inequality of the "≥" type is changed into an equality:**

  By the subtraction of a non negative surplus variable. By subtracting positive quantity s_i from the left hand side, the inequality constraint can, be written as:

  $k_{i1}x_1 + k_{i2}x_2 + \ldots\ldots + k_{in}x_n - s_i = b_i$ (i = 1, 2, …..m)

Solution Steps:

$Z = c_1x_1 + c_2x_2 + \ldots\ldots\ldots\ldots\ldots\ldots c_nx_n + 0s_1 + 0s_2 + \ldots\ldots\ldots\ldots\ldots + 0s_m$ (Objective function)

Subject to the constraints

$k_{11}x_1 + k_{12}x_2 + \ldots\ldots\ldots\ldots + k_{1n}x_n + s_1 = b_1$

$k_{21}x_1 + k_{22}x_2 + \ldots\ldots\ldots\ldots + k_{2n}x_n + s_2 = b_2$

$k_{m1}x_1 + k_{m2}x_2 + \ldots\ldots\ldots\ldots + k_{mn}x_n + s_m = b_m$

$x_1, x_2, \ldots x_n, s_1, s_2, \ldots s_m \geq 0$

- Note that *slack variables have been allocated zero coefficients in the objective function* as these variables typically contribute nil amount to the value of objective function.

INITIAL SIMPLEX TABLEAU

C_j (Contribution per Unit)			c_1	c_2	…..	c_n	0	0	…..	0	Minimum Ratio X_B / k_{ij}
C_B	Basic Variables (B)	Value of Basic Variables $b=(X_B)$	x_1	x_2	…..	x_n	s_1	s_2	…..	s_m	
			Coefficient Matrix				Identity Matrix				
C_{B_1}	s_1	$b_1 = x_{B_1}$	k_{11}	k_{12}	…..	k_{1n}	1	0	…..	0	
C_{B_2}	s_2	$b_2 = x_{B_2}$	k_{21}	k_{22}	…..	k_{2n}	0	1	…..	0	
-	-	-	-	-	…..	-	-	-	…..	-	
-	-	-	-	-	…..	-	-	-	…..	-	
C_{B_m}	s_m	$b_m = x_{B_m}$	k_{m1}	k_{m2}	…..	k_{mn}	0	0	…..	1	
$Z_j = \sum C_{Bi}X_j$			0	0	…..	0	0	0	…..	0	
Net Contribution *per unit* ($C_j - Z_j$)			$c_1 - z_1$	$c_2 - z_2$	…..	$c_n - z_n$	0	0	…..	0	

- **In the first row 'C_j'** we write the coefficients of the variables in the objective function (c_1, c_2,c_3 …….,c_n, 0,0…..0). These value will remain unchanged in the subsequent tableaus.
- **In the first column labeled 'C_B'**, we list the coefficients of the current basic variables in the objective function.
- **In the second column 'Basic Variables'** we place the basic variables (s_1, s_2, s_3 …..s_m)
- **'The Coefficient Matrix' (under x_1,x_2……x_n)** in the tableau represents the coefficients of the decision variables in the constraints set.
- **'The Identity Matrix' (under s_1, s_2…….s_m)** in the initial simplex tableau represents the coefficients of the slack variables in the constraint set.
- **To get Z_j row under a column**, we multiply the entries of that column by the corresponding entries of C_B column and add the product.
- The last row 'C_j – Z_j' called the 'Index Row or Net Evaluation Row'.
- *If all the elements or entries in the C_j – Z_j row are Negative or Zero then the* **current solution is optimum**.
- **If current solution is not optimum**, it can be further enhanced by eliminating one basic variable and replacing it by some non-basic one. For this
  - ✓ We now decide the **variable to enter** the solution mix. *Column with largest positive entry in the C_j – Z_j row is called* **'key Column'** *(indicated by ↑)*. The non-basic variable at the top of the column is the entering variable that will replace a basic variable.
  - ✓ Next step is to **determine 'leaving variable'** to be replaced. This can be trace by dividing each number in the X_B column by the corresponding number in key column. We compute ratio (b_1/k_{1j}, b_2/k_{2j} ….. b_m/k_{mj}). The row corresponding to the minimum of these ratios is key row (indicated by ⟵). Corresponding variable in the key row known as the leaving (departing) variable.
  - ✓ **'Key Element'** is the number that lies at the junction of the key column and key row.
  - ✓ Constructing Second Tableau.
    - ▪ **New values for the key row** are computed by simply dividing every element of key row by key element.
    - ▪ **New values of the elements in the remaining rows** for the new table can be obtained by performing elementary row operation on all rows so that all elements except the key element in the key column are zero. For each row other than the key row, we use following formula.

      New row number = Number in old rows-{(corresponding number in key row) × (Corresponding fixed ratio)}

      $$\text{Corresponding fixed ratio} = \frac{\text{Old row number in key column}}{\text{Key number}}$$

    Compute Z_j and C_j – Z_j rows. If all the numbers in C_j – Z_j rows are either negative or Zero, an optimum solution has been obtained. *If any of the numbers in C_j – Z_j row is positive repeat the steps as explained above until an optimum solution has been obtained.*

The process is illustrated with the help of the following example:

Illustration 3

Maximize $Z = 3x_1 + 4x_2$

Subject to $2x_1 + 3x_2 \leq 16$ *(machining time)*

$4x_1 + 2x_2 \leq 16$ *(assembly time)*

$x_1 \geq 0,\ x_2 \geq 0$

Solution

Step 1. *Obtaining an initial solution:* Obtain an initial solution that satisfies all the constraints of the problem. The simplex process requires that all constraints be expressed as equations. Therefore we must convert all inequalities into equalities.

Consider the constraint $2x_1 + 3x_2 \leq 16$

Value on the left hand side of the inequality represents the amount of machine time a particular solution uses, while the quantity on right side of inequality sign represents the total amount of machine time available. Let s_1 be a variable which represents the unused machine time in this solution so that

$$2x_1 + 3x_2 + s_1 = 16$$

Similarly, let s_2 represent the amount of assembly time that is available but not used so that

$$4x_1 + 2x_2 + s_2 = 16$$

The variables s_1 and s_2 are referred to as slack variables.

Thus slack variables represent the quantity of a resource not used by a particular solution, and they are necessary to convert the constraint inequalities to equalities.

As we proceed with the *simplex method,* it will be helpful to place certain information in a table known as a *simplex tableau.*

The simplex solution is as follows.

SIMPLEX TABLEAU-I

C_j (Contribution per Unit)				**3**	**4**	**0**	**0**	**Replacement Ratio***
Fixed Ratio	**Basic Variables (B)**	**Profit per Unit (C_B)**	**Qty [Value of Basic Variables b(=X_B)]**	**Coefficient Matrix**		**Identity Matrix**		
				x_1	**x_2**	**s_1**	**s_2**	
	s_1	0	16	2	3	1	0	← 16/3
2/3	s_2	0	16	4	2	0	1	8
			$Z_j = \sum C_{Bi}X_j$	0	0	0	0	
Net Contribution *per unit* ($C_j - Z_j$)				3	4 ↑	0	0	

*Minimum ratio

SIMPLEX TABLEAU-II

C_j (Contribution per Unit)				3	4	0	0	Replacement Ratio
Fixed Ratio	Basic Variables (B)	Profit per Unit (C_B)	Qty [Value of Basic Variables b(=X_B)	Coefficient Matrix		Identity Matrix		
				x_1	x_2	s_1	s_2	
1/4	x_2	4	16/3	2/3	1	1/3	0	8
	s_2	0	16/3	8/3	0	-2/3	1	← 2
			$Z_j = \sum C_{Bi}X_j$	8/3	4	4/3	0	
Net Contribution *per unit* ($C_j - Z_j$)				1/3 ↑	0	-4/3	0	

SIMPLEX TABLEAU-III

C_j (Contribution per Unit)				3	4	0	0	Replacement Ratio
Fixed Ratio	Basic Variables (B)	Profit per Unit (C_B)	Qty [Value of Basic Variables b(=X_B)	Coefficient Matrix		Identity Matrix		
				x_1	x_2	s_1	s_2	
	x_2	4	4	0	1	1/ 2	-1/4	
	x_1	3	2	1	0	-1/4	3/8	
			$Z_j = \sum C_{Bi}X_j$	3	4	5/4	1/8	
Net Contribution *per unit* ($C_j - Z_j$)				0	0	-5/4	-1/8	

The main simplex table, aside from headings, is seen divided into 3 horizontal sub-tables, I to III. There are 6 main headings 1 to 6 as explained below for sub-table I.

Col. 1. Fixed ratio—explanation deferred.

Col. 2. Program [Basic Variables (B)]—it contains the basic variables that are included in the solution. Initially, in sub-table 1, there are slack variables, this means we do not produce x_1 and x_2 products, thus the profit is zero.

Col. 3. In the column labeled 'Profit' are listed the objective co-efficients of the basic variables that are included in the specific program. Thus, the profit co-efficients of s_1 and s_2 are all zero [C_B].

Col. 4. In the column labelled 'QTY' are listed the values of the basic variables b(=X_B) included in the solution. Since our initial program consists of 16 units of s_1, and 16 units of s_2, these values are listed in 'QTY' column. Any variable which is not listed under the 'Program' column is known as the *non-basic variable,* the value of each non-basic variable is zero.

The total profit contribution resulting from a specific program can be calculated by multiplying corresponding entries in the "profit per unit" column and the "quantity" column and adding the product. Thus, total profit contribution in our first program is 16(0) + 16(0) i.e. zero.

Col. 5. This heading is divided in as many sub-headings as the number of real and slack variables. It indicates each of the variables with profit/unit as read from the objective function at its top. The body of the sub-table carries the co-efficients of the variables as read from the constraints. *That the co-efficients of the basic variables constitute a unit matrix in this initial sub-table is to be noted.* The numbers under the non basic variables represent substitution ratio. For example, the number 2 gives the rate of substitution between x_1, and s_1. If we wish to produce 1 unit of x_1, 2 unit s_1, must be sacrificed. The number 4 has similar interpretation. To produce 1 unit of x_1, 4 units of s_2 must be sacrificed. By using the same explanation, to produce 1 unit of x_2, we must sacrifice 3 units of s_1 and 2 units of s_2.

Col. 6. Replacement ratio as discussed earlier.

The numbers in the *net evaluation* row (NER) $C_j – Z_j$ give the net effect of exchange between one unit of each variable and basic variable. They are always zero under the basic variables. Under the non-basic variables, they can be positive, negative or zero. *Thus numbers in NER, under each column represent the opportunity cost of not having one unit of the respective column variable in the solution, in other words, the number represent the potential improvement in the objective function that will result by introducing into the program one unit of the respective column variable.*

Test for Optimality: In so far as the total profit contribution (obtained by multiplying corresponding entries in the 'Profit' column and 'QTY' column and adding the product) resulting from the initial program is zero, it can be improved and hence is not optimal program. **In maximization problem, the program is optimal if each $C_j – Z_j$, is either zero or negative.**

Computational aspects for the derivation of improved solutions are explained step by step below.

Step 1. *Select the incoming variable* in sub-table I in the NER. The NER entries in the first sub-table are simply the profit unit figures as read from the objective function or copied from headings 5. *However there is a more rigorous method of making these entries which alone has to be followed in making entries in the subsequent NER's and explanation on which is deferred.* Selection of the incoming variable is simple indeed. **The positive ($C_j – Z_j$) in NER indicate the magnitude of opportunity cost of not including 1 unit of variables x_1 and x_2 respectively at this stage. We select the one with highest entry in NER as incoming variable** and this happens to be x_2, with a value of 4. We note this selection by putting an arrow below 4 in the NER of sub-table I. The column with the arrow is known as the **key column.**

Step 2. *Select the outgoing variable*, for this we have to calculate how many units of x_2 can be brought in without exceeding the existing capacity of any one of the resources? Thus, we must calculate the maximum allowable number of units of x_2, that can be brought into the program without violating the non-negative constraints. For this, we compute the replacement ratio of col. 6. This is done by dividing col. 4 by the key column and we get 16/3 and 8 and as the ratio. The variable s_2, against the *least ratio* of 16/3 is selected as the outgoing variable and the fact is noted by putting an arrow against the least ratio of 16/3. This is maximum quantity of x_2 that can be produced at this stage without violating the non negative constraint. This row with the arrow is called the **key row.** The element at the intersection of the key row and the key column is known as the **pivot** or key element and it is encircled.

Step 3. Having noted the incoming variable (x_2) and the outgoing variable (s_1) we are ready to perform the row operations on sub-table-I and fill in columns 4 and 5 of sub-table II.

Before that however, we fill in cols. 2 and 3 of sub-table II which is straight forward; s_1 is replaced by x_2 in col. 2, s_2 stays in it and in col. 3 the profit/unit figure corresponding to x_2 is copied from heading while other figures remain the same.

Transformation of key row : The rule for transforming the key row is: Divide all the numbers in the key row by the key number. The resulting numbers form the corresponding row in the next table.

The key row of sub-table-I (under col. 4 and 5) is divided by the pivot element and this becomes the corresponding row of the sub-table-II. It reads 16/3, 2/3, 1, 1/3, 0

Transformation of the non-key rows : The rule for transforming a non-key row is: Subtract from the old row number (in each column) the product of the corresponding key-row number and the corresponding fixed ratio formed by dividing the old row number in the key column by the key number. The result will give the corresponding new row number.

This rule can be placed in the following equation form:

$$\text{Fixed ratio} = \frac{\text{Old row number in key column}}{\text{Key number}}$$

These are entered in the non key row under column I in the sub-table-I itself.

$$\begin{aligned}\text{New row number} &= \text{old row number} \\ &\quad - (\text{corresponding number in key row} \times \text{corresponding fixed ratio})\end{aligned}$$

The key row in sub-table-I is multiplied by the fixed ratio of the non key row. This leads to

$$16\times\frac{2}{3}=\frac{32}{3},\ \ 2\times\frac{2}{3}=\frac{4}{3},\ \ 3\times\frac{2}{3}=2,\ \ 1\times\frac{2}{3}=\frac{2}{3},\ \ \ 0\times\frac{2}{3}=0,$$

This, however, is a rough work. Entries are not made any where. The result of this multiplication are subtracted from the non-key row of sub-table-I to yield the non key row of the sub-table-II as below:

$$16-\frac{32}{3}=\frac{16}{3},\ \ 4-\frac{4}{3}=\frac{8}{3},\ \ 2-2=0,\ \ \ 0-\frac{2}{3}=-\frac{2}{3},\ \ 1-0=1$$

These entries are made for the non-key row of sub-table-II.

Step 4. This consists of deriving the NER of sub-table-II. Each of its elements is derived by multiplying col. 3 of sub-table-II with col. 5, summing these up and subtracting the sum from the profit/unit is heading 5. Computations for each element in the 2nd NER are shown below:

$C_j - Z_j =$

$$3-\left(4\times\frac{2}{3}+0\times\frac{8}{3}\right)=\frac{1}{3} \qquad 4-(4\times 1+0\times 0)=0$$

$$0-\left(4\times\frac{1}{3}+0\times\frac{-2}{3}\right)=-\frac{4}{3} \qquad 0-(4\times 0+0\times 1)=0$$

By applying these 4 steps on sub-table-II we derive sub-table-III. Since its NER does not have any positive element *i.e.* opportunity cost, we stop here. This is an indication of the optimal solution with x_1=2 and x_2 = 4 (and Z = 22), for if we derive another sub-table by bringing in x_3 or x_4 into the solution we shall decrease the profit. To sum up, when, in a maximization problem, all the elements in an NER are negative or zero we stop further iterations. The student may himself derive sub-table-III as an exercise.

It is obvious that the student needs a plenty of practice to be thorough with the computationally tedious though otherwise very elegant and versatile simplex method. *As a tip*, the *fractions in the simplex tables must be retained and by trying to decimalise them the student may land himself in trouble.* Numerous examples and exercises follow to give understanding and practice to the students.

10.5 Simplex Method for Minimization Problems

The simplex algorithm applies to both maximization and minimization problems. The only difference in the algorithm involves the selection of the incoming variable. In the maximization problem it is the one with highest +ve NER element. Conversely, it is the most –ve variable that is selected as the incoming variable in a minimization problem. And if all elements in the NER are either positive or zero, it is the indication for the optimal solution and we stop there.

Let us take an example straightway to explain the solution procedure.

Illustration 4

A small township of 15,000 people requires, on the average, 300,000 gallons of water daily. The city is supplied water from a central water-works where the water is purified by such conventional methods as filteration and chlorination. In addition, two different chemical compounds: (i) softening chemical and (ii) health chemical are needed for softening the water and for health purposes. The waterworks plans to purchase two popular brands that contain these chemicals. One unit of Chemico Corporation's product gives 8 pounds of softening chemical and 3 pounds of health chemical. One unit of Indian Chemical's product contains 4 pounds and 9 pounds per unit, respectively, for the same purposes.

To maintain the water at a minimum level of softness and to meet a minimum programme of health protection, experts have decided that 150 and 100 pounds of the two chemicals that make up each product must be added to water daily. At a cost of ₹ 8 and ₹ 10 per unit respectively for Chemico's and Indian Chemical's products, what is the optimal quantity of each product that should be used to meet the minimum level of softness and minimum health standard?

Solution

The relevant data may be tabulated as below:

	Brand		
	Chemico	**Indian**	**Daily Requirement**
Chemical			
(i) Softening	8	4	150
(ii) Health	3	9	100
Cost/Unit of each brand (₹)	8	10	

Let us formulate the LPP as below

Minimize (cost), $Z = 8x + 10y$

Subject to

$$8x + 4y \geq 150$$
$$3x + 9y \geq 100$$

and $x \geq 0, y \geq 0$

Here x and y represents units of softening and health chemical units respectively.

Subtracting surplus variables to convert inequalities into equalities.

$$8x + 4y - s_1 = 150$$
$$3x + 9y - s_2 = 100$$

Physical Interpretation of Surplus Variables: Let s_1 and s_2 represent the extra units (if any) of the two chemicals over 150 and 100 units respectively. Their magnitude being such as to just convert the inequalities into equalities, thereby restricting them to non-negative value. The problem may then be restated as below:

Minimise (cost) $Z = 8x + 10y + 0s_1 + 0s_2$

Subject to $8x + 4y - s_1 = 150$

$3x + 9y - s_2 = 100$

and $s_1 \geq 0,\ s_2 \geq 0,\ x \geq 0,\ y \geq 0.$

Artificial Slack Variables : If x and y are set equal to zero s_1 and s_2 turn out to be negative, violating the non-negativity restriction. Therefore, to circumvent this, we introduce another similar device of artificial slack variables. Let us represent these by A_1 and A_2 respectively. The problem then becomes :

Minimize cost Z = $8x + 10y + 0s_1 + 0s_2 + MA_1 + MA_2$

$$\text{Subject to } \left\{\begin{array}{l} 8x + 4y - s_1 + A_1 = 150 \\ 3x + 9y - s_2 + A_2 = 100 \end{array}\right\} \text{All variables} \geq 0$$

Physical Interpretation of the Artificial Variables.

These are imaginary brands, each unit containing 1 unit of the pertinent chemical. Both are restricted to non-negatives. Whereas surplus variables have zeros as their cost co-efficient, each artificial variable is assigned **an infinitely large cost co-efficient (usually denoted by M).**

Uses of artificial variables

1. It is our basic assumption that none of the basic variables in the L.P. problem can have a negative value. Thus, artificial variable is added to act as basic variable in a particular equation and hence it avoids the possibility of getting negative values for basic variables.
2. These artificial variables are such that their objective function co-efficients impose a huge and hence unacceptable penalty. In the case of maximization problem, the objective function is modified by subtracting a quantity (MA_1) where **M is arbitrarily large value** and A_1 is the artificial variable. For minimization problem, the objective function is modified by adding the quantity MA_1. Thus artificial variables enable us to make a convenient and correct start in obtaining an initial solution.
3. The artificial variables with the high penalty once replaced by a real variable will never enter the optimal program. Hence the solution to the modified problem will give the optimal solution to the original problem.

Designing the Initial Program : It is obtained by letting each of the variables *x,y,* s_1 and s_2 assume value of zero, *i.e.,* retaining artificial variables in the initial solution.

SIMPLEX TABLEAU-I

C_j				8	10	0	0	M	M	Replacement Ratio
Fixed Ratio	Basic Variables (B)	C_B	Value of Basic Variables b(=X_B)	x	y	s_1	s_2	A_1	A_2	
4/9	A_1	M	150	8	4	-1	0	1	0	150/4
	A_2	M	100	3	9	0	-1	0	1	← 100/9
			$Z_j = \sum C_{Bi}X_j$	11M	13M	-M	-M	M	M	
			$C_j - Z_j$	8-11M	(10-13M)* ↑	M	M	0	0	

***** Largest negative value

Initial Solution :

$\Delta = C_j - Z_j$

$$\left.\begin{array}{ll} x & 8-(8M+3M)=8-11M \\ y & 10-(4M+9M)=10-13M \\ s_1 & 0-(-M+0)=M \\ s_2 & 0-(0-M)=M \\ A_1 & M-(M+0)=0 \\ A_2 & M-(0+M)=0 \end{array}\right] \quad (A)$$

10–13M is the most – ve and, therefore, the column, under 'y' is the key column. Also, the ratio 100/9 is less than 150/4 (on extreme right). The pivot element, then, is 9. The outgoing variable is A_2 being replaced by *y*.

The key row is revised by dividing it through by 9, the pivot element and the results are as follows:

$\frac{100}{9}$, $\frac{3}{9}$, 1, 0, $\frac{-1}{9}$, 0, $\frac{1}{9}$.

The fixed ratio for revising the non-key row is 4/9. The revised figures for this row are computed below:

$$150 - 100 \times \frac{4}{9} = \frac{1350-400}{9} = \frac{950}{9}$$

$$8 - 3 \times \frac{4}{9} = \frac{72-12}{9} = \frac{60}{9}$$

$$4 - 9 \times \frac{4}{9} = 0$$

$$-1 - 0 = -1$$

$$0 + \frac{4}{9} = \frac{4}{9}$$

$$1 - 0 = 1$$

$$0 - 1 \times \frac{4}{9} = \frac{-4}{9}$$

The new table is shown below:

SIMPLEX TABLEAU-II

C_j				8	10	0	0	M	M	Replacement Ratio
Fixed Ratio	**Basic Variables (B)**	C_B	**Value of Basic Variables $b(=X_B)$**	**x**	**y**	s_1	s_2	A_1	A_2	
3/60	A_1	M	950/9	60/9	0	-1	4/9	1	-4/9	950/60 ←
	y	10	100/9	3/9	1	0	-1/9	0	1/9	100/3
			$Z_j = \sum C_{Bi}X_j$	$\frac{60}{9}M+\frac{30}{9}$	10	-M	$\frac{4}{9}M-\frac{10}{9}$	M	$\frac{-4}{9}M+\frac{10}{9}$	
			$C_j - Z_j$	$\left(\frac{42}{9}-\frac{60M}{9}\right)^*$ ↑	0	M	$\frac{10}{9}-\frac{4M}{9}$	0	$\frac{(13M-10)}{9}$	

* Largest negative value

Revising the key row by dividing it through by $\frac{60}{9}$ *i.e.* multiply by $\frac{9}{60}$, we get

$$\frac{950}{60},\ 1,\ 0,\ \frac{-9}{60},\ \frac{4}{60},\ \frac{9}{60},\ \frac{-4}{60}$$

For the non-key row the fixed ratio is 3/9 ¸ 60/9 = 3/60. Revising the non-key row.

$$\frac{100}{9}-\left[\left(\frac{950}{9}\times\frac{3}{9}\right)\times\frac{9}{60}\right]=\frac{35}{6}$$

$$\frac{3}{9}-\left[\left(\frac{3}{9}\times\frac{60}{9}\right)\times\frac{9}{60}\right]=0$$

$$1-\left[\left(0\times\frac{3}{9}\right)\times\frac{9}{60}\right]=1$$

$$0-\left[\left(-1\times\frac{3}{9}\right)\times\frac{9}{60}\right]=\frac{1}{20}$$

$$\frac{-1}{9}-\left[\left(\frac{4}{9}\times\frac{3}{9}\right)\times\frac{9}{60}\right]=\frac{-2}{15}$$

$$0-\left[\left(-1\times\frac{3}{9}\right)\times\frac{9}{60}\right]=\frac{-1}{20}$$

$$\frac{1}{9}-\left[\left(\frac{-4}{9}\times\frac{3}{9}\right)\times\frac{9}{60}\right]=\frac{2}{15}$$

The result is tabulated below:

SIMPLEX TABLEAU-III

C_j				8	10	0	0	M	M	Replace-ment Ratio
Fixed Ratio	Basic Variables (B)	C_B	Value of Basic Variables $b(=X_B)$	x	y	s_1	s_2	A_1	A_2	
	x	8	95/6	1	0	$\frac{-3}{20}$	$\frac{1}{15}$	$\frac{3}{20}$	$\frac{-1}{15}$	
	y	10	35/6	0	1	$\frac{1}{20}$	$\frac{-2}{15}$	$\frac{-1}{20}$	$\frac{2}{15}$	
			$Z_j = \sum C_{Bi}X_j$	8	10	$\frac{-7}{10}$	$\frac{-4}{5}$	$\frac{7}{10}$	$\frac{4}{5}$	
			$C_j - Z_j$			$\frac{7}{10}$	$\frac{4}{5}$	$M-\frac{7}{10}$	$M-\frac{4}{5}$	

Since all entries in $C_j - Z_j$ now are either zero or positive, the current solution is the optimum on and is given by:

$$x = \frac{95}{6},\ y = \frac{35}{6}$$

Hence the optimal cost is $\frac{95}{6} \times 8 + \frac{35}{6} \times 10$

$$= \frac{760+350}{6} = \frac{1{,}110}{6} = ₹185$$

10.6 Marginal Value of a Resource

Let us consider illustration 3 under section 10.4 once again to explain this concept. In the said example there are two resources machining and assembly. The optimal tableau of that example is reproduced below. We see in its NER –5/4 and –1/8 below s_1 and s_2. The marginal value of the machining and assembly resources are then respectively 5/4 and 1/8. The implication of this is that if now (at the optimal stage) we wish to bring in s_1 in the solution, the total profit will be reduced from 22 (the optimal) by 5/4 times the no. of units of s_1 brought in the program. This is demonstrated below where we bring s_1 in this program.

Optimal table of first illustration 3 under section 10.4 follows (rest omitted) s_1 is brought in the next to optimal table.

				x_1	x_2	s_1	s_2	
	x_2	4	4	0	1	1/ 2	-1/4	8 ←
1/2	x_1	3	2	1	0	-1/4	3/8	- 8
Optimal	NER	22*		0	0	-5/4	-1/8	Bring s_1 in solution
	s_1	0	8	0	2	1	-1/2	
	x_1	3	4	1	1/2	0	1/4	
Optimal	NER	12*	0	0	5/2	0	-3/4	

Please note that marginal value of resource is synonymous with *opportunity cost or shadow price.*

10.7 Some Remarks

1. It may be desired to convert a maximization problem into a minimization one and *vice versa.* Mathematically, this can be accomplished by reversing signs though of just the objective function.
2. *Inequalities in the wrong direction: Consider the problem:*

   Maximize $Z = x_1 + 5x_2$

   Subject to $3x_1 + 4x_2 \leq 6$ ... (i)

   $x_1 + 3x_2 \geq 2$ ... (ii)

   $x_1 , x_2 \geq 0$

   (Whether to introduce slack or surplus or artificial variables depends on the type of inequality and has nothing to do with type of the problem i.e., maximization or minimization)

   The 2nd inequality is in the *wrong* direction. Upon introducing the "surplus" variable.

   $x_1 + 3x_2 - s_2 = 2$

   If s_2 is taken in the initial solution it would be –ve when x_1 and x_2 are zero. To circumvent this, an artificial variable is also introduced in this inequality. The problem becomes:

   Maximize $Z = x_1 + 5x_2 - MA_2 + 0s_1 + 0s_2$

   Subject to $3x_1 + 4x_2 + s_1 = 6$

   $x_1 + 3x_2 - s_2 + A_2 = 2$

   $x_1 , x_2, s_1, s_2, A_2 \geq 0$

---

* Profit has been reduced from 22 to 12 because 8 units of s_1 have been brought in the new solution, thereby the net reduction of profit 10 naturally equals 8 × 5/4

(Note that in maximization problems M always has –ve sign and in minimization problems M always has a +ve sign in the objective function).

The initial solution consists of s_1 and A_2. Several examples on inequalities in the wrong direction follow. Surplus variables can never come in initial solution.

3. Any linear programming problem can be re-formulated into what is known as its dual. Any of the primal (the original) or the dual may be selected for iterating by the simplex method. The selection is made on the basis of computational burden. Also the dual provides interesting insights into the methodology of the LP solution.

4. If two or more variables share the maximum positive co-efficient in the net evaluation row any one may be chosen for introduction for the new solution arbitrarily, viz., in $Z = 2x_1 + 2x_2 + x_3$ it matters little if x_1 or x_2 is chosen.

5. Lower bounds may be specified in an LPP. For example, over and above to the three usual constraints, it may be stipulated that x_1 cannot be less than 25 or 40 or l_1.i.e., $x_1 \geq l_1$. This can be handled quite easily by introducing a variable y_1 such that $x_1 = l_1 + y_1$. Substitute $x_1 = l_1 + y_1$ wherever it occurs and solve the LPP. Computations would be greatly reduced.

6. In all the simplex tables there is bound to be a unit matrix of size p × p where p is the no. of rows (excluding net evaluation row). The columns that constitute such a unit matrix need not be adjacent.

7. In view of the tediousness of computational aspects it is useful to make a check at each iteration. This can be done by deriving the net evaluation row in two ways. (i) just like any other row in the simplex tableau by deriving its fixed ratio (ii) by summing the product of the quantities column with the profit/cost column and subtracting this sum from the original profit contribution or cost co-efficient of variable. These should tally. Also, having obtained the optimal solution it is desirable to verify it if it obeys the given constraints.

8. The simplex method, the graphical and trial and error methods, the dual approach provide several ways of doing an LPP. The student may want to do each LPP in more than one way for the sake of verification of the answer and practice.

10.8 Practical Applications of Linear Programming

In addition to its wide use in industrial and administrative applications, linear programming has extensive application to agricultural, aircraft and several military problems. These are briefly discussed below.

Industrial Applications: These are basically product-mix problems in which the general objective is to derive the optimal production and procurement plan for the time period under consideration. The measure of effectiveness is either a defined return that is sought to be maximized or a defined cost that is to be minimized.

Production Planning - Product Mix: An industrial concern has available a certain production capacity on various manufacturing processes and has the opportunity to utilize this capacity to manufacture various products. Typically, different products will have different selling prices, will require different amounts of production capacity at the several processes, and therefore will have different unit profits; there may also be stipulations on maximum and/or minimum production levels. The problem is to determine the optimal mix so that the total profit is maximized.

Blending Problems: These problems are likely to arise when a product can be made from a variety of available raw materials of various compositions and prices. The manufacturing process involves blending (mixing) some of these materials in varying quantities to make a product conforming to given specifications. The supply of raw materials and specifications serve as constraints in obtaining the minimum cost material blend. The solution would state the number of units of each raw material which are to be blended to make one unit of product.

Diet Problems: Diet Problems are not much different from the blending problem.

Trim Problems: Trim Problems are applicable to paper industry where paper of standard width has to be cut into smaller width as per customer requirements with the objective of minimizing the waste produced.

Advertising-Mix: Advertising Mix problem is analogous to the product-mix problem.

Financial Applications: The investment portfolio selection problems can be satisfactorily handled by linear programming.

Agriculture Applications: These deal with somewhat different resource, but the objectives are nevertheless same - to maximize the return from the allocation activity or to minimize some defined cost.

Flight Scheduling Applications: Linear programming has been effectively applied to a variety of operational scheduling problems.

Other applications of linear programming include structural design, scheduling military tanker fleet, minimizing the number of carriers to meet a fixed schedule, the least ballast shipping required to meet a specific shipping program, cost cutting in business, fabrication scheduling, computations of maximum flows in network, steel production scheduling, stocks and flows the balancing of assembly lines, etc.

As such linear programming has contributed greatly to the enhancement of productivity at the firm, industry and national levels and continues to be promising in this regard.

10.9 Miscellaneous Illustrations

LPP-Formulation

Illustration 5

WELL TYPE Manufacturing Company produces three types of typewriters; Manual type-writer, Electronic typewriters, and Deluxe Electronic typewriters. All the three models are required to be machined first and then assembled. The time required for the various models are as follows:

Type	*Machine Time (in hour)*	*Assembly Time (in hour)*
Manual Typewriter	*15*	*4*
Electronic Typewriter	*12*	*3*
Deluxe Electronic Typewriter	*14*	*5*

The total available machine time and assembly time are 3,000 hours and 1,200 hours respectively. The data regarding the selling price and variable costs for the three types are :

	Manual	*Electronic*	*Deluxe Electronic*
Selling Price (₹)	*4,100*	*7,500*	*14,600*
Labour, Material and other variable costs (₹)	*2,500*	*4,500*	*9,000*

The company sells all the three types on credit basis, but will collect the amounts on the first of next month. The labour, material and other variable expenses will have to be paid in cash. This company has taken a loan of ₹ 40,000 from a co-operative bank and this company will have to repay it to the bank on 1st April, 2003. The TNC bank from whom this company has borrowed ₹ 60,000 has expressed its approval to renew the loan.

The Balance Sheet of this Company as on 31.03.2012 is as follows:

Liabilities	*₹*	*Assets*	*₹*
Equity Share capital	*1,50,000*	*Land*	*90,000*
Capital Reserve	*15,000*	*Building*	*70,000*
General Reserve	*1,10,000*	*Plant & Machinery*	*1,00,000*
Profit & Loss a/c	*25,000*	*Furniture & Fixtures*	*15,000*
Long term loan	*1,00,000*	*Vehicles*	*30,000*
Loan from TNC Bank	*60,000*	*Inventory*	*5,000*
Loan from Co-op. Bank	*40,000*	*Receivables*	*50,000*
		Cash	*1,40,000*
Total	*5,00,000*	*Total*	*5,00,000*

The company will have to pay a sum of ₹ 10,000 towards the salary from top management executives and other fixed overheads for the month. Interest on long term loans is to be paid *every month at 24% per annum. Interest on loans from TNC and Co-operative Banks may be taken to be ₹ 1,200 for the month. Also this company has promised to deliver 2 Manual typewriters and 8 Deluxe Electronic typewriters to one of its valued costomers next month.*

Also make sure that the level of operations in this company is subject to the availability of cash next month. This company will also be able to sell all their types of typewriter in the market. The Senior Manager of this company desires to know as to how many units of each typewriter must be manufactured in the factory next month so as to maximize the profits of the company. Formulate this as a linear programming problem. The formulated problem need not be solved.

Solution

Let X_1, X_2 and X_3 denote the number of Manual, Electronic and Deluxe Electronic typewriters respectively to be manufactured in the factory next month. We are given the following data.

	Manual Typewriter	Electronic Typewriter	Deluxe Electronic Typewriter
Selling Price p.u. (₹)	4,100	7,500	14,600
Labour, Material and other variable costs p.u. (₹)	2,500	4,500	9,000
Profit contribution p.u. (₹)	1,600	3,000	5,600

The objective of the company is to maximize the profit hence the objective function is given by

Maximize $Z = 1,600\ X_1 + 3,000\ X_2 + 5,600\ X_3 - (₹\ 10,000 + ₹\ 2,000 + ₹\ 1,200)$

From the data given for time required for various models and the total number of hours available for machine time and assembly time, we get the following constraints:

$$15X_1 + 12\ X_2 + 14X_3 \leq 3,000 \text{ (machine time restriction)}$$

and $$4X_1 + 3X_2 + 5X_3 \leq 1,200 \text{ (assembly time restriction)}$$

The level of operations in the company is subject to the availability of cash next month. That is, the cash required for manufacturing various models should not exceed the cash available for the next month.

The cash requirements for X_1 units of Manual, X_2 units of Electronic and X_3 units of Deluxe Electronic typewriters are;

$$2,500\ X_1 + 4,500\ X_2 + 9,000\ X_3 \qquad \text{...(i)}$$

The cash availability for the next month from the balance sheet is as below:

Cash availability (₹) = Cash balance (₹ 1,40,000) + Receivables (₹ 50,000)

– Loan to repay to co-operative bank (₹ 40,000)

– Interest on loan from TNC & co-operative banks (₹ 1,200)

– Interest on long term loans $\left(\dfrac{0.24 \times ₹\ 1,00,000}{12}\right)$

– (Top management salary and fixed overheads) (₹ 10,000)

Or Cash availability = ₹ 1,40,000 + ₹50,000 – (₹40,000 + ₹1,200 + ₹2,000 + ₹10,000)

= ₹ 1,90,000 – ₹ 53,200

= ₹ 1,36,800 ...(ii)

From (i) & (ii), we get

$2,500 X_1 + 4,500 X_2 + 9,000 X_3 \leq$ ₹ 1,36,800

Further, the company has promised to deliver 2 Manual typewriters and 8 Deluxe Electronic Typewriters to one of its customers.

Hence, $X_1 \geq 2$, $X_2 \geq 0$, and $X_3 \geq 8$

The linear programming formulation of the given problem is as follows:

Maximize $Z = 1,600 X_1 + 3,000 X_2 + 5,600 X_3$ – (₹ 10,000 + ₹ 2,000 + ₹ 1,200)

Subject to the constraints:

$15X_1 + 12X_2 + 14X_3 \leq 3,000$

$4X_1 + 3X_2 + 5x_3 \leq 1,200$

$2,500X_1 + 4,500X_2 + 9,000X_3 \leq 1,36,800$

$X_1 \geq 2$, $X_2 \geq 0$, $X_3 \geq 8$,

X_1, X_2 and X_3 can take only positive integral values.

Illustration 6

Consider a company that must produce two products over a production period of three months of duration. The company can pay for materials and labour from two sources: company funds and borrowed funds.

The firm faces three decisions :

(1) How many units should it produce of Product 1?

(2) How many units should it produce of Product 2?

(3) How much money should it borrow to support the production of the two products?

In making these decisions, the firm wishes to maximize the profit contribution subject to the conditions stated below:

(i) Since the company's products are enjoying a seller's market, it can sell as many units as it can produce. The company would therefore like to produce as many units as possible subject to production capacity and financial constraints. The capacity constraints, together with cost and price data, are given in Table-1.

TABLE-1 Capacity, Price and cost data

Product	*Selling Price (₹ per unit)*	*Cost of Production (₹ per unit)*	*Required Hours per unit in Department*		
			A	*B*	*C*
1	*14*	*10*	*0.5*	*0.3*	*0.2*
2	*11*	*8*	*0.3*	*0.4*	*0.1*
Available hours per production period of three months			*500*	*400*	*200*

(ii) The available company funds during the production period will be ₹ 3 lakhs.

(iii) A bank will give loans upto ₹ 2 lakhs per production period at an interest rate of 20 percent per annum provided the company's acid (quick) test ratio is at least 1 to 1 while the loan is outstanding. Take a simplified acid-test ratio given by

$$\frac{\text{Surplus cash on hand after production} + \text{Accounts receivable}}{\text{Bank Borrowing} + \text{Interest accrued thereon}}$$

(iv) Also make sure that the needed funds are made available for meeting the production costs.

Formulate the above as a Linear Programming Problem.

Solution

Let x_1 = No. of units of product 1 produced

x_2 = No of units of product 2 produced

x_3 = Amount of money borrowed

The profit contribution per unit of each product is given by the selling price minus the variable cost of production. Total profit may be computed by summing up the profit from producing the two products minus the cost associated with borrowed funds (if any):

The objective function is thus stated as

Maximize Z $= (14-10)\ x_1 + (11-8)\ x_2 - 0.05x_3$ (Note that the interest rate is 20% per annum, hence 5% for a period of three months)

$= 4x_1 + 3x_2 - 0.05\ x_3$

Subject to the following constraints:

The production capacity constraints for each department as given by table 1 are:

$0.5x_1 + 0.3x_2 \le 500$ ...(1)

$0.3x_1 + 0.4x_2 \le 400$ ...(2)

$0.2x_1 + 0.1x_2 \le 200$ ...(3)

The funds available for production include both ₹ 3,00,000 cash that the firm possesses and any borrowed funds maximum up to ₹ 2,00,000. Consequently production is limited to the extent that funds are available to pay for production costs. The constraint expressing this relationship is

Funds required for production ≤ Funds available

i.e. $10x_1 + 8x_2 \leq ₹\ 3{,}00{,}000 + x_3$

or $10x_1 + 8x_2 - x_3 \leq ₹\ 3{,}00{,}000$ ...(4)

The borrowed funds constraint (from condition (iii) of the Question) is

$x_3 \leq ₹\ 2{,}00{,}000$ ...(5)

The constraint based on the acid – test condition is developed as follows:

$$\frac{\text{Surplus cash on hand after production} + \text{Accounts receivable}}{\text{Bank Borrowing} + \text{Interest accrued thereon}} \geq 1$$

$$\frac{(3{,}00{,}000 + x_3 - 10x_1 - 8x_2) + 14x_1 + 11x_2}{(x_3 + 0.05x_3)} \geq 1$$

or $3{,}00{,}000 + x_3 + 4x_1 + 3x_2 \geq (x_3 + 0.05x_3)$

or $-4x_1 - 3x_2 + 0.05x_3 \leq 3{,}00{,}000$ .....(6)

Thus, the linear programming problem is given by

Maximize $Z = 4x_1 + 0.3x_2 - 0.05x_3$

subject to

$0.5x_1 + 0.3x_2 \leq 500$ .....(1)

$0.3x_1 + 0.4x_2 \leq 400$ .....(2)

$0.2x_1 + 0.1x_2 \leq 200$ .....(3)

$10x_1 + 8x_2 - x_3 \leq ₹3{,}00{,}000$ .....(4)

$x_3 \leq ₹2{,}00{,}000$ .....(5)

$-4x_1 - 3x_2 + 0.05x_3 \leq ₹3{,}00{,}000$ .....(6)

Where $x_1, x_2, x_3 \geq 0$

Illustration 7

North-East Aircraft company, which operates out of a central terminal has 8 aircraft of Type I, 15 aircraft of Type II, and 12 aircraft of Type III available for to-day's flights. The tonnage capacities (in thousands of tons) are 4.5 for Type 1,7 for type II and 4 for Type III.

The company dispatches its planes to cities A and B. Tonnage requirements in thousands of tons are 20 at city A and 30 at city B: excess tonnage capacity supplied to a city has no value. A plane can fly once only during the day.

The cost of sending a plane from the terminal to each city is given by the following table:

	Type I	*Type II*	*Type III*
City A	*23*	*5*	*1.4*
City B	*58*	*10*	*3.8*

Formulate the LPP model to minimize the air-transportation cost

Solution

Let x_{11} be the I plane coming to A, x_{12} the II planes coming to A; and so on.

City	Type-I	Type-II	Type-III
A	x_{11}	x_{12}	x_{13}
B	x_{21}	x_{22}	x_{23}

Minimize Z = $23x_{11} + 5x_{12} + 1.4x_{13} + 58x_{21} + 10x_{22} + 3.8x_{23}$

Tonnage requirements of City A and B

$$4.5x_{11} + 7x_{12} + 4x_{13} = 20$$

$$4.5x_{21} + 7x_{22} + 4x_{23} = 30$$

Central Terminal has 8 aircraft of Type I, 15 aircraft of Type II, and 12 aircraft of Type III

$$x_{21} + x_{11} \le 8$$

$$x_{22} + x_{12} \le 15$$

$$x_{23} + x_{13} \le 12$$

$$x_{ij} \ge 0$$

Illustration 8

The Voltex Company produces an air conditioner/heating unit. The company currently has firm orders for 6 months into the future. The company can schedule its production over the next 6 months to meet orders on either a regular or on overtime basis. Consider orders and the associated production costs for the next 6 months as follows:

Month	***Jan.***	***Feb.***	***Mar.***	***Apr.***	***May***	***Jun.***
Orders Cost/unit (₹)	*590*	*610*	*650*	*700*	*500*	*700*
Regular Production Cost/Unit (₹)	*50*	*52*	*51*	*55*	*47*	*50*
Overtime Production	*62*	*58*	*63*	*60*	*55*	*52*

Maximum no. of units which can be produced on regular and overtime basis are 500 and 300 respectively.

With 75 air conditioners in stock at the beginning of January, the company wishes to have at least 100 air conditioners in stock at the end of June. The inventory-carrying cost for air-conditioners is ₹ 10 per unit per month.

Solution

Let x_{ij} = number of units produced in month i (i = 1, 2 ......, 6),

on a regular or overtime basis (j = 1, 2)

y_i – number of units of ending inventory in month i (i = 1,2...., 6)

Minimize (cost)

$$Z = 50x_{11} + 62x_{12} + 52x_{21} + 58x_{22} + 51x_{31} + 63x_{32} + 55x_{41} + 60x_{42} + 47x_{51} + 55x_{52} + 50x_{61} + 52x_{62} + 10(y_1 + y_2 + y_3 + y_4 + y_5 + y_6)$$

Subject to

$$\left.\begin{aligned} 75 + x_{11} + x_{12} - 590 &= y_1 \\ y_1 + x_{21} + x_{22} - 610 &= y_2 \\ y_2 + x_{31} + x_{32} - 650 &= y_3 \\ y_3 + x_{41} + x_{42} - 700 &= y_4 \\ y_4 + x_{51} + x_{52} - 500 &= y_5 \\ y_5 + x_{61} + x_{62} - 700 &= y_6 \end{aligned}\right\} \text{Monthly Inventory constraints}$$

$y_6 \geq 100$ (Ending Inventory Constraints)

$x_{11} \leq 500$; $x_{12} \leq 300$

$x_{21} \leq 500$; $x_{22} \leq 300$

$x_{31} \leq 500$; $x_{32} \leq 300$

$x_{41} \leq 500$; $x_{42} \leq 300$

$x_{51} \leq 500$; $x_{52} \leq 300$

$x_{61} \leq 500$; $x_{62} \leq 300$

$x_{ij} \geq 0$, all i and j

$y_i \geq 0$, all i

Illustration 9

A refinery makes 3 grades of petrol (A, B, C) from 3 crude oils (d, e, f). Crude can be used in any grade but the others satisfy the following specifications.

Grade	*Specifications*	*Selling Price per litre*
A	*Not less than 50% crude d* *Not more than 25% crude e*	*8.0*

B	*Not less than 25% crude d* *Not more than 50% crude e*	*6.5*
C	*No specifications*	*5.5*

There are capacity limitations on the amount of the three crude elements that can be used;

Crude	***Capacity('000)***	***Price per litre (₹)***
d	*500*	*9.5*
e	*500*	*5.5*
f	*300*	*6.5*

It is required to produce the maximum profit.

Formulation

Let there be x_1 litres of *d* in A

x_2 litres of *e* in A

x_3 litres of *f* in A

y_1 litres of *d* in B

y_2 litres of *e* in B

y_3 litres of *f* in B

z_1 litres of *d* in C

z_2 litres of *e* in C

z_3 litres of *f* in C

Then $\frac{x_1}{x_1+x_2+x_3} \geq \frac{1}{2}$ i.e. $-x_1 + x_2 + x_3 \leq 0$

$\frac{x_2}{x_1+x_2+x_3} \leq \frac{1}{4}$ i.e. $-x_1 + 3x_2 - x_3 \leq 0$

$\frac{y_1}{y_1+y_2+y_3} \geq \frac{1}{4}$ i.e. $-3y_1 + y_2 + y_3 \leq 0$

$\frac{y_2}{y_1+y_2+y_3} \leq \frac{1}{2}$ i.e. $-y_1 + y_2 - y_3 \leq 0$

Also $x_1 + y_1 + z_1 \leq 500{,}000$

$x_2 + y_2 + z_2 \leq 500{,}000$

$x_3 + y_3 + z_3 \leq 300{,}000$

Sales Price

$$\left[\left(x_1+x_2+x_3\right)\times 8+\left(y_1+y_2+y_3\right)\times 6.5+\left(z_1+z_2+z_3\right)\right]$$

Cost

$$-\left[\left(x_1+y_1+z_1\right)\times 9.5\right]$$

$$-\left[\left(x_2+y_2+z_2\right)\times 5.5\right]$$

$$-\left[\left(x_3+y_3+z_3\right)\times 6.5\right]$$

Profit = $-1.5x_1 + 2.5x_2 + 1.5x_3 - 3.0y_1 + 1.0\,y_2 + 0.y_3 - 4.0z_1 + 0.z_2 - 1.0z_3$

Max Z = $-1.5x_1 + 2.5x_2 + 1.5x_3 - 3.0y_1 + 1.0\,y_2 + 0.y_3 - 4.0z_1 + 0.z_2 - 1.0z_3$

Subject to

$-x_1 + x_2 + x_3 \le 0$

$-x_1 + 3x_2 - x_3 \le 0$

$-3y_1 + y_2 + y_3 \le 0$

$-y_1 + y_2 - y_3 \le 0$

$x_1 + y_1 + z_1 \le 500{,}000$

$x_2 + y_2 + z_2 \le 500{,}000$

$x_3 + y_3 + z_3 \le 300{,}000$

Illustration 10

The vitamins V and W are found in two different foods, F_1 and F_2. The amount of vitamin in each of the two foods, respective prices per unit of each food, and the daily vitamin requirements are given in the following table. The data indicate that one unit of F_1 contains 2 units of vitamin V and 3 units of vitamin W. Similarly one unit of F_2 contains 4 units of vitamin V and 2 units vitamin W. Daily requirements of vitamin V is at least 40 units and vitamin W of at least 50 units.

The problem is to determine optimal quantities of foods F_1 and F_2 to be bought so that the daily vitamin requirements are met and, simultaneously the cost of buying the goods is minimized.

Vitamin	***Food***		***Daily Requirements***
	F_1	***F_2***	
V	*2*	*4*	*40*
W	*3*	*2*	*50*
Cost/Unit of food (₹)	*3*	*2.5*	

Formulation:

Let x_1 and x_2 be quantities of F_1 and F_2 respectively.

Minimize $3x_1 + 2.5x_2$

subject to $2x_1 + 4x_2 \geq 40$

$3x_1 + 2x_2 \geq 50$

$x_1 \geq 0$

$x_2 \geq 0$

Illustration 11

The Fine Paper Company produces rolls of paper used in cash registers. Each roll of paper is 500 ft. in length and can be produced in widths of 1, 2, 3 and 5 inch. The company's production process results in 500' rolls that are 12 inches in width. Thus the company must cut its 12 inch roll to the desired width. It has six basic cutting alternatives as follows:

Cutting Alternative	No. of Rolls				Waste (Inches)
	1"	2"	3"	5"	
1	6	3	0	0	0
2	0	3	2	0	0
3	1	1	1	1	1
4	0	0	2	1	1
5	0	4	1	0	1
6	4	2	1	0	1

The maximum demand requirements for the four rolls are as follows:

Roll Width (inches)	Demand Requirements (Rolls)
1	3,000
2	2,000
3	1,500
5	1,000

The company wishes to minimize the waste generated by its production meeting its demand requirements. Formulate the LP model.

Formulation. Let x_j be the number of times cutting alternative (j = 1,2, ...6) is employed.

Minimise (waste produced) $Z = 1x_3 + 1x_4 + 1x_5 + 1x_6$ subject to

$6x_1 + 1x_3 + 4x_6 \leq 3{,}000$

$3x_1 + 3x_2 + 1x_3 + 4x_5 + 2x_6 \leq 2{,}000$

$2x_2 + 1x_3 + 2x_4 + 1x_5 + 1x_6 \leq 1{,}500$

$1x_3 + 1x_4 \leq 1{,}000$

$x_j \geq 0$, for all j

Illustration 12

The Agro Promotion Bank is trying to select investment portfolio for a cotton farmer. The bank has chosen a set of five investment alternatives, with subjective estimates of rates of return and risk, as follows:

Investment	*Annual Rate of Retrun*	*Risk*
Tax-free municipal bonds	*6.0*	*1.3*
Corporate bonds	*8.0*	*1.5*
High grade common stock	*5.0*	*1.9*
Mutual fund	*7.0*	*1.7*
Real estate	*15.0*	*2.7*

The bank officer in charge of the portfolio would like to maximize the average annual rate of return on the portfolio. However, the wealthy investor has specified that the average risk of the portfolio should not exceed 2.0; and does not want more than 20% of the investment to be put into real estate. Formulate an LP model for the problem.

Formulation

Let x_j be the percent of portfolio allocated to investment j (j = 1,2,3,4,5).

Maximize (average annual rate of return on the portfolio) $Z = 6\%x_1 + 8\%x_2 + 5\%\ x_3 + 7\%x_4 + 15\%\ x_5$

subject to (Average risk constraint) $1.3x_1 + 1.5x_2 + 1.9x_3 + 1.7x_4 + 2.7x_5 \leq 2.0$

(Real estate constraint) $x_5 \leq 0.2$

(Total investment constraint) $x_1 + x_2 + x_3 + x_4 + x_5 = 1.0$

$x_1, x_2, x_3, x_4, x_5 \geq 0.$

Illustration 13

A farmer has 1,000 acres of land on which he can grow corn, wheat or soyabeans. Each acre of corn costs ₹ 100 for preparation, requires 7 man-days of work and yields a profit of ₹ 30. An acre of wheat costs ₹ 120 for preparation, requires 10 man-days of work and yields a profit of ₹ 40. An acre of soyabeans costs ₹ 70 to prepare, requires 8 man days of work and yields a profit of ₹ 20. If the farmer has ₹ 100,000 for preparation and can count on 8000 man-days of work, how many acres should be allocated to each crop to maximize profits? Formulate an LP model.

Formulation: Let x_1, x_2 and x_3 designate the average of corn, wheat and soyabeans respectively.

Maximize $Z = 30x_1 + 40x_2 + 20x_3$

subject to (Money) $100x_1 + 120x_2 + 70x_3 \leq ₹100{,}000$

(Man-day) $7x_1 + 10x_2 + 8x_3 \leq ₹\ 8000$

(Acreage) $x_1 + x_2 + x_3 \le$ ₹ 1000

$x_1, x_2, x_3 \ge 0$

Illustration 14

The managers of several cattle feed lots are interested in determining how many of each of several types of livestock feed to purchase in order to satisfy the nutritional requirements for their livestock. They wish to purchase these foods in a manner that minimizes the cost of feeding their livestock. Relevant costs and nutritional data is as below.

Required Nutrient	***Units of Nutritional Elements***				***Minimum Nutrient Requirement***
	Alfa	***Corn***	***Soyabeans***	***Sorghum***	
Nutrient A	*40*	*50*	*30*	*60*	*500*
Nutrient B	*30*	*60*	*35*	*40*	*750*
Nutrient C	*25*	*30*	*25*	*50*	*600*
Cost per unit	*₹1.00*	*₹ 1.25*	*₹0.95*	*₹1.35*	

Formulation

Let x_j to be number of units of food type j = (1,2,3,4) used

Minimize (Cost) = $1.00x_1 + 1.25x_2 + 0.95x_3 + 1.35x_4$

Subject to (Nutrient A) $40x_1 + 50x_2 + 30x_3 + 60x_4 \geq 500$

(Nutrient B) $30x_1 + 60x_2 + 35x_3 + 40x_4 \geq 750$

(Nutrient C) $25x_1 + 30x_2 + 25x_3 + 50x_4 \geq 600$

$x_1, x_2, x_3, x_4 \ge 0$

Illustration 15

The owner of Fancy Goods Shop is interested to determine, how many advertisements to release in the selected three magazines A,B and C. His main purpose is to advertise in such a way that exposure to principal buyers of his goods is maximized. Percentages of readers for each magazine are known. Exposure in any particular magazine is the number of advertisements released multiplied by the number of principal buyers. The following data are available:

Particulars	***Magazines***		
	A	***B***	***C***
Readers	*1.0 lakh*	*0.6 lakh*	*0.4 lakh*
Principal Buyers	*20%*	*15%*	*8%*
Cost per Advertisement (₹)	*8,000*	*6,000*	*5,000*

The budgeted amount is at the most 1.0 lakh for the advertisement. The owner has already decided that magazine A should have no more than 15 advertisements and that B and C each gets at least 8 advertisements. Formulate a Linear Programming Model for this problem.

Solution

Let x_1, x_2 and x_3 denote the number of advertisements to be released in three magazines A, B and C respectively. Let Z denote the total exposure to the principal buyers of the goods.

Since the exposure in any magazine is the number of advertisements multiplied by the number of principal buyers, therefore, the value of Z is given by :

$$Z = (0.20 \times 1,00,000)\, x_1 + (0.15\times 60,000)\, x_2 + (.08 \times 40,000)\, x_3$$

$$= 20,000\, x_1 + 9,000\, x_2 + 3,200\, x_3$$

The owner of Fancy Goods shop wishes to maximise the total exposure to principal buyers, hence the objective function is given by

Maximize $Z = 20,000\, x_1 + 9,000\, x_2 + 3,200\, x_3$

subject to the following constraints :

(i) The budgeted amount for the advertisement is at the most ₹ 1,00,000.

Hence, $8,000\, x_1 + 6,000\, x_2 + 5,000\, x_3 \leq 1,00,000$.

(ii) Also, the magazine A should have no more than 15 advertisements, B and C each should get at least 8 advertisements.

Hence, $x_1 \leq 15$

and $x_2 \geq 8$, $x_3 \geq 8$

Thus, the linear programming model for the problem is

Maximise $Z = 20,000\, x_1 + 9,000\, x_2 + 3,200\, x_3$

subject to

$8,000\, x_1 + 6,000\, x_2 + 5,000\, x_3 \leq 1,00,000$

$x_1 \leq 15$, $x_2 \geq 8$, $x_3 \geq 8$

Where x_1, x_2, and $x_3 \geq 0$

Illustration 16

A Mutual Fund Company has ₹ 20 lakhs available for investment in Government Bonds, blue chip stocks, speculative stocks and short-term deposits. The annual expected return and risk factor are given below:

Type of investment	***Annual Expected Return (%)***	***Risk Factor (0 to 100)***
Government Bonds	*14*	*12*
Blue Chip Stocks	*19*	*24*
Speculative Stocks	*23*	*48*
Short-term Deposits	*12*	*6*

Mutual fund is required to keep at least ₹ 2 lakhs in short-term deposits and not to exceed average risk factor of 42. Speculative stocks must be at most 20 percent of the total amount invested. How should mutual fund invest the funds so as to maximize its total expected annual return? Formulate this as a Linear Programming Problem. Do not solve it.

Solution

Let x_1, x_2, x_3 and x_4 denote the amount of funds to be invested in government bonds, blue chip stocks, speculative stocks and short term deposits respectively. Let Z denote the total expected return.

Since the Mutual Fund Company has ₹ 20 lakhs available for investment,

$$x_1 + x_2 + x_3 + x_4 \leq 20{,}00{,}000 \qquad \text{... (i)}$$

Also, Mutual fund is required to keep at least ₹ 2 lakhs in short-term deposits,

Hence, $x_4 \geq 2{,}00{,}000$ ...(ii)

The average risk factor is given by

$$\frac{12x_1 + 24x_2 + 48x_3 + 6x_4}{x_1 + x_2 + x_3 + x_4}$$

Since the average risk factor Mutual Fund should not exceed 42, we get the following constraint.

$$\frac{12x_1 + 24x_2 + 48x_3 + 6x_4}{x_1 + x_2 + x_3 + x_4} \leq 42$$

or $12x_1 + 24x_2 + 48x_3 + 6x_4 \leq 42\,(x_1+x_2+x_3+x_4)$

or $-30x_1 - 18x_2 + 6x_3 - 36x_4 \leq 0$ ...(iii)

Further, speculative stock must be at most 20 per cent of the total amount invested, hence

$$x_3 \leq 0.20\,(x_1 + x_2 + x_3 + x_4)$$

or $-0.2x_1 - 0.2\,x_2 + 0.8x_3 - 0.2x_4 \leq 0$ ...(iv)

Finally, the objective is to maximise the total expected annual return, the objective function for Mutual Fund can be expressed as

Maximise $Z = 0.14x_1 + 0.19x_2 + 0.23x_3 + 0.12x_4$ ...(v)

Summarising equations (i) to (v), the linear programming model for the Mutual Fund company is formulated as below:

Objective function :

Maximize $Z = 0.14x_1 + 0.19x_2 + 0.23x_3 + 0.12x_4$

Subject to the constraints

$$x_1 + x_2 + x_3 + x_4 \leq 20,00,000$$

$$x_4 \geq 2,00,000$$

$$-30x_1 - 18x_2 + 6x_3 - 36x_4 \leq 0$$

$$-0.2x_1 - 0.2x_2 + 0.8x_3 - 0.2x_4 \leq 0$$

where $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ and $x_4 \geq 0$

Illustration 17

A firm produces three products A, B and C. It uses two types of raw materials I and II of which 5,000 and 7,500 units respectively are available. The raw material requirements per unit of the products are given below:

Raw Material	***Requirements per unit of Product***		
	A	***B***	***C***
I	*3*	*4*	*5*
II	*5*	*3*	*5*

The labour time for each unit of product A is twice that of product B and three times that of product C. The entire labour force of the firm can produce the equivalent of 3,000 units. The (marks) minimum demand of the three products is 600, 650 and 500 units respectively. Also, the ratios of the number of units produced must be equal to 2 : 3 : 4. Assuming the profits per unit of A, B and C as ₹ 50, 50 and 80 respectively.

Formulate the problem as a linear programming model in order to determine the number of units of each product which will maximize the profit.

Solution

Let the firm produce x_1 units of product A, x_2 units of products B and x_3 units of product C.

The profit per unit of products A, B and C is ₹ 50, and ₹ 80 respectively. Since the objective of the firm is to maximize the profit, therefore, the objective function is given by

Maximise $Z = 50x_1 + 50x_2 + 80x_3$

The firm uses two types of raw materials I and II of which 5,000 and 7,500 units respectively are available. As per the given data, the raw material constraints can be formulated as given below:-

$$3x_1 + 4x_2 + 5x_3 \leq 5,000 \qquad \text{...(i)}$$

and $$5x_1 + 3x_2 + 5x_3 \leq 7,500 \qquad \text{...(ii)}$$

The labour time for each unit of product A is twice that of product B and three times that of product C. Also the entire labour force can produce the equivalent of 3000 units.

$$\therefore x_1 + \frac{x_2}{2} + \frac{x_3}{3} \leq 3,000$$

Or $6x_1 + 3x_2 + 2x_3 \leq 18000$ ...(iii)

The minimum demand of the three products is 600, 650 and 500 units respectively.

Hence, $x_1 \geq 600$, $x_2 \geq 650$ and $x_3 \geq 500$ ...(iv)

Since the ratios of the number of units produced must be equal to 2 : 3 : 4, therefore,

$\frac{1}{2} x_1 = \frac{1}{3} x_2$, and $\frac{1}{3} x_2 = \frac{1}{4} x_3$

or $3x_1 = 2x_2$ and $4x_2 = 3x_3$ ...(v)

The linear programming model can be formulated as follows:

Maximise $Z = 50x_1 + 50x_2 + 80x_3$

Subject to the constraints:

$3x_1 + 4x_2 + 5x_3 \leq 5,000$

$5x_1 + 3x_2 + 5x_3 \leq 7,500$

$6x_1 + 3x_2 + 2x_3 \leq 18,000$

$3x_1 = 2x_2$ and $4x_2 = 3x_3$

$x_1 \geq 600$, $x_2 \geq 650$ and $x_3 \geq 500$.

Illustration 18

The Delhi Florist Company is planning to make up floral arrangements for the upcoming festival. The company has available the following supply of flowers at the costs shown:

Type	***Number available***	***Cost per flower***
Red roses	*800*	*₹ 0.20*
Gardenias	*456*	*₹ 0.25*
Carnations	*4,000*	*₹ 0.15*
White roses	*920*	*₹ 0.20*
Yellow roses	*422*	*₹ 0.22*

These flowers can be used in any of the four popular arrangements whose makeup and selling prices are as follows:

Arrangement	*Requirements*	*Selling price*
Economy	*4 red roses*	*₹ 6*
	2 gardenias	
	8 carnations	

May time	*8 white roses*	*₹ 8*
	5 gardenias	
	10 carnations	
	4 yellow roses	
Spring colour	*9 red roses*	*₹ 10*
	10 carnations	
	9 white roses	
	6 yellow roses	
Deluxe rose	*12 red roses*	*₹ 12*
	12 white roses	
	12 yellow roses	

Formulate a linear programming problem which allows the florist company to determine how many units of each arrangement should be made up in order to maximize profits assuming all arrangements can be sold.

The formulated LP problem is not required to be solved.

Solution

The profits for each arrangement are:

Economy	= 6.00 – 4 (0.20) – 2(0.25) – 8 (0.15)	= ₹ 3.50
May time	= 8.00 – 8 (0.20) – 5 (0.25) – 10 (0.15) – 4 (0.22)	= ₹ 2.77
Spring colour	= 10.00 – 9 (0.20) – 10 (0.15) – 9 (0.20) – 6 (0.22)	= ₹ 3.58
Deluxe rose	= 12.00 – 12 (0.20) – 12 (0.20) – 12 (0.22)	= ₹ 4.56

Let x_1, x_2, x_3, x_4 be number of units arrangements of type Economy, May time, Spring colour & Deluxe rose.

Then the objective is Maximise $Z = 3.5x_1 + 2.77x_2 + 3.58x_3 + 4.56x_4$

subject to

$$4x_1 + 9x_3 + 12x_4 \le 800$$
$$2x_1 + 5x_2 \le 456$$
$$8x_1 + 10x_2 + 10x_3 \le 4000$$
$$8x_2 + 9x_3 + 12x_4 \le 920$$
$$4x_2 + 6x_3 + 12x_4 \le 422$$
$$\text{All } x_i\text{'s} \ge 0.$$

Graphical Method

Illustration 19

A company produces two types of presentation goods A and B that require gold and silver. Each unit of type A requires 3 gms. of silver and 1 gm. of gold while that of B requires 1 gm. of silver and 2 gms. of gold. The company can procure 9 gms of silver and 8 gms. of gold. If each unit of type A brings a profit of ₹ 40 and that of types B ₹ 50, determine the number of units of each type that the company should produce to maximize the profit. What is the maximum profit?

Solution

Let x_1 be the number of units of type A of presentation goods to be produced and x_2 is that of type B. The formulation of L.P.P. based on the given data may be stated as follows:

Maximize Z = 40 x_1 + 50 x_2

Subject to the constraints

$$3x_1 + x_2 \leq 9$$
$$x_1 + 2x_2 \leq 8$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

For the line $3x_1 + x_2 = 9$

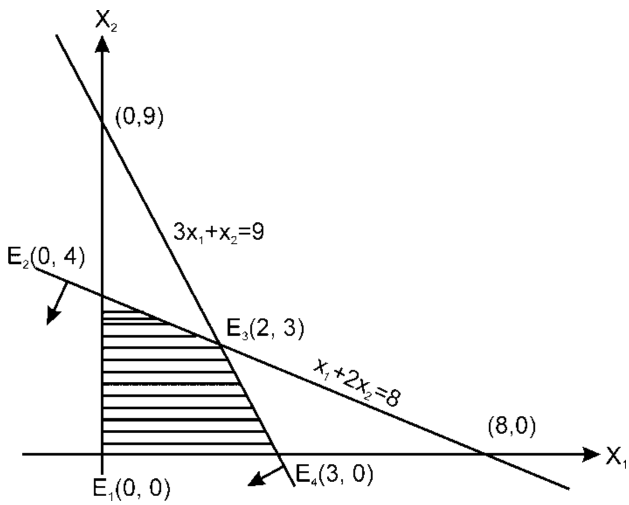
Let $x_1 = 0$, so that $x_2 = 9$

Let $x_2 = 0$, so that $x_1 = 3$

For the line $x_1 + 2x_2 = 8$

Let $x_1 = 0$, so that $x_2 = 4$

Let $x_2 = 0$, so that $x_1 = 8$



The shaded portion in the diagram represent the feasible region, and the matrix of the extreme points Ei, i = 1, 2, 3, 4 is

$$E = \begin{array}{c} \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & 4 \\ 2 & 3 \\ 3 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}$$

The column vector for the objective function is C = $\begin{bmatrix} 40 \\ 50 \end{bmatrix} \begin{array}{l} x_1 \\ x_2 \end{array}$

The column vector for the values of the objective function is given by

$$EC = \begin{bmatrix} 0 & 0 \\ 0 & 4 \\ 2 & 3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 40 \\ 50 \end{bmatrix} = \begin{bmatrix} 0\times40+0\times50 \\ 0\times40+4\times50 \\ 2\times40+3\times50 \\ 3\times40+0\times50 \end{bmatrix} = \begin{bmatrix} 0 \\ 200 \\ 230 \\ 120 \end{bmatrix} \begin{array}{l} E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}$$

Since 230 is the maximum value in EC, this value is reached at the extreme point E_3 whose coordinates are (2,3).

Thus, for maximising profit, the company should produce 2 units of type A and 3 units of type B of presentation goods.

The maximum profit is ₹ 230.

Illustration 20

A dealer wishes to purchase a number of fans and sewing machines. He has only ₹ 5,760 to invest and has space utmost for 20 items. A fan costs him ₹ 360 and a sewing machine ₹ 240. His expectation is that he can sell a fan at a profit of ₹ 22 and a sewing machine at a profit of ₹ 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit? Formulate this problem as a linear programming problem and then use graphical method to solve it.

Solution

Let x_1 be the number of fans and x_2 be the number of sewing machines purchased by this dealer

The formulation of the problem as per given information is as under.

Maximize Z = 22 x_1 + 18 x_2

Subject to the constraints

$$x_1 + x_2 \le 20$$
$$360x_1 + 240x_2 \le 5{,}760$$
$$x_1 \ge 0,\ x_2 \ge 0$$

For the line $x_1 + x_2 = 20$

let $x_1 = 0$, so that $x_2 = 20$

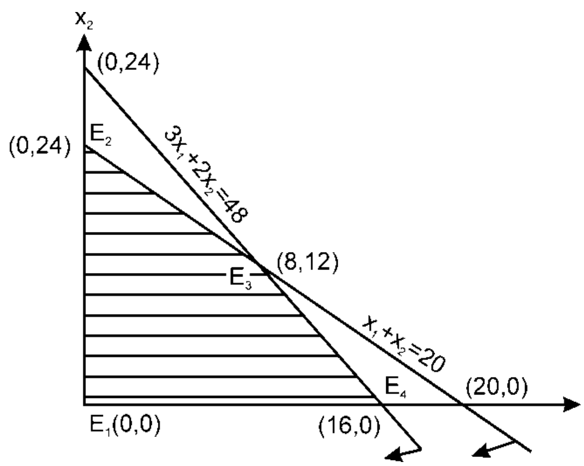
let $x_2 = 0$, so that $x_1 = 20$

For the line $360x_1 + 240x_2 = 5{,}760$

i.e. $3x_1 + 2x_2 = 48$

let $x_1 = 0$, so that $x_2 = 24$

let $x_2 = 0$, so that $x_1 = 16$



The shaded portion in the diagram is the feasible region, and the matrix E of the extreme points, E_i, i=1, 2, 3, 4 is given by

$$E = \begin{array}{c} \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & 20 \\ 8 & 12 \\ 16 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}$$

The column vector for the objective function is

$$C = \begin{bmatrix} 22 \\ 18 \end{bmatrix} \begin{array}{c} x_1 \\ x_2 \end{array}$$

The column vector for the values of the objective function is given by

$$EC = \begin{bmatrix} 0 & 0 \\ 0 & 20 \\ 8 & 12 \\ 16 & 0 \end{bmatrix} \begin{bmatrix} 22 \\ 18 \end{bmatrix} = \begin{bmatrix} 0\times 22+0\times 18 \\ 0\times 22+20\times 18 \\ 8\times 22+12\times 18 \\ 16\times 22+0\times 18 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 0 \\ 360 \\ 392 \\ 352 \end{bmatrix} \end{array} \begin{array}{l} E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}$$

Since 392 is the largest element of matrix EC, therefore the maximum profit is reached at the extreme point E_3 whose coordinates are (8,12). Thus, to maximize his profit the dealer should buy 8 fans and 12 sewing machines.

Illustration 21

A company produces two products X and Y, each of which requires three types of processing. The length of time for processing each unit and the profit per unit are given in the following table :

	Product X(hr/unit)	***Product Y (hr/unit)***	***Available capacity per day (hr.)***
Process I	*12*	*12*	*840*
Process II	*3*	*6*	*300*
Process III	*8*	*4*	*480*
Process per unit (₹)	*5*	*7*	

How many units of each product should the company produce per day in order to maximize profit?

Solution

Let *x* be the number of units of type X and *y* be the number of units of type Y products to be produced. The formulation of the problem based on the given data is as follows:

Maximize Z = $5x + 7y$

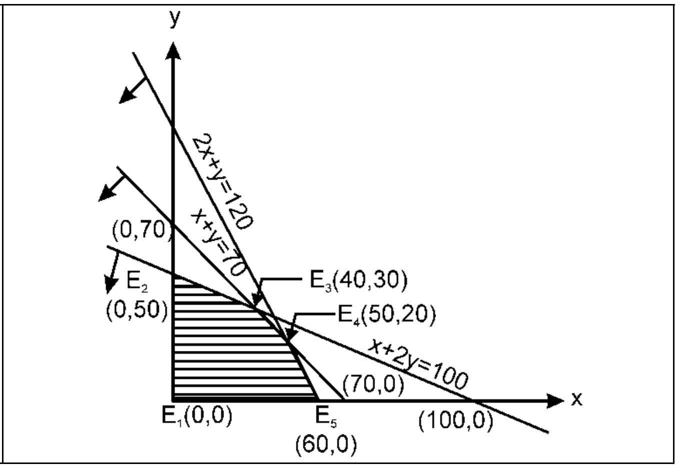
Subject to the constraints

$12x + 12y \le 840$

$3x + 6y \le 300$

$8x + 4y \le 480$

$x \ge 0,\ y \ge 0$



The line 12x + 12*y* = 840 is equivalent to

x + y = 70

x	40	50
y	30	20

The line 3x + 6y = 300 is equivalent to

x + 2y = 100

x	0	50
y	50	25

The line 8x + 4y = 480 is equivalent to

2x + y = 120

x	60	40
y	0	40

The shaded portion in the diagram (on previous page) represents the feasible region, and the matrix of the extreme points

$E_i, i = 1, 2, 3, 4, 5$ is

The column vector for the values of the objective function is given by

$$EC = \begin{bmatrix} 0 & 0 \\ 0 & 50 \\ 40 & 30 \\ 50 & 20 \\ 60 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 0\times5+0\times7 \\ 0\times5+50\times7 \\ 40\times5+30\times7 \\ 50\times5+20\times7 \\ 60\times5+0\times7 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 0 \\ 350 \\ 410 \\ 390 \\ 300 \end{bmatrix} \end{array} \begin{array}{c} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{array}$$

Since 410 is the largest element in EC, the maximum value is achieved at the extreme point E_3 whose coordinates are (40, 30).

Thus to maximize profit, the company should produce 40 units of X and 30 units of Y.

Illustration 22

A company that produces soft drinks has a contract that requires that a minimum of 80 units of the chemical A and 60 units of the chemical B into each bottle of the drink. The chemicals are available in a prepared mix from two different suppliers. Supplier X_1 has a mix of 4 units of A and 2 units of B that costs ₹ 10, and supplier X_2 has a mix of 1 unit of A and 1 unit of B that costs ₹ 4. How many mixes from company X_1 and company X_2 should the company purchase to honour contract requirement and yet minimize cost?

Solution

The problem may be summarized as follows:

	Chemical A	Chemical B	Cost per mix (₹)
Supplier X_1	4	2	10
Supplier X_2	1	1	4
Units required	80	60	

Let x_1 be the number of mixes to be purchased from supplier X_1 and x_2 be of those to be purchased from supplier X_2.

The conditions of the problem when symbolised, take the form:

Minimize Z = $10\ x_1 + 4\ x_2$

Subject to the restrictions

$$4x_1 + x_2 \geq 80$$

$$2x_1 + x_2 \geq 60$$

$$x_1 \geq 0,\ x_2 \geq 0$$

For the line $4x_1 + x_2 = 80$

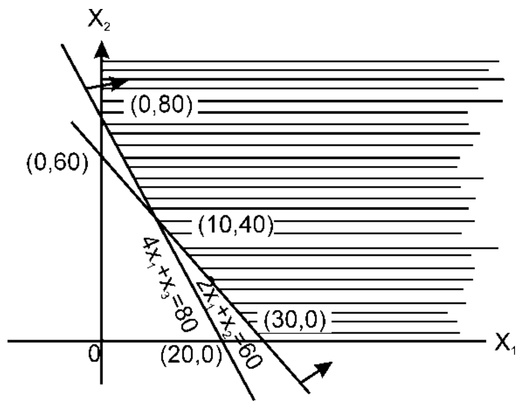
let $x_1 = 0$, so that $x_2 = 80$

let $x_2 = 0$, so that $x_1 = 20$

For the line $2x_1 + x_2 = 60$

let $x_1 = 0$, so that $x_2 = 60$

let $x_2 = 0$, so that $x_1 = 30$



Feasible region is shaded in the diagram which appears to be unbounded. We now try to determine the additional hidden conditions in the problem for which the feasible region becomes bounded.

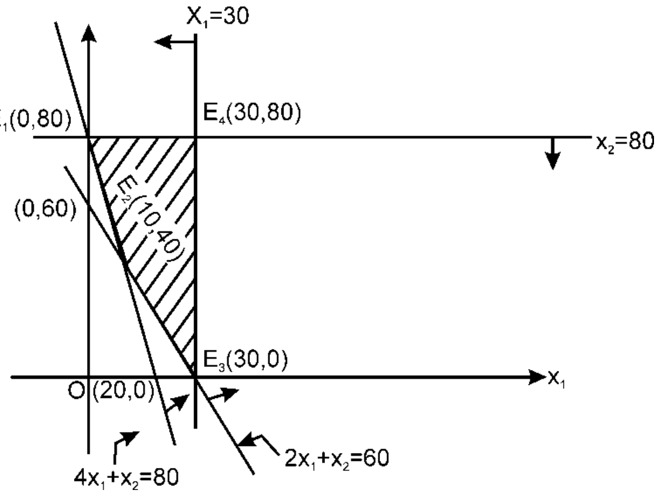
Each bottle of the drink contains 80 units of the chemical A and 60 units of the chemical B. If the drinks are made from the mixes of company X_1 only, each bottle would require 20 mixes to meet the chemical A requirement and 30 mixes to meet the chemical B requirement. Therefore, 30 mixes would meet all minimum requirements. Any lesser amount of mixes would not meet the minimum requirement for chemical B.

Similarly, if the drinks are made from the mixes of company X_2 only, each bottle would require 80 mixes to meet both the chemical requirements.

Since the company that manufactures drinks wants to make a blending of the mixes purchased from both the companies X_1 and X_2, it is possible that the company would need less than the maximum amount of both supplies to meet the guaranteed minimum.

Thus we add the two additional hidden conditions $x_1 \leq 30$, $x_2 \leq 80$.

With these two additional restrictions the above diagram takes the form :



The matrix of the extreme points, in this case, is $E = \begin{array}{c} \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{bmatrix} 0 & 80 \\ 10 & 40 \\ 30 & 0 \\ 30 & 80 \end{bmatrix} \end{array} \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}$

The column vector for the objective function is C = $\begin{bmatrix} 10 \\ 4 \end{bmatrix} \begin{array}{c} x_1 \\ x_2 \end{array}$

The column vector for the values of the objective function is given by

$$EC = \begin{bmatrix} 0 & 80 \\ 10 & 40 \\ 30 & 0 \\ 30 & 80 \end{bmatrix} \begin{bmatrix} 10 \\ 4 \end{bmatrix} = \begin{bmatrix} 0\times10+80\times4 \\ 10\times10+40\times4 \\ 30\times10+0\times4 \\ 30\times10+80\times4 \end{bmatrix} = \begin{bmatrix} 320 \\ 260 \\ 300 \\ 620 \end{bmatrix} \begin{array}{l} E_1 \\ E_2 \\ E_3 \\ E_4 \end{array}$$

Since 260 is the smallest element in EC, the minimum value is reached at the extreme point E_2, whose coordinates are (10,40).

Thus, to honour the contract and yet to minimize cost, the company should purchase 10 mixes from X_1 and 40 mixes from X_2.

Illustration 23

A local travel agent is planning a charter trip to a major sea resort. The eight day/seven-night package includes the fare for round-trip travel, surface transportation, board and lodging and selected tour options. The charter trip is restricted to 200 persons and past experience indicates that there will not be any problem for getting 200 persons. The problem for the travel agent is to determine the number of Deluxe, Standard, and Economy tour packages to offer for this charter. These three plans each differ according to seating and service for the flight, quality of accommodation, meal plans and tour options. The following table summarises the estimated prices for the three packages and the corresponding expenses for the travel agent. The travel agent has hired an aircraft for the flat fee of ₹ 2,00,000 for the entire trip.

Price and costs for tour packages per person

Tour Plan	*Price (₹)*	*Hotel Costs (₹)*	*Meals & other Expenses (₹)*
Deluxe	*10,000*	*3,000*	*4,750*
Standard	*7,000*	*2,200*	*2,500*
Economy	*6,500*	*1,900*	*2,200*

In planning the trip, the following considerations must be taken into account :

(i) At least 10 percent of the packages must be of the deluxe type.

(ii) At least 35 percent but not more than 70 percent must be of the standard type.

(iii) At least 30 percent must be of the economy type.

(iv) The maximum number of deluxe packages available in any aircraft is restricted to 60.

(v) The hotel desires that at least 120 of the tourists should be on the deluxe and standard packages together.

The travel agent wishes to determine the number of packages to offer in each type so as to maximize the total profit.

(a) Formulate the above as a linear programming problem.

(b) *Restate the above linear programming problem in terms of two decision variables, taking advantage of the fact that 200 packages will be sold.*

(c) *Find the optimum solution using graphical methods for the restated linear programming problem and interpret your results.*

Solution

Let x_1, x_2, x_3 denote the number of Deluxe, Standard & Economy tour packages to be offered to 200 persons that will maximize the profit. In other words, the total number of tours of three types offered by the concern is restricted to 200 only to maximize its profits.

The contribution (per person) arising out of each type of tour package offered is as follows:

Packages offered	Price (₹) (1)	Hotel Costs (₹) (2)	Meals & other Expenses (₹) (3)	Net Profit (₹) (4)=(1)–[(2)+(3)]
Deluxe	10,000	3,000	4,750	2,250
Standard	7,000	2,200	2,500	2,300
Economy	6,500	1,900	2,200	2,400

The travel agent has to pay the flat fee of ₹ 2,00,000 for the chartered aircraft for the entire trip. Consequently the objective function (profit function) will be:

Max Z = ₹ 2,250 x_1 + ₹ 2,300x_2 + ₹ 2,400x_3 – ₹ 2,00,000

The objective of the profit function is to maximize the profit.

The constraints based on the given conditions (i) to (v) are as folllows:

$x_1 \geq 20$ from condition (i) $\quad$ $x_3 \leq 60$ from condition (iii)

$x_2 \geq 70$ from condition (ii) $\quad$ $x_1 \leq 60$ from conditions (iv)

$x_2 \leq 140$ from condition (ii) $\quad$ $x_1 + x_2 \geq 120$ from condition (v)

Also $x_1 + x_2 + x_3 = 200$, $\quad$ $x_1, x_2, x_3 \geq 0$

On combining suitably, the above constraints reduce to the following constraints:

$$20 \leq x_1 \leq 60$$
$$70 \leq x_2 \leq 140$$
$$x_3 \geq 60$$
$$x_1 + x_2 \geq 120$$
$$x_1 + x_2 + x_3 = 200 \ \& \ x_1, x_2, x_3 \geq 0$$

(a) The linear programming problem formulated from the given data is (₹):

Max Z = ₹ 2,250 x_1 + ₹ 2,300x_2 + ₹ 2,400x_3 – ₹ 2,00,000

Subject to constraints

$$20 \leq x_1 \leq 60$$
$$70 \leq x_2 \leq 140$$

$x_3 \geq 60$

$x_1 + x_2 \geq 120$

$x_1 + x_2 + x_3 = 200$

$x_1, x_2, x_3 \geq 0$

(b) Since $x_1 + x_2 + x_3 = 200$ or $x_3 = 200 - (x_1 + x_2)$

Substitute the value of x_3 in the relations mentioned under (a), the linear programming problem reduces to the following:

Maximize Z = $-150x_1 + 100x_2 + 28,00,000$

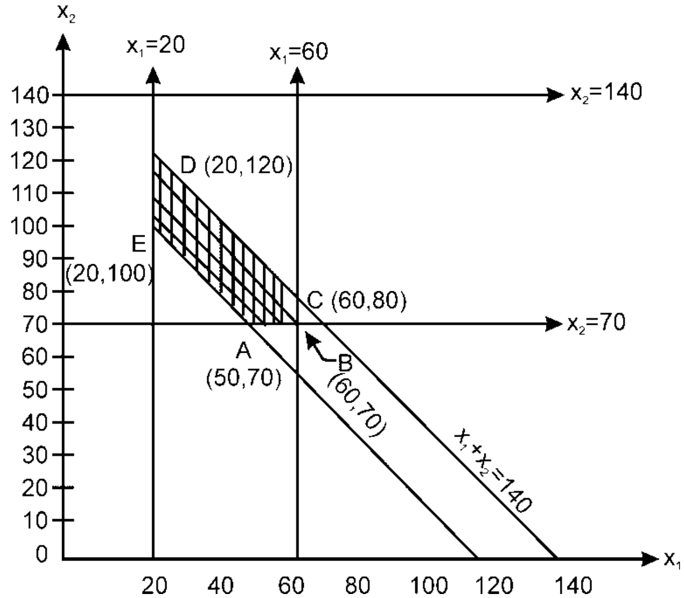
Subject to $20 \leq x_1 \leq 60$

$70 \leq x_2 \leq 14$

$120 \leq x_1 + x_2 \leq 140$

& $x_1, x_2 \geq 0$

(c) Refer to the figure drawn for the optimum solution of the restated L.P. problem under (b).



Point	*Co-ordinates of the corner points of the feasible region (values of x_1 and x_2)*	*Values of the objective function Z = – 150x_1 – 100x_2+ 2,80,000*
A	*(50,70)*	*₹ 2,65,500*
B	*(60,70)*	*₹ 2,64,000*
C	*(60,80)*	*₹ 2,63,000*
D	*(20,120)*	*₹ 2,65,000*
E	*(20,100)*	*₹ 2,67,000*

The maximum profit is attained at the corner point E whose co-ordinates are (20,100)

Interpretation: The profit of the objective function stated under (a) becomes maximum i.e. ₹ 2,67,000 when

$x_1 = 20$; $x_2 = 100$ and $x_3 = 80 \left\{\because x_3 = 200 - (x_1 + x_2) = 200 - (20 + 100)\right\}$

Simplex Method-Maximization

Illustration 24

An electronics firm is undecided as to the most profitable mix for its products. The products now manufactured are transistors, resistors and carbon tubes with a profit (per 100 unit) of ₹ 10, ₹ 6 and ₹ 4 respectively. To produce a shipment of transistors containing 100 units requires 1 hour of engineering, 10 hours of direct labours and 2 hours of administrative service. To produce 100 resistors are required 1 hour, 4 hours and 2 hours of engineering, direct labour and administrative time respectively. To produce on shipment of the tubes (100 units) requires 1 hour of engineering, 5 hours of direct labour and 6 hours of administration. There are 100 hours of engineering services available, 600 hours of direct labour and 300 hours of administration. What is the most profitable mix?

Solution

Let us tabulate the data in convenient manner.

	Products			
	Transistors	**Resistors**	**Carbon Tubes**	**Availability**
Engineering	1	1	1	100
Labour	10	4	5	600
Administration	2	2	6	300
Profit (per 100 units) (₹)	10	6	4	

Symbolically, Maximize $10x + 6y + 4z$

here x, y, z represents units of Transistors, Resistors and Carbon tubes (in 100 units)

Subject to $1x + 1y + 1z \leq 100$

$10x + 4x + 5z \leq 600$

$2x + 2y + 6z \leq 300$

$x \geq 0, y \geq 0, z \geq 0$

Introducing slack variables

Maximize $Z = 10x + 6y + 4z + 0s_1 + 0s_2 + 0s_3$

Subject to $x + y + z + s_1 = 100$

$10x + 4y + 5z + s_2 = 600$

$2x + 2y + 6z + s_3 = 300$

$x \geq 0, y \geq 0, z \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0$

SIMPLEX TABLEAU-I

C_j				10	6	4	0	0	0	Replacement Ratio
Fixed Ratio	**Basic Variables (B)**	**C_B**	**Value of Basic Variables $b(=X_B)$**	**x**	**y**	**z**	**s_1**	**s_2**	**s_3**	
1/10	s_1	0	100	1	1	1	1	0	0	100
	s_2	0	600	10	4	5	0	1	0	← 60
1/5	s_3	0	300	2	2	6	0	0	1	150
			$Z_j = \sum C_{Bi}X_j$	0	0	0	0	0	0	
			$C_j - Z_j$	10 ↑	6	4	0	0	0	

SIMPLEX TABLEAU-II

C_j				10	6	4	0	0	0	Replacement Ratio
Fixed Ratio	**Basic Variables (B)**	**C_B**	**Value of Basic Variables $b(=X_B)$**	**x**	**y**	**z**	**s_1**	**s_2**	**s_3**	
	s_1	0	40	0	6/10	5/10	1	-1/10	0	←400/6=67
2/3	x	10	60	1	4/10	5/10	0	1/10	0	600/4=150
2	s_3	0	180	0	12/10	5	0	-2/10	1	150
			$Z_j = \sum C_{Bi}X_j$	10	4	5	0	1	0	
			$C_j - Z_j$	0	2 ↑	-1	0	-1	0	

SIMPLEX TABLEAU-III

C_j				10	6	4	0	0	0	Replacement Ratio
Fixed Ratio	**Basic Variables (B)**	**C_B**	**Value of Basic Variables $b(=X_B)$**	**x**	**y**	**z**	**s_1**	**s_2**	**s_3**	
	y	6	400/6	0	1	5/6	10/6	-1/6	0	
	x	10	100/3	1	0	1/6	-2/3	1/6	0	
	s_3	0	100	0	0	4	-2	0	1	
			$Z_j = \sum C_{Bi}X_j$	10	6	20/3	10/3	+2/3	0	
			$C_j - Z_j$	0	0	-8/3	-10/3	-2/3	0	

$C_j - Z_j$ Since all entries in inidex row ($C_j - Z_j$) are –ve or zero. Current solution is optimum.

Hence the most profitable mix is y = $\frac{400}{6}$ Resistors and x = $\frac{100}{3}$ Transistors. The maximum profit is $400 + \frac{1000}{3} =$ ₹ $733\frac{1}{3}$.

Illustration 25

For a company engaged in the manufacture of three products viz., X, Y and Z, the available data are give in Tables 1, 2 and 3 below:

Table 1 : Minimum sales requirements

Product	Minimum Sales Requirements per month
X	10
Y	20
Z	30

Table 2 : Operations, Required Processing Times and Capacity

Operations	Time (Hrs) required per item of			Total available hours per month
	X	Y	Z	
1	1	2	2	200
2	2	1	1	220
3	3	1	2	180

Table 3 : Profit (₹) per unit

Product	Profit (₹)/Unit
X	10
Y	15
Z	8

Find out the product mix to maximize profit.

Solution

Let *x, y* and *z* denote the number of units produced per month for the products X, Y and Z respectively.

Minimum sales requirements give following constraints:

$x \geq 10$

$y \geq 20$ where $x, y, z \geq 0$

$z \geq 30$

Operations, processing times and capacity lead to following constraints:

$x + 2y + 2z \leq 200$

$2x + y + z \leq 220$

$3x + y + 2z \leq 180$

The objective function is

Maximize $10x + 15y + 8z$

Thus, we have to solve the following problem

Maximize $P = 10x + 15y + 8z$

Subject to $x + 2y + 2z \leq 200$ ....(i)

$2x + y + z \leq 220$ ....(ii)

$3x + y + 2z \leq 180$ ....(iii)

$x \geq 10$ ....(iv)

$y \geq 20$ ....(v)

$z \geq 30$ ....(vi)

$x, y, z \geq 0$

Let us make following substitutions

$x = a + 10$

$y = b + 20$

$z = c + 30$

Where $a, b, c \geq 0$

Substituting these values in the objective function and constraints (i), (ii), & (iii), the problem becomes

Maximize $P = 10a + 15b + 8c + 640$

Subject to $(a + 10) + 2(b + 20) + 2(c + 30) \leq 200$

$2(a + 10) + (b + 20) + (c + 30) \leq 220$

$3(a + 10) + (b + 20) + 2(c + 30) \leq 180$

Where, $a,b,c \geq 0$

Or, Maximize $P = 10a + 15b + 8c + 640$

Subject to $a + 2b + 2c \leq 90$

$2a + b + c \leq 150$

$3a + b + 2c \leq 70$

Where, $a,b,c \geq 0$

Adding slack variables s_1, s_2, and s_3, we get

Maximize $P = 10a + 15b + 8c + 640 + 0s_1 + 0s_2 + 0s_3$

Subject to $a + 2b + 2c + s_1 = 90$

$2a + b + c + s_2 = 150$

$3a + b + 2c + s_3 = 70$

The initial feasible solution is obtained by setting a = b = c = 0. The solution is s_1 = 90, s_2 = 150, s_3 = 70, P = 640. This solution and further improved solutions are presented in following matrix form:

SIMPLEX TABLEAU-I

	C_j		10	15	8	0	0	0	Minimum Ratio
C_B	Basic Variable (B)	Value of Basic Variables $b(=X_B)$	a	b	c	s_1	s_2	s_3	
0	s_1	90	1	2	2	1	0	0	45 ←
0	s_2	150	2	1	1	0	1	0	150
0	s_3	70	3	1	2	0	0	1	70
		$Z_j = \sum C_{Bi}X_j$	0	0	0	0	0	0	
		$C_j - Z_j$	10	15↑	8	0	0	0	

SIMPLEX TABLEAU-II

	C_j		10	15	8	0	0	0	Minimum Ratio
C_B	Basic Variable (B)	Value of Basic Variables $b(=X_B)$	a	b	c	s_1	s_2	s_3	
15	b	45	1/2	1	1	½	0	0	90
0	s_2	105	3/2	0	0	-1/2	1	0	70
0	s_3	25	5/2	0	1	-1/2	0	1	10 ←
		$Z_j = \sum C_{Bi}X_j$	15/2	15	15	15/2	0	0	
		$C_j - Z_j$	5/2↑	0	-7	-15/2	0	0	

SIMPLEX TABLEAU-III

C_j			10	15	8	0	0	0	Minimum Ratio
C_B	Basic Variable (B)	Value of Basic Variables $b(=X_B)$	a	b	c	s_1	s_2	s_3	
15	b	40	0	1	4/5	3/5	0	-1/5	
0	s_2	90	0	0	-3/5	-1/5	1	-3/5	
10	a	10	1	0	2/5	-7/5	0	2/5	
		$Z_j = \sum C_{Bi} X_j$	10	15	16	7	0	1	
		$C_j - Z_j$	0	0	-8	-7	0	-1	

Since all NER elements are less than or equal to zero, the previous table gives the optimal solution. The solution is

a = 10, b = 40 and c = 0

Substituting these values

x = 10 + 10 = 20, y = 40 + 20 = 60,

z = 0 + 30 = 30

and the value of objective function is given by

P = ₹ 10 × 20 + ₹ 15 × 60 + ₹ 8 × 30 = ₹ 200 + ₹ 900 + ₹ 240 = ₹ 1,340

The optimal product mix is to produce 20 units of X, 60 units of Y and 30 units of Z to get a profit of ₹ 1,340.

Simplex Method-Minimization

Illustration 26

You are required to solve following linear programming problem.

Minimize $Z=30\,x_1+40\,x_2$

Subject to the constraints

$10\,x_1+15\,x_2 \geq 450$

$20\,x_1+15\,x_2 \geq 600$

$x_1, x_2 \geq 0$

Solution

Expressing the problem in the standard form by adding surplus variables s_1 and s_2 and artificial variables A_1 and A_2, M is a large number.

Minimize $Z=30\,x_1+40\,x_2+0s_1 + 0s_2 +M\,A_1+M\,A_2$

Subject to the constraints

$10x_1+15x_2-s_1+A_1=450$

$20x_1+15x_2-s_2+A_2=600$

$x_1, x_2, s_1, s_2, A_1, A_2 \geq 0$

SIMPLEX TABLEAU-I

C_B	Basic Variable (B)	Value of Basic Variables $b(=X_B)$	30	40	0	0	M	M	Minimum Ratio
			x_1	x_2	s_1	s_2	A_1	A_2	
M	A_1	450	10	15	-1	0	1	0	45
M	A_2	600	20	15	0	-1	0	1	← 30
	Z_j		30M	30M	-M	-M	M	M	
	C_j-Z_j		30-30M ↑	40-30M	M	M	0	0	

30-30M is largest –ve

SIMPLEX TABLEAU-II

C_B	Basic Variable (B)	Value of Basic Variables $b(=X_B)$	30	40	0	0	M	M	Minimum Ratio
			x_1	x_2	s_1	s_2	A_1	A_2	
M	A_1	150	0	15/2	-1	½	1	-1/2	20 ←
30	x_1	30	1	3/4	0	-1/20	0	1/20	40
	Zj		30	15/2M+45/2	-M	M/2-3/2	M	-M/2+3/2	
	Cj-Zj		0	-15/2M+35/2 ↑	M	-M/2+3/2	0	3/2M-3/2	

-15/2M+35/2 is largest –ve

SIMPLEX TABLEAU-III

C_B	Basic Variable (B)	Value of Basic Variables $b(=X_B)$	30	40	0	0	M	M	Minimum Ratio
			x_1	x_2	s_1	s_2	A_1	A_2	
40	x_2	20	0	1	-2/15	1/15	2/15	-1/15	
30	x_1	15	1	0	1/10	-1/10	-1/10	1/10	
	Zj		30	40	-7/3	-1/3	7/3	1/3	
	Cj-Zj		0	0	7/3	1/3	M-7/3	M-1/3	

Since all entries in C_j-Z_j row are either zero or positive in table III, the current solution is the optimum one and is given by:

x_1=15, x_2=20, Minimum Z=1,250

10.10 Limitations of Linear Programming

Important limitations of linear programming problems are as follows:

(i) A primary requirement of linear programming is that the objective function and every constraint function must be linear. This requires that the measure of effectiveness and resource usage must be proportional to the level of each activity conducted individually. However, programming problems which are non-linear arise quite frequently. It is occasionally possible to reformulate a non-linear programming problem into the linear programming format so that the simplex method can be used. This is, however, the fortunate exception rather than the rule.

(ii) It may not be possible to solve those problems using linear programming, in which non-linearity arises because of joint interactions between some of the activities regarding the total measure of effectiveness or total usage of some resource. Hence linear programming problem requires that the total measure of effectiveness and total resource usage resulting from the joint performance of the activities must equal the respective sums of these quantities resulting from each activity being performed individually.

In some situations, it may not be true. For example, consider a situation where by a product is produced with the scrap material from the primary product. The material would still have to be procured if only one of the two products were produced. However, the total material requirements if both products are produced is less than the sum of requirements is each were produced individually. It may not be possible to handle such situation with linear programming problems.

(iii) In linear programming problem, fractional values are permitted for the decisions variables. However, many decision problems require that the solution for decision variable should be obtained in non-fractional values. Rounding-off the values obtained by linear programming techniques may not result into an optimal solution in such cases.

(iv) In linear programming problem, coefficients in the objective function and the constraint equations must be completely known and they should not change during the period of study i.e. they should be known constraints. In practical situation, it may not be possible to state all coefficients in the objective function and constraints with certainty. Furthermore, these coefficients may actually be random variables, each with an underlying probability distribution for the values. Such problems cannot be solved using linear programming.

11

The Transportation Problem

LEARNING OBJECTIVES

After studying this chapter students will understand.

- Formulate a transportation problem
- Find initial basic feasible solution by following various methods
- Find minimum transportation cost schedule
- Ascertain minimum transportation cost schedule
- Discuss appropriate method to make unbalanced transportation problems balanced
- Examine prohibited and preferred routes
- Formulate and solve transportation problem

11.1 Introduction

This chapter is devoted to special problems that belong to the so called transportation class. These special problems are quite important from the practical point of view. Their practical importance arises because several real situations can be described by systems of equations that fall into the transportation class. Sizeable applications of linear programming problems have been made in this field.

A typical transportation problem is concerned with selecting routes in a production distribution network among manufacturing plants and distribution warehouses or among regional distribution warehouses and local distribution outlets. The objective is to schedule transportation of products from sources to destination in such a way as to minimise the total transportation cost.

A transportation problem can be paraphrased by considering 'm' factories which supply to 'n' warehouses or distribution centers. The factories produce goods at level $a_1, a_2, \ldots a_m$ and the demand are requirements of the distribution centers for these goods are $b_1, b_2 \ldots b_n$ respectively. If the unit cost of shipping from i-th factory to warehouse j is c_{ij} what shipping pattern minimizes the transportation cost?

Let x_{ij} denote number of units transported from factory i to destination j

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, \ldots, m \qquad \ldots (1)$$

$\sum_{i=j}^{m} x_{ij} = b_j j = 1,....,n$ ... (2)

Minimize Z = $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}, x_{ij}$ ... (3)

Where, $x_{ij} \geq 0$ for all i,j ... (4)

Equation 1 is interpreted as the sum of what leaves each factory (or origin) for the various warehouses (or destinations) is equal to what is produced at the factory, (2) implies that the sum of what arrives at each warehouse from the various origins in equal to the demand at the warehouse. The double sum of equation 3 represents the total transportation cost. The non-negative conditions (4) arise because negative values for any x_{ij} have no physical meaning.

Solution to the problem described by equations 1 to 4 is given under the condition that

$$\sum_{i=j}^{m} a_i = \sum_{j=i}^{n} a_j$$

From the physical point of view this condition means that the system of equations is in balance i.e., total production is equal to the total requirements. Equations (1) and (2) may be expanded as below.

$x_{11} + x_{12} ... + x_{1n} = a_1$

$x_{12} + x_{22} + ... + x_{2n} = a_2$

$x_{m1} + x_{m2} + x_{mn} = a_m$

$x_{11} + x_{12} + ... + x_{m1} = b_1$

$x_{12} + x_{22} + ... + x_{m2} = b_2$

:

:

:

$x_{1n} + x_{2n} + ... x_{mn} = b_n$

This is a system of (m+n) equations in mn unknown; but the equations are not independent. Two important observations about the system of equations are worth noting.

(i) The co-efficient of x_{ij}'s are either 1 or 0

(ii) Any x_{ij} appears only once in the first m equations and once in the last n equations.

Our problem is to determine x_{ij}, the quantity that is to be shipped from the i-th origin to the j-th destination in such a way that the total transportation cost is minimum. The quantities of interest can be tabulated as below.

Destinations

Origins	D_1	D_2...	D_i....	D_a	**Available**
0_1 0_2 : 0_i 0_m	c_{11} c_{21} : c_{i1} c_{m1}	c_{12}.... c_{22}.... : C_{i2}.... C_{m_2}....	C_{1j}... C_{2j}... : C_{ij}.... C_{mj}...	c_{1n} c_{2n} : c_{in} c_{mn}	a_1 a_2 : a_i a_m
Required	b_1	b_2...	b_j...	b_n	Total

The method for solving the class of problems consist of finding a basic feasible solution. If it is not optimal (an interactive procedure is used to improve it) then optimality test is applied to make it an optimal solution.

11.2 Methods of finding Initial Basic Feasible solution to transportation problems

In this chapter we will learn three methods of finding initial Basic Feasible solution.

11.2.1 North-West corner Rule: The idea is to find an initial to find an initial basic feasible solution i.e., a set of allocations that satisfied the row and column totals. This method simply consists of making allocations to each row in turn, apportioning as much as possible to its first cell and proceeding in this manner to its following cells until the row total in exhausted.

The algorithm involved under north-west corner rule consists for the following steps:

Steps:

1. Before allocation, ensure that the total of availability and requirement is equal. If not then make the same equal.

2. The first allocation is made in the cell occupying the upper left hand corner of the matrix. The assignment is made in such a way that either the resource availability is exhausted or the demand at the first destination is satisfied.

3. (a) If the resource availability of the row one is exhausted first, we move down the second row and first column to make another allocation which either exhausts the resource availability of row two or satisfies the remaining destination demand of column one.

   (b) If the first allocation completely satisfies the destination demand of column one, we move to column two in row one, and make a second allocation which either exhausts the remaining resource availability of row one or satisfies the destination requirement under column two.

4. The above procedure is repeated until all the row availability and column requirements are satisfied. Consider, for example, the following sample problem. This method does not use transportation costs which we shall bring in later in the other method.

							Available
							10
							12
							4
							18
							20
Required	8	8	16	3	8	21	

Row wise allocation as above, is made below. The maximum that can be allocated.

8	2					10/2/0
	6	6				12/6/0
		4				4/0
		6	3	8	1	18/12/9/1/0
					20	20/0
8	8	16	3	8	21	
0	6	10	0	0	21	
	0	6			0	
		0				

In cell (1, 1) 8 is allocated. This satisfies completely the requirements of column 1, but availabilities of row 1 are not completely exhausted. Therefore, we proceed to cell (1,2) in row 1 and allocate the remaining 2 units. We can start at cell (2,2) only since first column's requirements have been completely satisfied, there is nothing that we can allocate in its first cell (2,1). By the aforesaid procedure, we allocate 6 units to cell (2,2) and another 6 units to cell (2,3) exhausting completely the availabilities of row 2. This process is continued until we reach the cell (5,6).

11.2.2 The Least cost method:

(i) Before starting the process of allocation, ensure that the total of availability and demand is equal. The least cost method starts by making the first allocation in the cell whose shipping cost (or transportation cost) per unit is lowest.

(ii) This lowest cost cell is loaded or filled as much as possible in view of the origin capacity of its row and the destination requirements of its column.

(iii) We move to the next lowest cost cell and make an allocation in view of the remaining capacity and requirement of its row and column. In case there is a tie for the lowest cost cell during any allocation, we can exercise our judgment and we arbitrarily choose cell for allocation.

(iv) The above procedure is repeated till all row requirements are satisfied.

11.2.3 Vogel's Approximation Method (VAM) : The Vogel's Approximation Method (VAM) is considered to be superior to the North-West Corner rule in that it usually provides an initial solution that is optimal or nearly so. Therefore, we shall also stick to it for the discussion ahead. However, the readers may like to try their hand on the following solved examples by the North-West corner rule and Least-cost method for sake of practice. But we here apply VAM method.

VAM entails the following steps :

Step 1: For each row of the transportation table identify the smallest and next smallest costs. Find the difference between the two costs and display it to the right of that row as "Difference" (Diff.). Likewise, find such a difference for each column and display it below that column. In case two cells contain the same least cost then the difference will be taken as zero.

							Avail.	Diff.
	4	6	9	2	7	8	10	2
	3	5	4	8	10	0 (12)	12/0	3
	2	6	9	8	4	13	4	2
	4	4	5	9	3	6	18	1
	9	8	7	3	2	14	20	1
Req.	8	8	16	3	8	21/9		
Diff.	1	1	1	1	1	6		

Step 2: From amongst these row and column differences, select the one with the largest difference. Allocate the maximum possible to the least cost cell in the selected column or row. If there occurs a tie amongst the largest differences, the choice may be made for a row or column which has least cost. In case there is a tie in cost cell also, choice may be made for a row or column by which maximum requirement is exhausted. Hatch that column or row containing this cell whose totals have been exhausted so that this column or row is ignored in further consideration.

Step 3: Recompute the column and row differences for the reduced transportation table and go to step 2. Repeat the procedure until the entire column and row totals are exhausted.

The VAM is applied to the previous problem on the table above as an illustration. Entered in the table are the given unitshipping costs.

The arrow indicates that the last column having the largest difference from amongst all rows and columns is selected for allocation. Cell (2, 6) with the least cost is picked up and allocated 12 units. Row 2 is hatched since its total is exhausted.

							Avail.	Diff.
	4	6	9	2	7	8	10	2
	3	5	4	8	10	0 [12]	–	–
	2 [4]	6	9	8	4	13	4/0	2
	4	4	5	9	3	6	18	1
	9	8	7	3	2	14	20	1
Req.	8/4	8	16	3	8	9		
Diff.	2	2	2	1	1	2		

							Avail.	Diff.
	4	6	9	2 [3]	7	8	10/7	2
	3	5	4	8	10	0 [12]	–	–
	2 [4]	6	9	8	4	13	–	–
	4	4	5	9	3	6	18	1
	9	8	7	3	2	14	20	1
Req.	4	8	16	3/0	8	9		
Diff.	0	2	2	1	1	2		

This procedure is continued in the following tables until all the columns and rows are hatched.

							Avail.	Diff.
	4	6	9	2 $\boxed{3}$	7	8	7	2
	3	5	4	8	10	0 $\boxed{12}$	–	–
	2 $\boxed{4}$	6	9	8	4	13	–	–
	4	4	5	9	3	6	18	1
	9	8	7	3	2 $\boxed{8}$	14	20/12	5
Req.	4	8	16	–	8/0	9		
Diff.	0	2	2	–	1	2		

							Avail.	Diff.
	4	6	9	2 $\boxed{3}$	7	8	7	2
	3	5	4	8	10	0 $\boxed{12}$	–	–
	2 $\boxed{4}$	6	9	8	4	13	–	–
	4	4 $\boxed{8}$	5	9	3	6	18/10	0
	9	8	7	3	2 $\boxed{8}$	14	12	1
Req.	4	8/0	16	–	–	9		
Diff.	0	2	2	–	–	2		

							Avail.	Diff.
	4 $\boxed{4}$	6	9	2 $\boxed{3}$	7	8	7/3	4
	3	5	4	8	10	0 $\boxed{12}$	–	–
	2 $\boxed{4}$	6	9	8	4	13	–	–
	4	4 $\boxed{8}$	5	9	3	6	10	1
	9	8	7	3	2 $\boxed{8}$	14	12	2
Req.	4/0	–	16	–	–	9		
Diff.	0	–	2	–	–	2		

							Avail.	Diff.
	4 $\boxed{4}$	6	9	2 $\boxed{3}$	7	8	3	1
	3	5	4	8	10	0 $\boxed{12}$	–	–
	2 $\boxed{4}$	6	9	8	4	13	–	–
	4	4 $\boxed{8}$	5	9	3	6	10	1
	9	8	7 $\boxed{12}$	3	2 $\boxed{8}$	14	12/0	7
Req.	–	–	16/4	–	–	9		
Diff.	–	–	2	–	–	2		

4 [4]	6	9	2 [3]	7	8 [3]
3	5	4	8	10	0 [12]
2 [4]	6	9	8	4	13
4	4 [8]	5 [4]	9	3	6 [6]
9	8	7 [12]	3	2 [8]	14

Since only one column is left in the table above there is no question of finding the differences and 16 units are allocated to the various cells of this column straightway as below

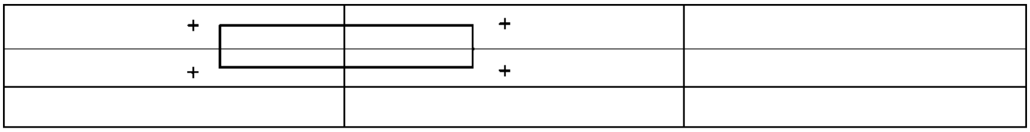
							Avail.
	4 [4]	6	9	2 [3]	7	8 [3]	10
	3	5	4	8	10	0 [12]	12
	2 [4]	6	9	8	4	13	4
	4	4 [8]	5 [4]	9	3	6 [6]	18
	9	8	7 [12]	3	2 [8]	14	20
Req.	8	8	16	3	8	21	

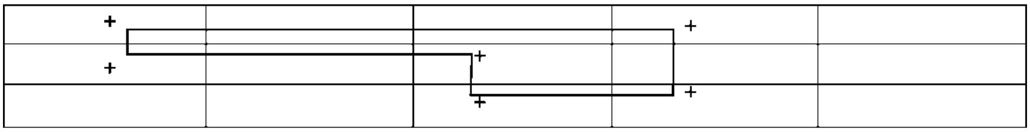
So many tables above have been drawn merely for exposition. All we shall see with the following examples the initial allocation by VAM can as well be obtained in just one table.

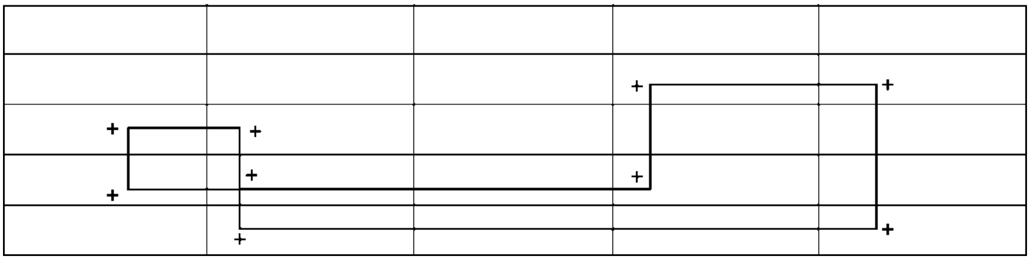
The superiority of VAM lies in the fact that, unlike the Northwest Corner rule, not only availabilities and requirements are taken into account but also due regard is paid to the unit costs. A row and column difference actually indicates the minimum unit penalty incurred by failing to make an allocation to the smallest cost cell in the row or column.

11.3 Optimality Test

Now we know how to obtain the initial basic feasible solution but it remains to be tested whether it is optimal or not if it is not so, how do we go about deriving the optimal solution? Before we taken up this matter it is necessary to explain the concept of **independence and non-independence** amongst allocations. Towards this, consider the following transportation tables of different problems. The allocations in these are shown by plus signs.







The lines constitute, in these allocation pattern what is known as loops. A loop may or may not involve all the allocations. It consists of (at least 4) horizontal and vertical lines with an allocation at each corner which, in turn is a join of a horizontal and vertical line. At this stage the last loop above is to be particularly noted. Here, two lines intersect each other at cell (2,4) and do not simply join; therefore, this is not to be regarded as a corner. Such allocations in which a loop can be formed are known as non-independent, whereas those in which a loop cannot be formed as regarded as independent. Towards explanation of this nomenclature consider the following allocation pattern with a loop in it.

4	6		10
8	2		10
		5	5
12	8	5	

It is possible to progressively adjust the allocations along the corners of the loop without violating the row and column totals. One adjusted allocations pattern for the above table is show below as an example.

4-2=2	6+2=8	
8+2=10	2-2=0	
		5

But such a possibility of reallocation does not exist where a loop cannot be formed i.e., allocations are independent.

Coming back now to the optimality test to be described shortly it can be applied to a transportation table if it satisfies the following conditions.

(1) It contains exactly m+n-1 allocations where m and n represent the number of rows and column of the table.

(2) These allocations are independent i.e. a loop can not be performed by them

Without giving the proof, we may also mention that whenever there are m+n-1 allocation and a tick is placed in an empty cell one and only one loop can be passed through the ticked cell and some are all of the m+n-1 allocation.

							Avail.
	4 [4]	6	9	2 [3]	7	8 [3]	10
	3	5	4	8	10	0 [12]	12
	2 [4]	6	9	8	4	13	4
	4	4 [8]	5 [4]	9	3	6 [6]	18
	9	8	7 [12]	3	2 [8]	14	20
Req.	8	8	16	3	8	21	

Cell Evaluations:

Consider the initial solution to the transportation problem as obtained by VAM and produced below for example.

The allocations are m+n-1 in number and independent. It may be stated in passing that initial allocations obtained by Northwest Corner rule or VAM are always in independent positions though they may be m+n-1 or less than m+n-1 in number. The motivation in the optimality test is to see if it is possible to improve upon the existing solution. In the simplex language, are any of the net evaluations negative? Before we attempt this for the problem on hand let us see what net evaluations mean in the context of transportation problems. As in the simplex method, we want to replace a basic variable with a non-basic variable i.e., want to zeroise an existing allocation cell and instead make some allocation in an empty cell. The problem then boils down to determination of an “outgoing allocation” and an “incoming allocation” that brings us closest to the optimal, if at all possible.

							Avail.
	4 (4)	6 (√)	9 (3)	2 (3)	7	8	10
	3	5	4	8	10	0 (12)	12
	2 (4)	6	9	8	4	13	4
	4	4 (8)	5 (1)	9	3	6 (9)	18
	9	8	7 (12)	3	2 (8)	14	20
Req.	8	8	16	3	8	21	

Towards this, we arbitrarily select the empty cell (1, 2), put a tick in it meaning that we want to make some allocation in it and zeroing one (or more) existing allocation. For reason mentioned above, one and only one loop can be formed in the allocation pattern. It is shown in the table above. If we bring one unit to the ticked cell, other allocations on the corners of the loop would be adjusted as below.

0+1	3-1
8-1	1+1

This would mean lowering of the cost by

6×1-9×1+5×1-4×1 = 2

This then is the net evaluation of the ticked cell and obviously it would pay to bring this cell into solution since the cost would be reduced by 2 per unit transferred to this along the loop. Likewise, we can compute cell evaluations for all the empty cells and select that as the incoming one which has the most –ve cell evaluation.

The procedure would consist of ticking each empty cell, forming a loop involving the tick and computing the cell evaluation for a unit transfer to the ticked cell along the loop. Obviously this would be a rather lengthy procedure. Fortunately there exists a much easier way of computing cell evaluations for all empty cells at one stroke. This is what we described as the optimality test above and it is applicable to transportation tables containing exactly m +n –1 independent allocations. It consists of the following steps:

1. Determine a set of m +n numbers

   u_i; i = 1, 2…m

   v_j ; j = 1, 2…n

such that, for each occupied cell

$c_{ij} = u_i + v_j$

2. Compute the cell evaluation Δ_{ij} for each empty cell (i, j) by the relationship

$$\Delta_{ij} = c_{ij} - (u_i + v_j)$$

Let us apply this optimality test to the problem on hand. Below is the table indicating just the unit costs in allocated cells in the problem on hand. We set $u_1 = 0$ for row 1. it could be assigned any other value but zero has been assigned for ease in subsequent computations. Also it is desirable to select that row or column which contains maximum number of allocations. In this case row 1 has a maximum of 3 allocations. Now, we wish to find the value of v_1, v_3 and v_4 for the column of these 3 allocations which can be accomplished as below.

$u_1 + v_1 = 4$ Since $u_1 = 0$ therefore, $v_1 = 4$

$u_1 + v_3 = 9$ $v_3 = 9$

$u_1 + v_4 = 2$ $v_4 = 2$

							u_1
	4		9	2			$0(u_1)$
						0	$-10(u_2)$
	2						$-2(u_3)$
		4	5			6	$-4(u_4)$
			7		2		$-2(u_5)$
v_j	$4(v_1)$	$8(v_2)$	$9(v_3)$	$2(v_4)$	$4(v_5)$	$10(v_6)$	

Values of $v_1 = 4$, $v_3 = 9$, $v_4 = 2$ have been entered at the foot of each volume. Now let us proceed with v_1, v_3 and v_4 in turn.

v_1: $u_3 + v_1 = 2$ since $v_1 = 4$, therefore, $u_3 = -2$ is entered

v_3: $u_4 + v_3 = 5$ since $v_3 = 9$, therefore, $u_4 = -4$ is entered

v_4: No more allocation in its column and we ignore it.

Having established and entered values of u_3 and u_4 let us take up these in turn.

u_2: No more allocation in its row and we ignore it.

u_4: $u_4 + v_2 = 4$ since $u_4 = -4$ therefore, $v_2 = 8$ is entered

$u_4 + v_6 = 6$ since $u_4 = -4$ therefore, $v_6 = 10$ is entered

Proceeding is this manner we fill up all u_i and v_j.

In the following table, we derive $u_i + v_j$ for each empty cell.

	8			4	10
−6	−2	−1	−8	−6	
	6	7	0	2	8
0			−2	0	
2	6		0		8

$u_j + v_j$

The following table gives the cell evaluations derived by subtracting above figures from the original unit costs.

	–2			3	–2
9	7	5	16	16	
	0	2	8	2	5
4			11	3	
7	2		3		6

$\Delta_{ij}+C_{ij}-(u_i+v_j)$

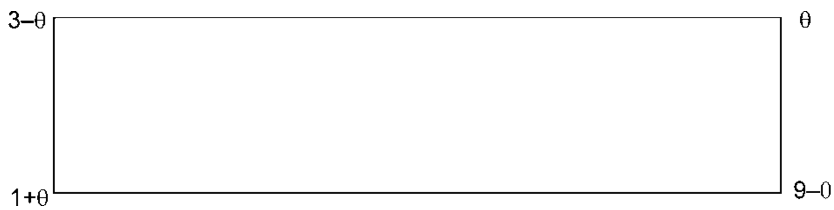
Since some of the Δ_{ij}s are –ve the initial solution is not optimal. It would pay the most to zeroise an existing allocation and allocate as much as possible to the most –veΔ_{ij} cell (6, 1). There are two cells having the same most –veΔ_{ij}. We pick up one of them arbitrarily for ticking.

This is done below is the allocation table by ticking cell (1,6) and involving it in the only loop.

4		3	3		
					12
4					
	8	1			9
		12		8	

Transferring the maximum amount to the ticked cell

Reallocation is done by transferring the maximum amount to the ticked cell. The rule for obtaining the maximum amount (say, θ_{max}) that can be transferred to the ticked cell is derived below:



Starting with the ticked cell θ is added to and subtracted from the corner allocations alternately.

θ_{max} = min of $3-\theta = 0$

$9-\theta = 0 = 3$ Thus$\theta_{max} = 3$

Reallocation along this loop is shown below for clarity:

Complete reallocations follow:

4			3		3	
					12	Reallocation matrix
4						
	8	4			6	(optimal as tested below)
		12		8		

The optimality test is performed below:

							u_j
	4			2		8	0
						0	–8
	2						–2
		4	5			6	–2
			7		2		0
v_j	4	6	7	2	2	8	

	6	7		2	
–4	–2	–1	–6	–6	
	4	5	0	0	6
2			0	0	
4	6		2		8

(u_i+v_j) matrix

	0	2		5	
7	7	5	14	16	
	2	4	8	4	7
2			9	3	
5	2		1		6

Δ_{ij} matrix

Since none of the Δijis –ve, this solution is optimal, though an alternative solution may exist.

Least cost	4×4	=	16	
	3×2	=	6	
	3×8	=	24	
	0×12	=	0	
	4×2	=	8	
	8×4	=	32	
	4×5	=	20	
	6×6	=	36	
	12×7	=	84	
	8×2	=	16	
			242	Answer

11.4 Special Cases

11.4.1 Unbalanced transportation problem

Illustration 1

A company has factories at A, B and C which supply warehouse at D, E, F and G. Monthly factory capacities are 160, 150 and 190 units respectively. Monthly warehouse requirements are 80, 90, 110 and 160 units respectively. Units shipping costs (in ₹) are as follows:

	To	D	E	F	G
	A	42	48	38	37
From	B	40	49	52	51
	C	39	38	40	43

Solution

Availabilities = 160 +150 +190 = 500

Requirements = 80 +90 +110 +160 = 440

Availabilities –Requirement = 500 –440 = 60

Therefore, a dummy warehouse H is introduced, and initial solution is obtained below by VAM in just one table.

	D	E	F	G	H	Available	Diff.
A	42	48	38	37 [160]	0 [e]	160/0	37/1/1/1
B	40 [80]	49	52 [10]	51	0 [60]	150/90/10/0	40/9/11/1
C	39	38 [90]	40 [100]	43	0	190/100/0	38/1/1/3
Req.	80/0	90/0	110/10/0	160/0	60/0		
Diff.	1	10	2	6	0		
	1	10	2	6	–		
	1	–	2	6	–		
	–	–	2	–	–		

11.4.2 Degeneracy : We know that a transportation problem has m +n –1 basic variables which means that the number of occupied cells is such a solution is one less than the number of rows plus the number of columns. It may happen sometimes that the number of occupied cells is less than m +n –1. Such a solution is called a degenerate solution. *We handle such a situation by introducing an infinitesimally small allocation e in the least cost and independent cell.*

In this example, since there are only 6 (one less than m+n –1) allocations, an infinitesimally small allocation 'e' is placed in the least cost and independent cell (1, 5). This solution is tested for optimality below. (N.B.: if allocations were m +n –2 we would place two e's i.e. e_1, e_2 which are virtually zero in the 2 least cost independent cells). This device enables us to apply to optimality test (m +n –1) allocations.

						u_i
				37	0	0
	40		52		0	0
		38	40			–12
v_j	40	50	52	37	0	

40	50	52			
	50		37		(u_i+v_j) matrix
28			25	-12	

2	–2	–14			
	–1		14		Δ_{ij}matrix
11			18	12	

Since there are –veΔ_{ij} 's the initial solution is not optimal. Reallocation is done below by ticking the most -veΔ_{ij} cell (1, 3) and involving it in the loop.

θ max = e			√	160	e
min10 – θ	80		10		60
min e – θ		90	100		

[Note that the maximum that can be transferred to the ticked cell is e. Since e is infinitesimally small it leaves other corner allocations unaffected. (Intermediate i.e. non corner allocations are never altered in the process of reallocations)].

		e	160		
80		10		60	Reallocation
	90	100			

This solution is tested for optimally below:

		38	37		38
40		52		0	52
	38	40			40
–12	–2	0	–1	–52	

26	36			–14	
	50		51		(u_i+v_j) matrix
28			39	–12	

16	12			14	
	–1		0		Δ_{ij} matrix
11			4	12	

Since there are –veθ, this solution too is not optimal. Reallocation is done below:

θ_{max}=min $\begin{pmatrix} 10 - \theta = 0 \\ 90 - \theta = 0 \end{pmatrix}$

		e	160	
80	√	10-θ		60
	90-?	100+θ		

		e	160		
80	10			60	Reallocation
	80	110			

This solution is tested for optimality as below:

					u_i
		38	37		–13
40	49			0	0
	38	40			–11
40	49	51	50	0	

v_j

27	36			–13	
		51	50		$(u_i + v_j)$
29			39	–11	

15	12			13	
		1	1		Δ_{ij}matrix
10			4	11	

Since all Δ_{ij}'s are +ve, this solution is optimal. The student may compute the minimum total cost.

11.4.3 Multiple solution case

Illustration 2

Solve the following transportation problem

	D1	D2	D3	D4	
Q1	5	3	6	2	19
Q2	4	7	9	1	37
Q3	3	4	7	5	34
	16	18	31	25	

Solution

The initial solution is obtained below by Vogel's method

	D1	D2	D3	D4	Available	Cost Diff.
Q1	5	3 [18]	6 [1]	2	19/1/0	1/2/2/1
Q2	4 [12]	7	9	1 [25]	37/12/0	3/3
Q3	3 [4]	4	7 [30]	5	34/30/0	1/1/1/4
Req.	16	18	31	25		
	4	0	30	0		
	0					
Cost Diff.	1	1	1	1		
	1	1	1	–		
	2	1	1	–		
	2	–	1	–		

Since there are 6 (i.e. m + n - 1) allocations the optimality test is performed below straightway:

					u_i
		3	6		0
	4			1	2
	3		7		1
v_j	2	3	6	–1	

2			-1
	5	8	
	4		0

(ui+vj) Matrix

	D1	D2	D3	D4
Q1	3			3
Q2		2	1	
Q3		0		5

Δ_{ij} matrix

Since all Δ_{ij} 's are positive; the initial solution is optimal. Also, since one Δ_{ij} is zero there exists other alternative optimal solution, too i.e. A zero Δ_{ij} for an unused square means that if this route were brought into the solution, the shipping assignments would change, yet the total transportation cost would be same. To determine what this alternative optimal solution is, we follow the same procedure used for bringing any route into the solution.

11.5 Maximization Transportation Problems

Illustration 3

A company has 3 factories manufacturing the same product and 5 sales agencies in different parts of the country. Production costs differ from factory to factory, and the sales prices from agency to agency. The shipping cost per unit product from each factory to any agency is known. Given the following data, find the production and distribution schedules most profitable to the company

Production cost/unit	***Max capacity No. of units***	***Factory***
20	*150*	*1*
22	*200*	*2*
18	*125*	*3*

1	*1*	*5*	*9*	*4*	*Shipping Cost*
9	*7*	*8*	*3*	*6*	
4	*5*	*3*	*2*	*7*	
1	*2*	*3*	*4*	*5*	*Agency j*
80	*100*	*75*	*45*	*125*	*Demand to be met*
30	*32*	*31*	*34*	*29*	*Sales price*

Solution

The profit matrix is derived below from the equation;

Profit = Selling Price –production cost –shipping cost.

Profit matrix

	1	**2**	**3**	**4**	**5**
1	9	11	6	5	5
2	–1	3	1	9	1
3	8	9	10	14	4

The above profit matrix is converted into a loss matrix so that minimization transportation algorithms can be applied.

This is done by subtracting each entry of the above table from 14 (the largest number in the table). Subsequently we get the following loss table.

	1	2	3	4	5
1	5	3	8	9	9
2	15	11	13	5	13
3	6	5	4	0	10

Since the problem is unbalanced, a dummy warehouse is added and the initial solution is obtained by VAM below:

	1	2	3	4	5	Dummy	Available	Diff.
1	5 (50)	3 (100)	8	9	9	0	150/50/0	3/2/2/2/4
2	15	11	13	5 (45)	13 (105)	0 (50)	200/150/105/0	5/6/2/2/2/2
3	6 (30)	5	4 (75)	0	10 (20)	0	125/50/20/0	0/4/1/1/4/4
Req.	80	100	75	45	125	50		
	30	0	0	0	105	0		
	0				0			
Diff.	1	2	4	5	1	0		
	1	2	4	5	1	–		
	1	2	4	–	1	–		
	1	2	–	–	1	–		
	1	–	–	–	1	–		
	9	–	–	–	3	–		

Since there are 8 (i.e. m + n - 1) allocations the optimality test is straightway performed below

							u_i
	5	3					–4
				5	13	0	0
	6		4		10		–3
v_j	9	7	7	5	13	0	

		3	1	9	–4	
9	7	7				(u_i+v_j) matrix
	4		2		–3	

		5	8	0	4	
6	4	6				Δij matrix
	1		–2		3	

max=min $\begin{pmatrix} 20-\theta=0 \\ 45-\theta=0 \\ \quad =20 \end{pmatrix}$

50	100				
			45-θ	105+?	50
30		75	√	20-?	

50	100					
			25	125	50	Reallocation Matrix
30		75	20			

This solution is tested for optimality test

							u_i
	5	3					–6
				5	13	0	0
	6		4	0			–5
v_j	11	9	9	5	13	0	

		3	-1	7	-6	
11	9	9				(u_i+v_j) matrix
	4			8	-5	

		5	10	2	6	
4	2	4				Δ_{ij}matrix
	1			2	5	

Since all Δ_{ij} are positive this solution is optimal

50×9	=	450
100×11	=	1,100
25×9	=	225
125×1	=	125
30×8	=	240
75×10	=	750
20×14	=	280
	₹3,170	Answer

Note: Alternatively, this Solution can be obtained by introducing dummy requirement in Profit-Matrix itself.

11.6 Prohibited routes

Sometimes in a given transportation problem, some routes may not be available. There could be several reasons for this such as bad road conditions or strike etc. In such situations, there is a restriction on the route available for transportation. To handle such type of a situation, a very large cost (or a negative profit for the maximization problem) represented by ∞ or 'M' is assigned to each of such routes which are not available. Due to assignment of very large cost, such routes would automatically be eliminated in the final solution. The problem is the solved in its usual way.

Illustration 4

Solve the following transportation problem:

	1	***2***	***3***	***4***	***5***	***6***	***Stock available***
1	*7*	*5*	*7*	*7*	*5*	*3*	*60*
2	*9*	*11*	*6*	*11*	*–*	*5*	*20*
3	*11*	*10*	*6*	*2*	*2*	*8*	*90*
4	*9*	*10*	*9*	*6*	*9*	*12*	*50*
Demand	*60*	*20*	*40*	*20*	*40*	*40*	

Note: *It is not possible to transport any quantity from factory 2 to godown 5. State whether the solution derived by you is unique.*

Solution

The initial solution is found by VAM below:

Factory	**Godowns**						**Avail.**	**Diff.**
	1	**2**	**3**	**4**	**5**	**6**		
1	7	5 [20]	7	7	5	3 [40]	60/40/0	2/4/0
2	9 [10]	11	6 [10]	11	M	5	20/10/0	1/3
3	11	10	6 [30]	2 [20]	2 [40]	8	90/70/30/0	0/4/2/5
4	9 [50]	10	9	6	9	12	50/0	3/0
Demand	60	20	40	20	40	40		
	50	0	10	0	0	0		
	0		0					
Diff.	2	5	0/1	4	3	2		

The above initial solution is tested for optimality. Since there are only 8 allocations and we require 9(m+n-1 =9) allocations, we put a small quantity in the least cost independent cell (2, 6) and apply the optimality test. Let u_3 = 0 and then we calculate remaining u_i and v_j

Factory	Godowns					
	1	2	3	4	5	6
1	7	5 [20]	7	7	5	3 [40]
2	9 [10]	11	6 [10]	11	M	5 [e]
3	11	10	6 [30]	2 [20]	2 [40]	8
4	9 [50]	10	9	6	9	12

Optimality test

							u_i
	7	5	4	0	0	3	–2
	9	7	6	2	2	5	0
	9	7	6	2	2	5	0
	9	7	6	2	2	5	0
v_j	9	7	6	2	2	5	

Note :_(u_i +v_j) matrix for allocated cells has been indicated in the shaded cells and the value of unshaded cells represent (u_i +v_j) matrix for unallocated cells._

Now we calculate $\Delta_{ij} = c_{ij} - (u_i + v_j)$ for non basic cells which are given in the table below:

C–(u_i+v_j) or ∆ij Matrix

0		3	7	5	
	4		9	M	
2	3				3
	3	3	4	7	7

Since all Δ_{ij} are positive, the initial solution found by VAM is an optimal solution. The final allocations are given below:

Factory	Godown	Unit	Cost (₹)	Value (₹)
1	2	20	5	100
1	6	40	3	120

2	1	10	9	90
2	3	10	6	60
3	3	30	6	180
3	4	20	2	40
3	5	40	2	80
4	1	50	9	450
			Total cost	1,120

The above solution is not unique because the opportunity cost of cell (1,1) is zero. Hence alternative solution exists. Students may find that the alternative solution is as given below:

Reallocation Table

+	20				40–
– 10		10			e +
		30	20	40	
50					

Re-allocated Table

10	20				30
		10			10
		30	20	40	
50					

Factory	Godown	Unit	Cost (₹)	Value (₹)
1	1	10	7	70
1	2	20	5	100
1	6	30	3	90
2	3	10	6	60
2	6	10	5	50
3	3	30	6	180
3	5	40	2	80
3	4	20	2	40
4	1	50	9	450
			Total cost	1,120

Illustration 5

The Link manufacturing company has several plants, three of which manufacture two principal products, Standard card table and Deluxe card table. A New Deluxe card table will be introduced which must be considered in term of selling price & costs. The selling prices are: Standard ₹14.95, Deluxe ₹ 18.95, and New Deluxe ₹ 21.95.

Requirements		***Variable costs***			***Available plant capacity***	
Model	***Quantity***	***Plant A***	***Plant B***	***Plant C***	***Plant***	***Capacity***
Standard	*450*	*8.00*	*7.95*	*8.10*	*A*	*800*
Deluxe	*1,050*	*8.50*	*8.60*	*8.45*	*B*	*600*
New Deluxe	*600*	*9.25*	*9.20*	*9.30*	*C*	*700*

Solve this problem by the transportation technique for the greatest contribution.

Solution

Contribution matrix

6.95	7.00	6.85
10.45	10.35	10.50
12.70	12.75	12.65

Contribution = Selling Price – Variable Cost

Now, we convert this contribution Matrix into loss matrix by subtracting each figure in the above contribution matrix from the highest figure of 12.75 and multiplying by 100 to get whole numbers for case in subsequent arithmetic.

				Req.	**Diff.**
	450 580	575	590	450/0	5/10
	350 230	240	700 225	1050/350/0	5/5
	e 5	600 0	10	600/0	5
Availability	800 450 0	600 0	700 0		
Diff.	225 350	240 –	215 365		

The above allocation has been obtained by Vogel's rule.

Since there are 4 allocation, which are less than (m+ n-1) i.e., 5 allocations, a small allocation 'e' is placed in the least cost independent cell (3, 1). The optimality test is performed below:

($u_i + v_j$) for allocated cells

			u_i
580			580
230		225	230
5	0		5
v_j 0	–5	–5	

($u_i + v_j$) for unallocated cells

			u_i
	575	575	580
	225		230
		0	5
v_j 0	–5	–5	

Δ_{ij} matrix

		0	15
		15	
			10

Since, all values in Δ_{ij}matrix is non-negative, the initial solution is optimal. The largest contribution is given by.

6.95x450 +10.45x350 +12.75x600 +10.50x700 =21,785 answer.

Since, there is one zero, alternative solution exist.

Note: in the examples solved herein we see that sometimes we get allocation less than m +n-1 in the initial solution. We place 'e' in the least cost independent cells. It is also possible that we get less than m +n –1 allocations in an intermediate reallocation in this case we would place 'e' in the most –veΔ_{ij} independent cell.

11.7 Miscellaneous Illustrations

Illustration 6

A company wishes to determine an investment strategy for each of the next four years. Five investment types have been selected, investment capital has been allocated for each of the coming four years, and maximum investment levels have been established for each investment type. An assumption is that amounts invested in any year will remain invested until the end of the planning horizon of four years. The following table summaries the data for this problem. The values in the body of the table represent net return on investment of one rupee up to the end of the planning horizon. For example, a rupeeinvested in investment type B at the beginning of year 1 will grow to ₹1.90 by the end of the fourth year, yielding a net return of ₹0.90.

Investment made at the beginning of year	***Investment Type***					***₹ Available (in '000)***
	A	***B***	***C***	***D***	***E***	
	Net Return Data					
1	*0.80*	*0.90*	*0.60*	*0.75*	*1.00*	*500*
2	*0.55*	*0.65*	*0.40*	*0.60*	*0.50*	*600*
3	*0.30*	*0.25*	*0.30*	*0.50*	*0.20*	*750*
4	*0.15*	*0.12*	*0.25*	*0.35*	*0.10*	*800*
Maximum Rupee investment (in ₹ 000)	*750*	*600*	*500*	*800*	*1,000*	

The objective in this problem is to determine the amount to be invested at the beginning of each year in an investment type so as to maximize the net rupee return for the four-year period. Solve the above transportation problem and get an optimal solution. Also calculate the net return on investment for the planning horizon for four-year period.

Solution

We note that this transportation problem is an unbalanced one and it is a maximization problem. As a first step, we will balance this transportation problem.

Step 1:

Investment type

Years	A	B	C	D	E	Available ₹(in '000)
	Net Return Data					
1	0.80	0.90	0.60	0.75	1.00	500
2	0.55	0.65	0.40	0.60	0.50	600
3	0.30	0.25	0.30	0.50	0.20	750
4	0.15	0.12	0.25	0.35	0.10	800
Dummy	0	0	0	0	0	1,000
Maximum₹Investment(in '000)	750	600	500	800	1,000	3,650

Step 2: We shall now convert the above transportation problem (a profit matrix) into a loss matrix by subtracting all the elements from the highest value in the table viz. 1.00.

Investment type

Years	A	B	C	D	E	Available ₹ (in '000)
	Net Loss Data (₹)					
1	0.20	0.10	0.40	0.25	0	500
2	0.45	0.35	0.60	0.40	0.50	600
3	0.70	0.75	0.70	0.50	0.80	750
4	0.85	0.88	0.75	0.65	0.90	800
Dummy	1.00	1.00	1.00	1.00	1.00	1,000
Maximum₹ Investment(in '000)	750	600	500	800	1,000	3,650

For convenience, let us express the net loss data in the body of the above table in paise. Thereafter, we shall apply VAM to get an initial solution.

Investment type

Years	A	B	C	D	E	₹ available ('000's)	Difference
1	20	10	40	25	0 [500]	500/0	10
2	45	35 [600]	60	40	50	600/0	5 5
3	70	75	70	50 [750]	80	750/0	20 20 20
4	85 [250]	88	75 [500]	65 [50]	90	800/750/250/0	10 10 10 10 10
Dummy	100 [500]	100	100	100	100 [500]	1000/500/0	0 0 0 0 0
Maximum rupee investment in (000's)	750/500/0	600/0	500/0	800/50/0	1000/500/0		
Diff.	25	25	20	15	50		
	25	40	10	10	30		
	15	–	5	15	10		
	15	–	25	35	10		
	15	–	25	–	10		

The initial solution got above by VAM is given below

Years	A	B	C	D	E
1	20	10 [e]	40	25	0 [500]
2	45	35 [600]	60	40	50
3	70	75	70	50 [750]	80
4	85 [250]	88	75 [500]	65 [50]	90
Dummy	100 [500]	100	100	100	100 [500]

We will now test the above solution for optimality.

Total number of allocations should be m +n –1 = 9 but there are only 8 allocations in the above solution which are one less than 9, hence the initial solution found above is degenerate. We introduce a small quantity 'e' in an independent least cost cell which is (1, B) in this case, to make the total number of allocation equal to 9.

We introduce u_i's and v_j's such that $\Delta_{ij}=C_{ij}-(u_i+v_j)$ (for i, j = 1, -5). We assume that u_4 = 0 and various u_j's and v_j's and Δ_{ij}'s are calculated as below. Values of various Δ_{ij}'s are also written in the left hand corner of the box for non basic cells.

(u_i+v_j) matrix

						u_i
	0	10	–10	–20	0	–85
	25	35	15	5	25	–60
	70	80	60	50	70	–15
	85	95	75	65	85	0
	100	110	90	80	100	15
vj	85	95	75	65	85	

Note :(u_i+v_j) *matrix for allocated cells has been indicated in the shaded cells and the value of unshaded cells represent* (u_i+v_j) *matrix for unallocated cells.*

Δ_{ij} Matrix or C- (u_i+v_j)

20		50	45	
20		45	35	25
0	–5	10		10
	–7			5
	–10	10	20	

Since some of the Δ_{ij}'s are negative, the above initial solution is not optimal. Introducing in the cell (Dummy, B) with most negative Δ_{ij} an assignment e. The value of e and the new solution as obtained from above is shown below. The values of u_i's, v_j's are also calculated. The solution satisfies the conditions of optimality.The condition $\Delta_{ij}=C_{ij}-(u_i+v_j) \geq 0$ for non allocated cells is also fulfilled.

	e –			+500
	600			
			750	
250		500	50	
500	+			–500

Re-allocated Table

				500
	600			
			750	
250		500	50	
500	e			500

Now we will perform optimality test

					u_i
0	0	–10	–20	0	–85
35	35	25	15	35	–50
70	70	60	50	70	–15
85	85	75	65	85	0
100	100	90	80	100	15
85	85	75	65	85	

(last row: vj)

Note : *(u_i +v_j) matrix for allocated cells has been indicated in the shaded cells and the value of unshaded cells represent (u_i +v_j)matrix for unallocated cells.*

Δ_{ij} Matrix

20	10	50	45	
10		35	25	15
0	5	10		10
	3			5
		10	20	

Since all Δ_{ij}'s are positive, hence, the second solution obtained above is optimal.The allocation is given below.

In the year	Investment type	Amount (in '000)
1	E	500
2	B	600
3	D	750
4	A	250
4	C	500
4	D	50

The net return on investment for planning horizon of four years period is given by:

500x1.0+600 x0.65 +750 x0.50 +250 x0.15 +500x0.25 +50x0.35 = ₹1,445 thousands

Illustration 7

XYZ and Co. has provided the following data seeking your advice on optimum investment strategy:

Investment made at the beginning of year	*Net return data (in paise) of selected investments*				*Amount available (lakhs)*
	P	*Q*	*R*	*S*	
1	*95*	*80*	*70*	*60*	*70*
2	*75*	*65*	*60*	*50*	*40*
3	*70*	*45*	*50*	*40*	*90*
4	*60*	*40*	*40*	*30*	*30*
Maximum Investment (Lakhs)	*40*	*50*	*60*	*60*	*–*

The following additional information are also provided:

— *P, Q, R and S represents the selected investments.*

— *The company has decided to have four years investment plan.*

— *The policy of the company is that amount invested in any year will remain so until the end of the fourth year.*

— *The values (Paise) in the table represent net returns on investment of one Rupee till the end of the planning horizon (for example, a Rupee invested in investment P at the beginning of year 1 will grow to ₹1.95 by the end of the fourth year, yielding a return of 95 paise). Using the above, determine the optimum investment strategy.*

Solution

The given problem is an unbalanced transportation problem, which is converted into a balanced one by adding a dummy investment as given below:

Year	**Net return data (in paise) of investments**					**Amount available ₹ (lakhs)**
	P	**Q**	**R**	**S**	**Dummy**	
1	95	80	70	60	0	70
2	75	65	60	50	0	40
3	70	45	50	40	0	90
4	60	40	40	30	0	30
Maximum Investment	40	50	60	60	20	—

The values in the table represent net return on investment of one rupee till the end of the fourth year. The objective of the company is to maximize the net return. For achieving this objective, let us convert this maximization problem into minimization problem by subtracting all the elements of the above payoff matrix from the highest payoff i.e. 95, and apply Vogel's approximation method for finding the initial feasible solution.

Year	**P**	**Q**	**R**	**S**	**Dummy**	Amount available	Diff
1	0 $\boxed{40}$	15 $\boxed{30}$	25	35	95	70/30/0	15/10….
2	20	30 $\boxed{20}$	35 $\boxed{20}$	45	95	40/20/0	10/5/5/10
3	25	50	45 $\boxed{40}$	55 $\boxed{50}$	95	90/50/0	20/5/5/10/40
4	35	55	55	65 $\boxed{10}$	95 $\boxed{20}$	30/20/0	20/0/0/10/30
Maximum	40/0	50/20/0	60/40/0	60/10/0	20/0		
Diff.	20	15	10	10	0		
	–	15	10	10	0		
	–	20	10	10	0		
	–	–	10	10	0		

Total number of allocations are 8, which is equal to (m+n–1) i.e. 8

This initial solution is tested for optimality. There are 8 (=m +n–1) independent allocations. Let us introduce u_i, v_j, i = (1,2,3,4); j = (1, 2, 3, 4, 5) such that $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for allocated cells. We assume $u_1 = 0$ and remaining u_j's, v_j's and Δ_{ij}'s are calculated below.

$(u_i + v_j)$ Matrix

						u_i
	0	15	20	30	60	0
	15	30	35	45	75	15
	25	40	45	55	85	25
	35	50	55	65	95	35
v_j	0	15	20	30	60	

Note : *$(u_i + v_j)$ matrix for allocated cells has been indicated in the shaded cells and the value of unshaded cells represent $(u_i + v_j)$matrix for unallocated cells.*

C – $(u_i + v_j)$ Matrix

		5	5	35
5			0	20
0	10			10
0	5	0		

Since, in Δ_{ij} matrix, all sale values are non-negative. Hence, this is the optimal solution. Though alternative solution exist.

On calculating Δ_{ij}'s for non-allocated cells, we found that their values are positive, hence the initial solution obtained above is optimal. The optimal allocations are given below:

Year		Invest in	Net return	
1	Invest	₹40 lakhs in investment P	0.95x₹40 lakhs =	₹38,00,000
		₹30 lakhs in investment Q	0.80x₹30 lakhs =	₹24,00,000
2	Invest	₹20 lakhs in investment Q	0.65x₹20 lakhs =	₹13,00,000
		₹20 lakhs in investment R	0.60x₹20 lakhs =	₹12,00,000
3.	Invest	₹40 lakhs in investment R	0.50x₹40 lakhs =	₹20,00,000
		₹50 lakhs in investment S	0.40x₹50 lakhs =	₹20,00,000
4	Invest	₹10 lakhs in investment S	0.30x₹10 lakhs =	₹ 3,00,000
			Total	₹1,30,00,000

Illustration 8

A particular product is manufactured in factories A, B, C, and D; and is sold at Centres 1, 2 and 3. The cost in rupeesof product per unit and capacity in killogram per unit time of each plant is given below:

Factory	*Cost (₹) per unit*	*Capacity (kg) per unit*
A	*12*	*100*
B	*15*	*20*
C	*11*	*60*
D	*13*	*80*

The sales price in ₹Per unit and the demand in kg per unit time are as follows:

Sales Centre	*Sales price (₹) per unit*	*Demand (kgms) per unit*
1	*15*	*120*
2	*14*	*140*
3	*16*	*60*

Find the optimal sales distribution.

Solution

The given problem is a transportation problem. The profit matrix for various factories and sales counters is calculated below:

Factory	Sales Centres			Capacity (kg) per unit
	1	2	3	
A	3	2	4	100
B	0	–1	1	20
C	4	3	5	60
D	2	1	3	80
Demand (kg)	120	140	60	

Since this is an unbalanced transportation problem (demand >capacity), let us introduce a dummy factory with profit as ₹0 per until for various sales Centres and capacity equal to sixty units. The resulting matrix would be as below:

Factory	Sales Centres			Capacity (kg) per unit
	1	2	3	
A	3	2	4	100
B	0	–1	1	20
C	4	3	5	60
D	2	1	3	80
Dummy	0	0	0	60
Demand (kg)	120	140	60	

The above profit matrix can be converted into a loss matrix by subtracting all its elements from the highest payoff of the matrix i.e. 5. The loss matrix so obtained is given below.

Factory	Sales Centres			Capacity (kg) per unit
	1	2	3	
A	2	3	1	100
B	5	6	4	20
C	1	2	0	60
D	3	4	2	80
Dummy	5	5	5	60
Demand (kg)	120	140	60	

The initial solution is obtained by applying Vogel's approximation method.

Factory	Sales Centres 1	2	3	Capacity (kg) per unit	Difference
A	2 [100]	3	1	100/0	11–
B	5	6 [20]	4	20/0	111
C	1	2	0 [60]	60/0	1– –
D	3 [20]	4 [60]	2	80/60/0	111
Dummy	5	5 [60]	5	60/0	000
Demand	120/20/0	140/120/60/0	60/0		
Diff.	1	1	1		
	1	1	–		
	2	1	–		

The solution obtained by VAM is as given below:

Factory	Sales Centres 1	2	3	Capacity (kg) per unit
A	2 [100]	3	1 [e]	100
B	5	6 [20]	4	20
C	1	2	0 [60]	60
D	3 [20]	4 [60]	2	80
Dummy	5	5 [60]	5	60
Demand	120	140	60	

The initial solution is tested for optimality. The total number of independent allocations is 6 which is one less than (m +n –1) allocations. Let us introduce a small quantity 'e' in the least cost independent cell (A, 3). We also introduce u_i, v_j, i = (1,2,3,4,5); j = (1,2,3) such that $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for allocation cells.

We assume $v_2 = 0$ and remaining v_i's, v_j's and Δ_{ij} are calculated as below:

($u_i + v_j$) Matrix

				u_i
	2	3	1	3
	5	6	4	6
	1	2	0	2
	3	4	2	4
	4	5	3	5
v_j	–1	0	–2	

Note :*($u_i + v_j$) matrix for allocated cells has been indicated in the shaded cells and the value of unshaded cells represent ($u_i + v_j$)matrix for unallocated cells*

Δ_{ij} Matrix

	0	
0		0
0	0	
		0
1		2

Since, in Δ_{ij} matrix all cell values are non-negative hence this is the optimal solution. Though alternative solution exists.

The optimal solution for the given problem is given below:

Factory	Sales Centre	Quantity	Profit per unit (₹)	Total profit (₹)
A	1	100	3	300
B	2	20	-1	-20
C	3	60	5	300
D	1	20	2	40
D	2	60	1	60
Dummy	2	60	0	0
			Total profit	680

Illustration 9

Factory	*Destination*			*Supply to be exhausted*
	1	*2*	*3*	
A	*5*	*1*	*7*	*10*
B	*6*	*4*	*5*	*80*
C	*3*	*2*	*5*	*15*
Demand	*75*	*20*	*50*	

Since there is not enough supply, some of the demands at the three destinations may not be satisfied. For the unsatisfied demands, let the penalty costs be rupees 1,2 and 3 for destinations (1), (2) and (3) respectively. Find the optimal allocation that minimises the transportation and penalty cost.

Solution

Since demand (75+ 20 + 50 = 145) is greater than supply (10 + 80 +15 = 105) by 40 units, the given problem is an unbalanced one. We introduce a dummy factory with a supply of 40 units. It is given that for the unsatisfied demands, the penalty cost is rupees 1,2 and 3 for destinations (1), (2) and (3) respectively. Hence, the transportation problem becomes.

	Destination			Supply to be exhausted
	(1)	(2)	(3)	
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
Dummy	1	2	3	40
Demand	75	20	50	

The initial solution is obtained below by VAM method.

Factory	**Destination**			**Supply**	**Difference**
	1	**2**	**3**		
A	5	1 [10]	7	10/0	4– –
B	6 [20]	4 [10]	6 [50]	80/70/50/0	222
C	3 [15]	2	5	15/0	111
Dummy	1 [40]	2	3	40/0	11–
Demand	75/35/20/0	20/10/0	50/0		
Diff.	2	1	2		
	2	0	2		
	3	2	1		

The initial solution is given is the table below:

Factory	Destination 1	Destination 2	Destination 3	Supply
A	5	1 [10]	7	10
B	6 [20]	4 [10]	6 [50]	80
C	3 [15]	2	5	15
Dummy	1 [40]	2	3	40
Demand	75	20	50	

We now apply the optimality test to find whether the initial solution found above is optimal or not.

The number of allocations is 6 which is equal to the required m + n – 1 =6 allocations. Also, these allocations are independent. Hence, both the conditions are satisfied.

Let us now introduce u_i, and v_j; i = (1,2,3,4) and j = (1,2,3) such that $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for allocated cells. We assume that $u_2 = 0$ and remaining u_i's, v_j's and Δ_{ij}'s are calculated.

($u_i + v_j$) Matrix

				u_i
	3	1	3	–3
	6	4	6	0
	3	1	3	–3
	1	–1	1	–5
v_j	6	4	6	

Note : *($u_i + v_j$) matrix for allocated cells has been indicated in the shaded cells and the value of unshaded cells represent ($u_i + v_j$)matrix for unallocated cells*

Δ_{ij} Matrix

2		4
	1	2
	3	2

Since all Δ_{ij},'s for non basic cells are positive, therefore, the solution obtained above is an optimal one. The allocation of factories to destinations and their cost is given below:-

Factory	Destination	units	Cost	Total cost	
A	2	10	₹ 1	₹10	
B	1	20	₹6	₹120	Transportation cost
B	2	10	₹4	₹40	
B	3	50	₹6	₹300	
C	1	15	₹3	₹45	
Dummy	1	40	₹ 1	₹40	Penalty cost
				₹555	

Illustration 10

ABC enterprises is having three plants manufacturing dry-cells, located at different locations. Production cost differs from plant to plant. There are five sales offices of the company located in different regions of the country. The sales prices can differ from region to region. The shipping cost from each plant to each sales office and other data are given by following table:

Production cost per unit	***Production data table Maximum capacity in no. of units***	***Plant No.***
20	*150*	*1*
22	*200*	*2*
18	*125*	*3*

Shipping cost, demand & sales price table

Shipping cost

	Sales office 1	***Sales office 2***	***Sales office 3***	***Sales office 4***	***Sales office 5***
Plant 1	*1*	*1*	*5*	*9*	*4*
Plant 2	*9*	*7*	*8*	*3*	*6*
Plant 3	*4*	*5*	*3*	*2*	*7*

Demand & sales prices

	Sales office 1	***Sales office 2***	***Sales office 3***	***Sales office 4***	***Sales office 5***
Demand	*80*	*100*	*75*	*45*	*125*
Sales price	*30*	*32*	*31*	*34*	*29*

Find the production and distribution schedule most profitable to the company

Solution

The given information can be tabulated in following transportation problem:

Profit Matrix

Plant	Sales Offices 1	2	3	4	5	Capacity in units
1	9	11	6	5	5	150
2	–1	3	1	9	1	200
3	8	9	10	14	4	125
Demand	80	100	75	45	125	

Where entries in the cells of the above table indicate profit per unit received by selling one unit of item from plant(i =1,2,3) to the sales office (j= 1,2,3,4,5). The profit per unit is calculated using the following formula.

Profit = Sales price –(Production cost +Shipping cost)

The objective of the company is to maximize the profit. For achieving this objective, let us convert this maximization problem into minimization problem by subtracting all the elements of the above payoff matrix from the highest payoff i.e. ₹14.

Loss Matrix

Plant	Sales Offices 1	2	3	4	5	Capacity in units
1	5	3	8	9	9	150
2	15	11	13	5	13	200
3	6	5	4	0	10	125
Demand	80	100	75	45	125	

The problem is an unbalanced transportation problem since capacity (=475 units) is 50 units more than the demand. Hence a dummy sales office is added with cost equal to zero for all plants and demand equal to 50 units. Now, let us apply Vogel's Approximation method to the resultant balanced matrix for finding the initial feasible solution.

Plant	Sales Offices 1	2	3	4	5	Dummy	Capacity	Diff.
1	5 $\boxed{50}$	3 $\boxed{100}$	8	9	9	0	150/50/0	32224–
2	15	11	13	5 $\boxed{45}$	13 $\boxed{105}$	0 $\boxed{50}$	200/150/105/0	562222
3	6 $\boxed{30}$	5	4 $\boxed{75}$	0	10 $\boxed{20}$	0	125/50/20/0	041144
Demand	80/30/0	100/0	75/0	45/0	125/105/0	50/0		
Diff.	1	2	4	5	1	0		
	1	2	4	5	1	–		
	1	2	4	–	1	–		
	1	2	–	–	1	–		
	1	–	–	–	1	–		
	9	–	–	–	3	–		

The initial solution obtained by VAM which is tested for optimality.

(u_i +v_j) Matrix

							u_i
	5	3	3	1	9	–4	–4
	9	7	7	5	13	0	0
	6	4	4	2	10	–3	–3
v_j	9	7	7	5	13	0	

Note :(*u_i +v_j) matrix for allocated cells has been indicated in the shaded cells and the value of unshaded cells represent (u_i +v_j)matrix for unallocated cells*

Δ_{ij} Matrix

		5	8	0	4
6	4	6			
	1		–2		3

Since some of the Δ_{ij}'s are negative, therefore, the above solution is not optimal. Introduce in the cell (2,4) with the most negative Δ_{ij}, an assignment. The value of θ and reallocated solution as obtained from above is given below. The reallocated solution is again tested for optimally. Hence, the values u_i'sv_j's and Δ_{ij}'s are again calculated.

Re-allocation table

50	100				
			45 –	+ 105	50
30		75	+	– 20	

Re-allocation Matrix

50	100				
			25	125	50
30		75	20		

Now we will perform optimality test for the reallocation table

($u_i + v_j$) Matrix

							u_i
	5	3	3	–1	7	–6	–6
	11	9	9	5	13	0	0
	6	4	4	0	8	–5	–5
v_j	11	9	9	5	13	0	

Note :*($u_i + v_j$) matrix for allocated cells has been indicated in the shaded cells and the value of unshaded cells represent ($u_i + v_j$)matrix for unallocated cells*

Δ_{ij} Matrix

		5	10	2	6
4	2	4			
	1			2	5

Since all Δ_{ij}'s for non-basic cells are positive, therefore, the solution obtained above is an optimal one. The allocation of plants to sales officers and their profit amount is given below:

Plant	Sales Office	Units	Profit per unit (₹)	Profit (₹)
1	1	50	9	450
1	2	100	11	1,100
2	4	25	9	225
2	5	125	1	125
2	Dummy	50	0	0
3	1	30	8	240
3	3	75	10	750
3	4	20	14	280
			Total	3,170

Note: This problem can be solved alternatively by introducing dummy sales office with zero profit in the profit matrix itself.

Summary

- Finding of initial base feasible solution
  - ✓ North west corner method
  - ✓ Least cost method
  - ✓ Vogels approximation method (VAM)
- Optimality test can be applied if it satisfies the given two conditions:
  - ✓ It contains exactly m+n-1 allocations where m and n represent the number of rows and column of the table.
  - ✓ These allocations are independent i.e. a loop can not be performed by them.

12

The Assignment Problem

LEARNING OBJECTIVES:

After studying this unit, you will be able to :

- Use assignment technique, which is a special form of transportation problem.
- Use Hungarian Assignment Method.
- Negotiate with restrictions on assignments.
- Deal with unbalanced assignment problems.
- Prepare table with dummy rows and columns.

12.1 Introduction

The Assignment Problem is another special case of LPP. It occurs when n jobs are to be assigned to n facilities on a one-to-one basis with a view to optimising the resource required.

12.2 The Assignment Algorithm

The assignment problem can be solved by applying the following steps:

Step 1: Subtract the minimum element after row operation of each row from all the elements in that row. From each column of the matrix so obtained, subtract its minimum element. The resulting matrix is the starting matrix for the following procedure.

Step 2: Draw the minimum number of horizontal and vertical lines that cover all the zeros. If this number of lines is *n*, order of the matrix, optimal assignment can be made by skipping steps 3 and 4 and proceeding with step 5. If, however, this number is less than *n*, go to the next step.

Step 3: Here, we try to increase the number of zeros in the matrix. We select the smallest element out of these which do not lie on any line. Subtract this element from all such (uncovered) elements and add it to the elements which are placed at the intersections of the horizontal and vertical lines. Do not alter the elements through which only one line passes.

Step 4: Repeat steps 1, 2 and 3 until we get the minimum number of lines equal to *n*.

Step 5 (A) Starting with first row, examine all rows of matrix in step 2 or 4 in turn until a row containing exactly one zero is found. Surround this zero by □ , indication of an assignment there. Draw a vertical line through the column containing this zero. This eliminates any confusion of making any further assignments in that column. Process all the rows in this way.

(B) Apply the same treatment to columns also. Starting with the first column, examine all columns until a column containing exactly one zero is found. Mark □ around this zero and draw a horizontal line through the row containing this marked zero. Repeat steps 5A and B, until one of the following situations arises:

(i) No unmarked (□) or uncovered (by a line) zero is left,

(ii) There may be more than one unmarked zero in one column or row. In this case, put around one of the unmarked zero arbitrarily and pass 2 lines in the cells of the remaining zeros in its row and column. Repeat the process until no unmarked zero is left in the matrix.

Illustration 1

An Accounts Officer has 4 subordinates and 4 tasks. The subordinates differ in efficiency. The tasks also differ in their intrinsic difficulty. His estimates of the time each would take to perform each task is given in the matrix below. How should the tasks be allocated one to one man, so that the total man hours are minimized?

Subordinates	***Tasks***			
	I	***II***	***III***	***IV***
1	*8*	*26*	*17*	*11*
2	*13*	*28*	*4*	*26*
3	*38*	*19*	*18*	*15*
4	*19*	*26*	*24*	*10*

Let us apply the above steps take the above example.

Solution

Step 1

By subtracting the minimum element of each row from all its elements in turn, the given matrix reduces to

$$\begin{matrix} 0 & 18 & 9 & 3 \\ 9 & 24 & 0 & 22 \\ 23 & 4 & 3 & 0 \\ 9 & 16 & 14 & 0 \end{matrix}$$

Next we subtract the minimum element of each column from all elements in turn, obtaining,

$$\begin{matrix} 0 & 14 & 9 & 3 \\ 9 & 20 & 0 & 22 \\ 23 & 0 & 3 & 0 \\ 9 & 12 & 14 & 0 \end{matrix}$$

Step 2

We draw the minimum number of lines to cover all zeros in the last matrix above as follows. To do so the first line is row 3 that contains the highest number of zeros. It can be seen that 4(= n) lines cover all the zeros; hence optimal assignment is possible and it is obtained by the application of step 5 straight away below.

$$\begin{array}{cccc}
\boxed{0} & 14 & 9 & 3 \\
9 & 20 & \boxed{0} & 22 \\
23 & \boxed{0} & 3 & \cancel{0} \\
9 & 12 & 14 & \boxed{0}
\end{array}$$

The optimal assignment, then is

$1 \longrightarrow$ I, $\quad 2 \longrightarrow$ III, $\quad 3 \longrightarrow$ II, $\quad 4 \longrightarrow$ IV.

Minimum time taken = 8 + 4 + 19 + 10 = 41 hours.

Illustration 2

A manager has 5 jobs to be done. The following matrix shows the time taken by the j-th job (j = 1,2...5) on the i-th machine (i = I,II,III…V). Assign 5 jobs to the 5 machines so that the total time taken is minimized.

	Jobs				
Machines	***1***	***2***	***3***	***4***	***5***
I	*9*	*3*	*4*	*2*	*10*
II	*12*	*10*	*8*	*11*	*9*
III	*11*	*2*	*9*	*0*	*8*
IV	*8*	*0*	*10*	*2*	*1*
V	*7*	*5*	*6*	*2*	*9*

Solution

Subtracting the minimum element of each row from all its elements in turn, the given matrix reduces to

$$\begin{array}{ccccc}
7 & 1 & 2 & 0 & 8 \\
4 & 2 & 0 & 3 & 1 \\
11 & 2 & 9 & 0 & 8 \\
8 & 0 & 10 & 2 & 1 \\
5 & 3 & 4 & 0 & 7
\end{array}$$

Now subtracting the minimum element of each column from its column, the matrix reduces to

$$\begin{array}{ccccc}
3 & 1 & 2 & 0 & 7 \\
0 & 2 & 0 & 3 & 0 \\
7 & 2 & 9 & 0 & 7
\end{array}$$

4	0	10	2	0
1	3	4	0	6

Next, we draw the minimum number of lines as follows:

3	1	2	0	7
0	2	0	3	0
7	2	9	0	7
4	0	10	2	0
1	3	4	0	6

Matrix A

Since there are only 3 lines (less than 5) optimal assignment cannot be made as yet. We, therefore, perform step 3 on the last matrix above. The minimum uncovered element is 1. It is subtracted from all the uncovered elements and adding it to those at intersection of two lines, giving thereby the following matrix.

2	0	1	0	6
0	2	0	4	0
6	1	8	0	6
4	0	10	3	0
0	2	3	0	5

As can be seen below, minimum number of lines that cover all zeros is 5.

2	0	1	0	6
0	2	0	4	0
6	1	8	0	6
4	0	10	3	0
0	2	3	0	5

Optimal assignment is, therefore, possible and is made as per step 5 below:

2	$\boxed{0}$	1	0	6
0	2	$\boxed{0}$	4	0
6	1	8	$\boxed{0}$	6
4	0	10	3	$\boxed{0}$
$\boxed{0}$	2	3	0	5

Optimal assignment, then is

I → 2, II → 3, III → 4, IV → 5 and V → 1.

Minimum time = 3 + 8 + 0 + 1 + 7 = 19.

Illustration 3

5 salesmen are to be assigned to 5 districts. Estimates of sales revenue in thousands of rupees for each salesman are given below.

	A	B	C	D	E
1	*32*	*38*	*40*	*28*	*40*
2	*40*	*24*	*28*	*21*	*36*
3	*41*	*27*	*33*	*30*	*37*
4	*22*	*38*	*41*	*36*	*36*
5	*29*	*33*	*40*	*35*	*39*

Find the assignment pattern that maximizes the sales revenue.

Solution

In order to convert this maximization problem into a minimization problem to be able to apply the assignment algorithm, we subtract each element from the highest, 41 and obtain the following loss matrix.

9	3	1	13	1
1	17	13	20	5
0	14	8	11	4
19	3	0	5	5
12	8	1	6	2

Applying step 1 to the loss matrix we derive the following matrix, in which 4 lines are drawn to cover all zeros.

8	0	0	7	0
0	14	12	14	4
0	12	8	6	4
19	1	0	0	5
11	5	0	0	1

The minimum uncovered element is 4 that is subtracted from all elements and added to all elements at intersections. This yields the following matrix in which 5 lines are needed to cover all zeros.

12	0	0	7	0
0	10	8	10	0
0	8	4	2	0
23	1	0	0	5
15	5	0	0	1

Step 5 (A) and (B) is applied below to obtain the optimal assignment.

12	$\boxed{0}$	0	7	0
0	10	8	10	0
0	8	4	2	0
23	1	0	0	5
15	5	0	0	1

Condition (ii) of Step B arises above; therefore cell (2,1) is arbitrarily chosen and put around it and a line is also drawn in the second row.

$$\begin{array}{ccccc}
12 & \boxed{0} & 0 & 7 & 0 \\
\boxed{0} & 10 & 8 & 10 & 0 \\
0 & 8 & 4 & 2 & 0 \\
23 & 1 & 0 & 0 & 5 \\
15 & 5 & 0 & 0 & 1
\end{array}$$

This process is repeated below by putting □ around cell (3,4) arbitrarily chosen. Therefore, 2 lines are drawn to cover 4th row and 3rd column.

$$\begin{array}{ccccc}
12 & \boxed{0} & 0 & 7 & 0 \\
\boxed{0} & 10 & 8 & 10 & 0 \\
0 & 8 & 4 & 2 & 0 \\
23 & 1 & \boxed{0} & 0 & 5 \\
15 & 5 & 0 & 0 & 1
\end{array}$$

The same process is continued to yield the optimal pattern as shown below:

$$\begin{array}{ccccc}
12 & \boxed{0} & 0 & 7 & 0 \\
\boxed{0} & 10 & 8 & 10 & 0 \\
0 & 8 & 4 & 2 & \boxed{0} \\
23 & 1 & \boxed{0} & 0 & 5 \\
15 & 5 & 0 & \boxed{0} & 1
\end{array}$$

Optimal assignment, then is

1 $\longrightarrow$ B, 2 $\longrightarrow$ A, 3 $\longrightarrow$ E, 4 $\longrightarrow$ C and 5 $\longrightarrow$ D.

The maximum assignment profit is given by Z = 38 + 40 + 37 + 41 + 35 = 191 thousand rupees.

12.2.1 Rationale of the Assignment Algorithm

Step 1: The relative cost of assigning facility i to job j is not changed by the subtraction of a constant from either a column or a row of the original cost matrix.

Step 2: An optimal assignment exists if total reduced cost of the assignment is zero. This is the case when the minimum number of lines necessary to cover all zeros is equal to the order of the matrix. If, however, it is less than *n,* a further reduction of the cost matrix has to be undertaken.

Step 3: The underlying logic can be explained by means of Matrix A of Example 2, in which only 3(= *n*–2) lines can be drawn.

An optimal assignment is not possible. Further reduction is necessary. This reduction is made by subtracting the smallest non-zero element from all elements in the matrix which is 1.

This yields the following matrix:

2	0	1	–1	6
–1	1	–1	2	–1
6	1	8	–1	6
3	–1	9	1	–1
0	2	3	–1	5

This matrix contains -ve values. Since the objective is to obtain an assignment with the reduced costs of zero, the -ve numbers must be eliminated. This can be done by adding 1 to each of the rows and columns crossed by 3 lines shown in matrix A of Example 2. Doing so in the above table yields the following matrix:

2	0	1	0	6
0	2	0	4	0
6	1	8	0	6
4	0	10	2	0
0	2	3	0	5

All this, in fact, amounts to step 3 i.e., add the least non-zero uncovered element to elements at intersections, subtract it from all the uncovered elements and leave other elements unaltered.

12.3 Unbalanced Assignment Problems

Like the unbalanced transportation problems there could arise unbalanced assignment problems too. They are to be handled exactly in the same manner i.e., by introducing dummy jobs or dummy men, etc. The following unbalanced problem serves as an example.

Illustration 4

To stimulate interest and provide an atmosphere for intellectual discussion, a finance faculty in a management school decides to hold special seminars on four contemporary topics– leasing, portfolio management, private mutual funds, swaps and options. Such seminars should be held once a week in the afternoons. However, scheduling these seminars (one for each topic, and not more than one seminar per afternoon) has to be done carefully so that the number of students unable to attend is kept to a minimum. A careful study indicates that the number of students who cannot attend a particular seminar on a specific day is as follows:

	Leasing	*Portfolio Management*	*Private Mutual Funds*	*Swaps and Options*
Monday	*50*	*40*	*60*	*20*
Tuesday	*40*	*30*	*40*	*30*
Wednesday	*60*	*20*	*30*	*20*
Thursday	*30*	*30*	*20*	*30*
Friday	*10*	*20*	*10*	*30*

Find an optimal schedule of the seminars. Also find out the total number of students who will be missing at least one seminar.

Solution

This is an unbalanced minimization assignment problem. We first of all balance it by adding a dummy topic:

	Leasing	Portfolio Management	Private Mutual Funds	Swaps and Options	Dummy
Monday	50	40	60	20	0
Tuesday	40	30	40	30	0
Wednesday	60	20	30	20	0
Thursday	30	30	20	30	0
Friday	10	20	10	30	0

Subtracting the minimum element of each column from all the elements of that column, we get the following matrix:

	Leasing	Portfolio Management	Private Mutual Funds	Swaps and Options	Dummy
Monday	40	20	50	0	0
Tuesday	30	10	30	10	0
Wednesday	50	0	20	0	0
Thursday	20	10	10	10	0
Friday	0	0	0	10	0

The minimum number of lines to cover all zeros is 4 which is less than the order of the square matrix, (i.e. 5), the above matrix will not give the optimal solution. Subtract the minimum uncovered element (=10) from all uncovered elements and add it to the elements lying on the intersection of two lines, we get the following matrix:

	Leasing	Portfolio Management	Private Mutual Funds	Swaps and Options	Dummy
Monday	30	20	40	0	0
Tuesday	20	10	20	10	0
Wednesday	40	0	10	0	0
Thursday	10	10	0	10	0
Friday	0	10	0	20	10

Since the minimum number of lines to cover all zeros is 5 which is equal to the order of the matrix, the above matrix will give the optimal solution which is given below:

	Leasing	Portfolio Management	Private Mutual Funds	Swaps and Options	Dummy
Monday	30	20	40	$\boxed{0}$	0
Tuesday	20	10	20	10	$\boxed{0}$
Wednesday	40	$\boxed{0}$	10	0	0
Thursday	10	10	$\boxed{0}$	10	0
Friday	$\boxed{0}$	10	0	20	10

and the optimal schedule is

		No. of students missing
Monday:	Swaps and options	20
Tuesday:	No seminar	0
Wednesday:	Portfolio Management	20
Thursday:	Pvt. Mutual funds	20
Friday:	Leasing	<u>10</u>
		<u>70</u>

Thus, the total number of students who will be missing at least one seminar = 70

Illustration 5

A solicitor's firm employs typists on hourly piece-rate basis for their daily work. There are five typists for service and their charges and speeds are different. According to an earlier understanding only one job is given to one typist and the typist is paid for full hours even if he works for a fraction of an hour. Find the least cost allocation for the following data:

Typist	*Rate per hour (₹)*	*No. of pages typed/hour*
A	*5*	*12*
B	*6*	*14*
C	*3*	*8*
D	*4*	*10*
E	*4*	*11*

Job	*No. of pages*
P	*199*
Q	*175*
R	*145*
S	*298*
T	*178*

Solution

The following matrix gives the cost incurred if the *ith* typist (*i* = A, B, C, D, E) executes the *jth* job (*j* = P, Q, R, S, T):

Typist	Job				
	P	Q	R	S	T
A	85	75	65	125	75
B	90	78	66	132	78
C	75	66	57	114	69
D	80	72	60	120	72
E	76	64	56	112	68

Subtracting the minimum element of each row from all its elements in turn, the above matrix reduces to

Typist	Job				
	P	Q	R	S	T
A	20	10	0	60	10
B	24	12	0	66	12
C	18	9	0	57	12
D	20	12	0	60	12
E	20	8	0	56	12

Now subtract the minimum element of each column from all its elements in turn, the above matrix reduces to

2	2	0	4	0
6	4	0	10	2
0	1	0	1	2
2	4	0	4	2
2	0	0	0	2

Since there are only 4 lines (< 5) to cover all zeros, optimal assignment cannot be made. The minimum uncovered element is 2.

We subtract the value 2 from all uncovered elements, add this value to all junction values and leave the other elements undisturbed. The revised matrix looks as:

2	2	2	4	0
4	2	0	8	0
0	1	2	1	2
0	2	0	2	0
2	0	2	0	2

Since the minimum number of lines required to cover all the zeros is only 4 (< 5), optimal assignment cannot be made at this stage also.

The minimum uncovered element is 1. Repeating the usual process again, we get the following matrix:

2	1	2	3	0
4	1	0	7	0
0	0	2	0	2
0	1	0	1	0
3	0	3	0	3

Since the minimum number of lines to cover all zeros is equal to 5, this matrix will give optimal solution. The optimal assignment is made in the matrix below:

Typist	Job				
	P	Q	R	S	T
A	2	1	2	3	$\boxed{0}$
B	4	1	$\boxed{0}$	7	0
C	0	$\boxed{0}$	2	0	2
D	$\boxed{0}$	1	0	1	0
E	3	0	3	$\boxed{0}$	3

	Cost (₹)
Thus typist A is given job T :	75
Typist B is given job R :	66
Typist C is given job Q :	66
Typist D is given job P :	80
Typist E is given job S :	112
Total (₹)	399

Note: In this case the above solution is not unique. Alternative solution also exists.

Illustration 6

WELLDONE Company has taken the third floor of a multi-storied building for rent with a view to locate one of their zonal offices. There are five main rooms in this floor to be assigned to five managers. Each room has its own advantages and disadvantages. Some have windows, some are closer to the washrooms or to the canteen or secretarial pool. The rooms are of all different sizes and shapes. Each of the five managers was asked to rank their room preferences amongst the rooms 301, 302, 303,304 and 305. Their preferences were recorded in a table as indicated below:

MANAGER

M_1	M_2	M_3	M_4	M_5
302	*302*	*303*	*302*	*301*
303	*304*	*301*	*305*	*302*
304	*305*	*304*	*304*	*304*
	301	*305*	*303*	
		302		

Most of the managers did not list all the five rooms since they were not satisfied with some of these rooms and they have left off these from the list. Assuming that their preferences can be quantified by numbers, find out as to which manager should be assigned to which room so that their total preference ranking is a minimum.

Solution

Let us first formulate the preference ranking assignment problem.

MANAGERS

Room No.	M_1	M_2	M_3	M_4	M_5
301	–	4	2	–	1
302	1	1	5	1	2
303	2	–	1	4	–
304	3	2	3	3	3
305	–	3	4	2	–

We have to find an assignment so that total preference ranking is minimum. In a cell (-) indicates that no assignment is to be made in that particular cell. Let us assign a very large ranking value M to all such cells.

Step 1 : From each row, subtract the minimum element of that row, from all the elements of that row to get the following matrix.

MANAGERS

Room No.	M_1	M_2	M_3	M_4	M_5
301	M	3	1	M	0
302	0	0	4	0	1
303	1	M	0	3	M
304	1	0	1	1	1
305	M	1	2	0	M

Draw the minimum number of lines in the above table to cover all zeros. In this case the number of such lines is five, so the above matrix will give the optimal solution. The assignment is made as below:

MANAGERS

Room No.	M_1	M_2	M_3	M_4	M_5
301	M	3	1	M	$\boxed{0}$
302	$\boxed{0}$	0	4	0	1
303	1	M	$\boxed{0}$	3	M
304	1	$\boxed{0}$	1	1	1
305	M	1	2	$\boxed{0}$	M

Thus, the assignment is

$M_1 \longrightarrow 302$, $M_2 \longrightarrow 304$, $M_3 \longrightarrow 303$, $M_4 \longrightarrow 305$, and $M_5 \longrightarrow 301$

and the total minimum ranking = 1 + 2 + 1 + 2 + 1 = 7

Illustration 7

XYZ airline operating 7 days a week has given the following timetable. Crews must have a minimum layover of 5 hours between flights. Obtain the pairing flight that minimizes layover time away from home. For any given pairing the crew will be based at the city that results in the smaller layover.

Chennai-Mumbai			*Mumbai-Chennai*		
Flight Number	***Depart.***	***Arrive***	***Flight Number***	***Depart.***	***Arrive***
A1	*6 AM*	*8 AM*	*B1*	*8 AM*	*10 AM*
A2	*8 AM*	*10 AM*	*B2*	*9 AM*	*11 AM*
A3	*2 PM*	*4 PM*	*B3*	*2 PM*	*4 PM*
A4	*8 PM*	*10 PM*	*B4*	*7 PM*	*9 PM*

Solution

To begin with, let us first assume that the crew is based at Chennai. The flight A_1, which starts from Chennai at 6 AM, reaches Mumbai at 8 AM. The schedule time for the flight at Mumbai is 8 AM. Since the minimum layover time for crew is 5 hours, this flight can depart only on the next day i.e. the layover time will be 24 hours. Similarly, layover times for other flights are also calculated and given in the following table.

Flight No.	**Crew based at Chennai**			
	B_1	**B_2**	**B_3**	**B_4**
A_1	24	25	6	11
A_2	22	23	28	9
A_3	16	17	22	27
A_4	10	11	16	21

The layover times for various flight connections when crew is assumed to be based at Mumbai are similarly calculated in the following table.

Flight No.	Crew based at Mumbai			
	B_1	B_2	B_3	B_4
A_1	20	19	14	9
A_2	22	21	16	11
A_3	28	27	22	17
A_4	10	9	28	23

Now since the crew can be based at either of the places, minimum layover times can be obtained for different flight numbers by selecting the corresponding lower value out of the above two tables. The resulting table is as given below:

Flight No.	Flight No.			
	B_1	B_2	B_3	B_4
A_1	20*	19*	6	9*
A_2	22	21*	16*	9
A_3	16	17	22	17*
A_4	10	9*	16	21

A * with an entry in the above table indicates that it corresponds to layover time when the crew is based at Mumbai. We will now apply the assignment algorithm to find the optimal solution. Subtracting the minimum element of each row from all the elements of that row, we get the following matrix.

Flight No.	Flight No.			
	B_1	B_2	B_3	B_4
A_1	14	13	0	3
A_2	13	12	7	0
A_3	0	1	6	1
A_4	1	0	7	12

Since there is a zero in each column, there is no need to perform column reduction. The minimum number of lines to cover all zeros is four which is equal to the order of the matrix. Hence, the above table will give the optimal solution. The assignment is made below:

Flight No.	Flight No.			
	B_1	B_2	B_3	B_4
A_1	14	13	$\boxed{0}$	3
A_2	13	12	7	$\boxed{0}$
A_3	$\boxed{0}$	1	6	1
A_4	1	$\boxed{0}$	7	12

The optimal assignment is

From flight No.	To flight No.	Layover time
A_1	B_3	6
A_2	B_4	9
A_3	B_1	16
A_4	B_2*	9
		40 hours

Illustration 8

An organisation producing 4 different products viz. A, B, C and D having 4 operators viz. P,Q,R and S, who are capable of producing any of the four products, work effectively 7 hours a day. The time (in minutes) required for each operator for producing each of the product are given in the cells of the following matrix along with profit (₹ per unit).

Operator	*Product*			
	A	*B*	*C*	*D*
P	*6*	*10*	*14*	*12*
Q	*7*	*5*	*3*	*4*
R	*6*	*7*	*10*	*10*
S	*20*	*10*	*15*	*15*
Profit (₹/Unit)	*3*	*2*	*4*	*1*

Find out the assignment of operators to products which will maximize the profit.

Solution

Using the information that the factory works effectively 7 hours (= 420 minutes) a day, and the time required by each operator for producing each of the products, we obtain the following production and profit matrices.

Production Matrix (Units)				
Operator	***Product***			
	A	***B***	***C***	***D***
P	70	42	30	35
Q	60	84	140	105
R	70	60	42	42
S	21	42	28	28

Production Matrix (in ₹)				
Operator	***Product***			
	A	***B***	***C***	***D***
P	210	84	120	35
Q	180	168	560	105
R	210	120	168	42
S	63	84	112	28

In order to apply assignment algorithm for minimizing losses, let us first convert this profit matrix to a loss matrix by subtracting all the elements of the given matrix from its highest element which is equal to ₹ 560. The matrix so obtained is given below:

Operator	Product			
	A	B	C	D
P	350	476	440	525
Q	380	392	0	455
R	350	440	392	518
S	497	476	448	532

Now apply the assignment algorithm to the above loss matrix. Subtracting the minimum element of each row from all elements of that row, we get the following matrix:

Operator	**Product**			
	A	**B**	**C**	**D**
P	0	126	90	175
Q	380	392	0	455
R	0	90	42	168
S	49	28	0	84

Now subtract the minimum element of each column from all the elements of that column to get the following matrix:

Operator	**Product**			
	A	**B**	**C**	**D**
P	0	98	90	91
Q	380	364	0	371
R	0	62	42	84
S	49	0	0	0

Draw the minimum number of lines to cover all zeros. The minimum number of lines to cover all zeros is three which is less than the order of the square matrix (i.e. 4), thus the above matrix will not give the optimal solution. Subtract the minimum uncovered element (= 62) from all uncovered elements and add it to the elements lying on the intersection of two lines, we get the following matrix:

Operator	***Product***			
	A	***B***	***C***	***D***
P	0	36	90	29
Q	380	302	0	309
R	0	0	42	22
S	111	0	62	0

The minimum number of lines which cover all zeros is 4 which is equal to the order of the matrix, hence, the above matrix will give the optimal solution. Specific assignments in this case are as shown below:

Operator	***Product***			
	A	***B***	***C***	***D***
P	$\boxed{0}$	36	90	29
Q	380	302	$\boxed{0}$	309
R	0	$\boxed{0}$	42	22
S	111	0	62	$\boxed{0}$

Operator	***Product***	***Profit (₹)***
P	A	210
Q	C	560
R	B	120
S	D	28
Total	Profit (₹)	918

Illustration 9

A firm produces four products. There are four operators who are capable of producing any of these four products. The processing time varies from operator to operator. The firm records 8 hours a day and allow 30 minutes for lunch. The processing time in minutes and the profit for each of the products are given below:

Operators	***Products***			
	A	***B***	***C***	***D***
1	*15*	*9*	*10*	*6*
2	*10*	*6*	*9*	*6*
3	*25*	*15*	*15*	*9*
4	*15*	*9*	*10*	*10*
Profit (₹) per unit	*8*	*6*	*5*	*4*

Find the optimal assignment of products to operators.

Solution

The firm records 8 hours a day and allows 30 minutes for lunch, hence the net working time available per day is 7 hours and 30 minutes i.e. 450 minutes. The number of units of each product which could be produced in 450 minutes by the four operators is calculated in the table given below:

Operators	Products			
	A	B	C	D
1	30	50	45	75
2	45	75	50	75
3	18	30	30	50
4	30	50	45	45
Profit (₹) per unit	8	6	5	4

Since we are given the profit per unit of each product, the profit matrix is computed as given below:

Operators	Profits Matrix in (₹) of Products			
	A	B	C	D
1	240	300	225	300
2	360	450	250	300
3	144	180	150	200
4	240	300	225	180

The above profit matrix is converted into a loss matrix by subtracting all the elements of the profit matrix from its highest pay off ₹ 450. The loss matrix so obtained is given below:

Operators	Loss Matrix – of Products			
	A	B	C	D
1	210	150	225	150
2	90	0	200	150
3	306	270	300	250
4	210	150	225	270

Let us now apply the assignment algorithm that is 'Hungarian Rule' to the above loss matrix. Accordingly, subtract the minimum element of each row from all its elements in turn, the above matrix thus reduces to

Operators	Loss Matrix – Products			
	A	B	C	D
1	60	0	75	0
2	90	0	200	150
3	56	20	50	0
4	60	0	75	120

Subtract the minimum element of each column from all the elements of the column in turn. Draw the minimum number of lines in the resultant matrix so as to cover all zeros, we get

Operators	Loss Matrix – Products			
	A	B	C	D
1	4	0	25	0
2	34	0	150	150
3	0	20	0	0
4	4	0	25	120

Since the minimum number of lines to cover all zeros is three which is one less than the order of the matrix, we subtract the minimum uncovered element (= 4) from all uncovered elements and add it to the elements lying at the intersection of two lines. The matrix so obtained is given below:

Operators	Loss Matrix – Products			
	A	B	C	D
1	0	0	21	0
2	30	0	146	150
3	0	24	0	4
4	0	0	21	120

Since the minimum number of lines to cover all zeros is 4 which is equal to the order of the matrix, the above matrix will give optimal solution. The optimal assignments are given below:

Operators	Loss Matrix – Products			
	A	B	C	D
1	~~0~~	~~0~~	21	$\boxed{0}$
2	30	$\boxed{0}$	146	150
3	~~0~~	24	$\boxed{0}$	4
4	$\boxed{0}$	~~0~~	21	120

The optimal assignment is as shown below:

Operators	Products	Profit (₹)
1	D	300
2	B	450
3	C	150
4	A	240
		₹ 1,140

Illustration 10

A manufacturing company has four zones A, B, C, D and four sales engineers P, Q, R, S respectively for assignment. Since the zones are not equally rich in sales potential, therefore it is estimated that a particular engineer operating in a particular zone will bring the following sales:

Zone A	*:*	*4,20,000*
Zone B	*:*	*3,36,000*
Zone C	*:*	*2,94,000*
Zone D	*:*	*4,62,000*

The engineers are having different sales ability. Working under the same conditions, their yearly sales are proportional to 14, 9, 11 and 8 respectively. The criteria of maximum expected total sales is to be met by assigning the best engineer to the richest zone, the next best to the second richest zone and so on.

Find the optimum assignment and the maximum sales.

Solution

It is given that the yearly sales of four sales engineers are proportional to 14, 9, 11 and 8 respectively. The sum of proportions is 14+9+11+8=42.

Let us assume that ₹ 1,000 is equivalent to one unit. The sales units of four engineers in four different zones have been calculated as in the following table:

Sales Engineer	Zones			
	A	B	C	D
P	(14/42)×420= 140	(14/42)×336=112	(14/42)×294=98	(14/42)×462=154
Q	(9/42)×420=90	(9/42)×336=72	(9/42)×294=63	(9/42)×462=99
R	(11/42)×420=110	(11/42)×336=88	(11/42)×294=77	(11/42)×462=121
S	(8/42)×420=80	(8/42)×336=64	(8/42)×294=56	(8/42)×462=88

The problem here is to find the optimum assignment in the following sales table so as to maximise the total sales of the company.

Sales Engineer	Zones (Loss in thousands of rupees)			
	A	B	C	D
P	140	112	98	154
Q	90	72	63	99
R	110	88	77	121
S	80	64	56	88

In order to apply the assignment algorithm, we will first convert this maximisation problem into a minimisation problem by subtracting all elements of the above matrix from the highest element i.e. 154. The resultant loss matrix is given below:-

Sales Engineer	Zones (Loss in thousands of rupees)			
	A	B	C	D
P	14	42	56	0
Q	64	82	91	55
R	44	66	77	33
S	74	90	98	66

Now perform the row operations with each of the rows i.e. from all the elements of a row, subtract the minimum element of that row. The reduced matrix is a given below:

Sales Engineer	Zones (Loss in thousands of rupees)			
	A	B	C	D
P	14	42	56	0
Q	9	27	36	0
R	11	33	44	0
S	8	24	32	0

Now, from all the elements of a column, subtract the minimum element of that column. Repeat this operation with all the columns to get the following table:

Sales Engineer	Zones (Loss in thousands of rupees)			
	A	B	C	D
P	6	18	24	0
Q	1	3	4	0
R	3	9	12	0
S	0	0	0	0

The minimum number of lines drawn to cover all zeros is 2 which is less than the order of the matrix (i.e.4), hence we can not make assignments. Subtract the minimum uncovered element from all the uncovered elements and add it to the elements lying at the intersection of two lines, we get:

Sales Engineer	Zones (Loss in thousands of rupees)			
	A	B	C	D
P	5	17	23	0
Q	0	2	3	0
R	2	8	11	0
S	0	0	0	1

The minimum number of lines drawn again to cover all the zeros is 3 which is one less than the order of the matrix. Repeat the above step which gives the following table:

Sales Engineer	Zones (Loss in thousands of rupees)			
	A	B	C	D
P	5	15	21	0
Q	0	0	1	0
R	2	6	9	0
S	2	0	0	3

The minimum number of lines to cover all zeros is still one less than the order of the matrix. Repeat the above step again, which gives the following table:

Sales Engineer	Zones (Loss in thousands of rupees)			
	A	B	C	D
P	3	13	19	0
Q	0	0	1	2
R	0	4	7	0
S	2	0	0	5

The minimum number of lines drawn to cover all the zeros is 4 which is equal to the order of the matrix. Hence, the above table will give the optimum assignment. The assignments are as follows:

Sales Engineer	Zones (Loss in thousands of rupees)			
	A	B	C	D
P	~~3~~	13	19	$\boxed{0}$
Q	~~0~~	$\boxed{0}$	1	2
R	$\boxed{0}$	4	7	~~0~~
S	2	~~0~~	$\boxed{0}$	5

Engineers	Zones	Sales (in ₹)
P	D	1,54,000
Q	B	72,000
R	A	1,10,000
S	C	56,000
		3,92,000

It can be seen from the above assignments that the best engineer P is assigned to the richest zone D, the next best engineer R is assigned to second richest zone A, the next best engineer Q is assigned to zone B and so on. Hence, the optimum assignment matches the company's criteria of achieving the maximum expected total sales.

13

Critical Path Analysis

LEARNING OBJECTIVES

After studying this unit you will be able to :

- Explain the generic term network analysis
- Draw diagram & chart representing inter-relationships between various elements of the project and finally establishing critical path
- Identify Dummy Activities
- Identify earliest and latest event times of each event. Identify different types of Float Help in Decision Making about project times and resource allocation Calculating probability of completing network in a given time

13.1 Introduction

Planning, Scheduling and Controlling are three important functions of management. Planning involves the formulation of objectives and goals that are subsequently translated into specific plans and projects. Scheduling is concerned about the implementation of activities necessary to achieve the laid down plans. The function of control is to institute a mechanism that can trigger a warning signal if actual performance is deviating (in terms of time, cost and some other measures of effectiveness) from the plan. If such a deviation is unacceptable to the concerned manager, he will be required to take corrective action to bring performance in conformity with the plans. The PERT and CPM models are extremely useful for the purpose of planning, scheduling and controlling the progress and completion of large and complex projects or for carrying out the analysis of these three managerial functions. Before we describe the basic concepts used in the construction and analysis of these models, let us first understand the meaning of a project.

What is a project?

A project can be defined as a set of large number of activities or jobs that are performed in a certain sequence determined logically or technologically and it has to be completed within (i) a specified time, (ii) a specified cost and (iii) meeting the performance standards. Examples of a project from fairly diverse fields are given below:

1. Introducing a new product in the market.
2. Construction of a new bridge over a river or construction of a 25 storied building.
3. Executing a large and complex order on jobbing production.
4. Sending a spacecraft to the mars.

13.2 General Framework of PERT/CPM

A network is a graphical representation of a project, depicting the flow as well as the sequence of well-defined activities and events. Developed during the 1950s, both CPM (Critical Path Method) and PERT (Programme Evaluation and Review Technique) are network techniques/models. The network approach helps project managers in planning, scheduling and controlling. As a planning tool it helps the manager to estimate the requirements of resources viz., materials, equipment, manpower, cost and time for each activity or tasks of the project. This approach cannot make decisions by its own. It only provides additional information to executives to facilitate decision making process. Also it does not provide solution to every management problem. It certainly helps in identification of those activities, jobs or events which control the completion of the project.

13.2.1 Methodology of Critical Path Analysis

The working methodology of critical path analysis (CPA) which includes both CPM and PERT, consists of following five steps:

1. Analyse and break down the project in terms of specific activities and/ or events.
2. Determine the interdependence and sequence of specific activities and prepare a network.
3. Assign estimates of time, cost or both to all the activities of the network.
4. Identify the longest or critical path through the network.
5. Monitor, evaluate and control the progress of the project by re-planning, re-scheduling and re-assignment of resources.

The central task in the control aspect of these models is to identify the longest path through the network. **The longest path is the critical path because it equals the minimum time required to complete the project.** All other paths other than the critical path (i.e. non-critical or slack paths) offer flexibility in scheduling and transferring resources, because they take less time to complete than the critical path.

13.3 Advantages of Critical Path Analysis

There are a number of advantages in using critical path analysis.

1. It allows for a comprehensive view of the entire project. Because of the sequential and concurrent relationships, time scheduling becomes very effective. Identifying the critical activities keeps the executive alert and in a state of preparedness, with alternative plans ready in case these are needed. Breaking down the project into smaller components permits better and closer control.
2. Critical path analysis offers economical and effective system of control based on the principle of management by exception i.e. need for corrective action arises only in exceptional situations and in most of other cases, performance is in conformity with the plans.
3. It is a dynamic tool of management which calls for constant review, a reformulation of the network, and finding the current path of relevance and optimum resources allocation.

13.4 Fundamentals of a Critical Path Analysis (CPA) Network

❖ **Activity**

An activity is any portion of a project which consumes time or resources and has a definable beginning and ending. For example, "laying of pipe" is an activity requiring the use of resource mainly effort. Activity may involve labour, paper work, contractual negotiations, machinery operations, etc. Commonly used terms synonymous with "activity" are "task" and "job".

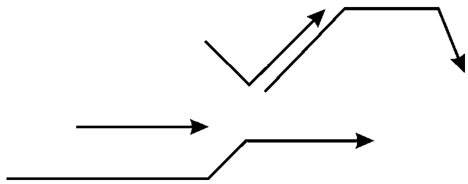


Figure 1

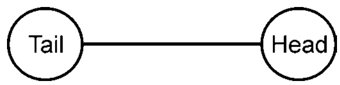


Figure 2

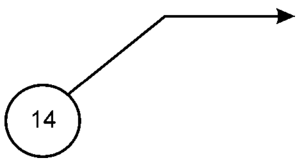


Figure 3

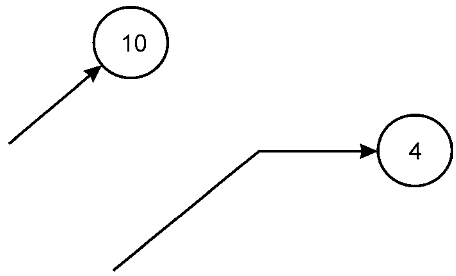


Figure 4

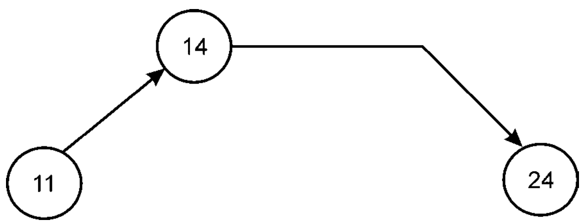


Figure 5

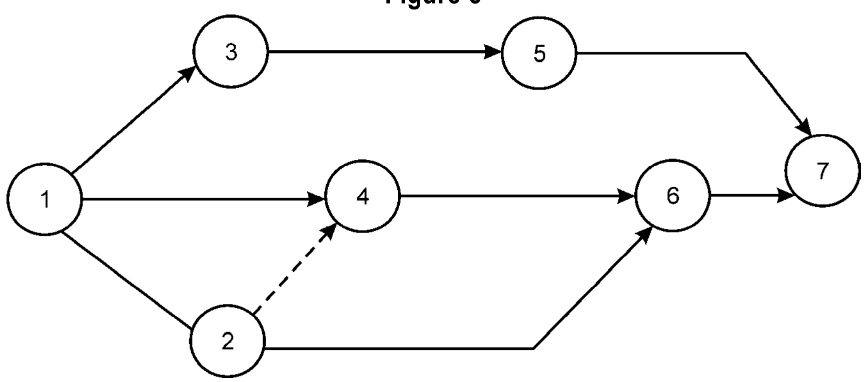
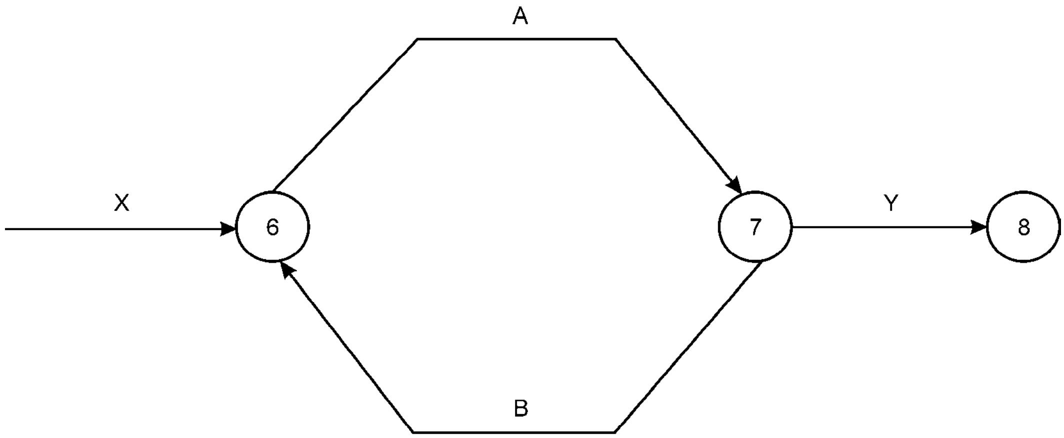


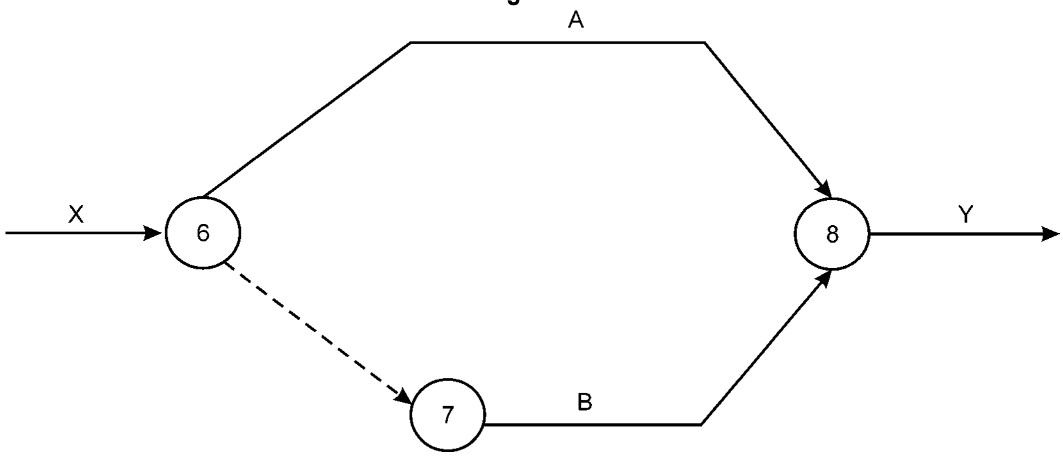
Figure 6

Activities are graphically represented by arrows, usually with description and time estimates written along the arrows. The tail of the arrow portraying an activity represents the starting point of the activity and its head represents its completion. The arrow may be straight slanting, or bent but not broken (see figure-1). The arrow is not a vector and need not be drawn to scale.



Duplicate Activities

Figure 7



Introduction of dummy activity

Figure 8

Normally an activity can be categorized into four activities:

(i) **Predecessor activity:** The activity or activities which immediately come before another activity without any intervening activities are called Predecessor activities.

(ii) **Successor activity:** The activity or activities which follow another activity without any intervening activities are called successor activity.

(iii) **Concurrent activity:** Activities or tasks that can be carried out concurrently with another activity are called concurrent activity. An activity may be concurrent with one or more than one activities.

(iv) **Dummy activity:** It is a hypothetical activity and does not consume any kind of resource. It is represented by dotted lines and is inserted in the network to clarify activity pattern under the following situations.

a. to make activity/activities with common starting and finishing points distinguishable.

b. to identify and maintain the proper precedence relationship between activities that are not connected by events.

c. to bring all "loose ends" to a single initial and a single terminal event in each network using dummies, if necessary. When in a network diagram situation of duplicate activity arise, this situation is corrected by introducing a dummy activity.

❖ **Events**

The beginning and ending points of an activity or a group of activities are called *events*. Synonyms of an event are "node" and "connectors" An event is often represented graphically by a numbered circle (see figure-2), although any geometric figure such as square, oval, rectangle etc. will serve the purpose. We shall, however, stick to the most commonly used convention for representing an event viz, the circle. A few examples of events are as follows: (i) Material procured, (ii) Design completed, (iii) Project started, (iv) Bricks laid, etc.

All activities in a network must commence from some event. Such events are called the *tail events* because they are connected to the tail of an activity. These are shown in figure 3. Similarly, all activities in a network must have terminal points called the *head event*, because it is at the head of an activity. These are shown in figure-4. Figure-5 depicts the tail and head events connected by arrows representing activities i.e. it depicts the dual role of an event. Event 14 is the head event for one activity and tail event for another.

In a network, symbol "*i*" is used for the tail event (also called preceding event) and "*j*" for the head event (or succeeding event) of an activity. The activity, then being *i-j*.

The events may be classified into three categories:

(i) *Merge event:* If an event represents the joint completion of more than one activity i.e., a particular point (termed as event) where two or more activities complete is called merge event (See figure–9)

(ii) *Burst event:* If an event represents joint initiation i.e., a particular point (termed as event) form where more than one activities starts, is called Burst event (see figure–10).

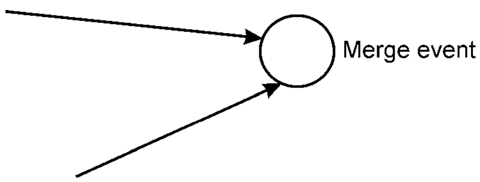


Figure 9

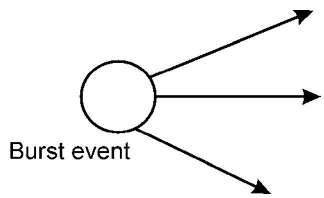


Figure 10

(iii) *Merge and Burst event: It* is an event from where more than one activity starts and more than one activity complete as well. In general language it is a starting point for some activities and ending point for some activities (see figure–11)

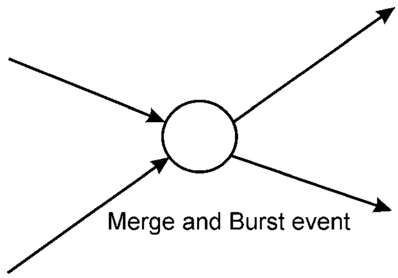


Figure 11

A network is, then, a graphical representation of a project plan, showing the inter-relationship of the various activities. Networks are also called *arrow diagrams* (see figure-6). When the results of time estimates and computations have been added to a network, it may be used as a project schedule.

13.4.1 Conventions adopted in drawing networks: There are two conventions normally adopted while drawing networks. In the early stages of network drawing, it is suggested that the conventions should be respected until sufficient experience has been gained to justify dropping them. These conventions are:

(a) Time flows from left to right.

(b) Head events always have a number higher than that of the tail events.

The above stated conventions allow activities to be referred uniquely by their tail and head event numbers, so that "activity 3-4" means only "the activity which starts from event 3 and proceeds to event 4"; it cannot mean "the activity which starts from event 4 and finishes at event 3".

13.4.2 Graphical representation of events and activities: Events are represented by numbers within circles. Activities are represented by arrows; the arrow-heads represent the completion of the activities. The length and orientation of the arrow are of no significance whatsoever (chosen only for the convenience of drawing). The activity of leaving place A and walking to place B can equally well be represented by figure-12.

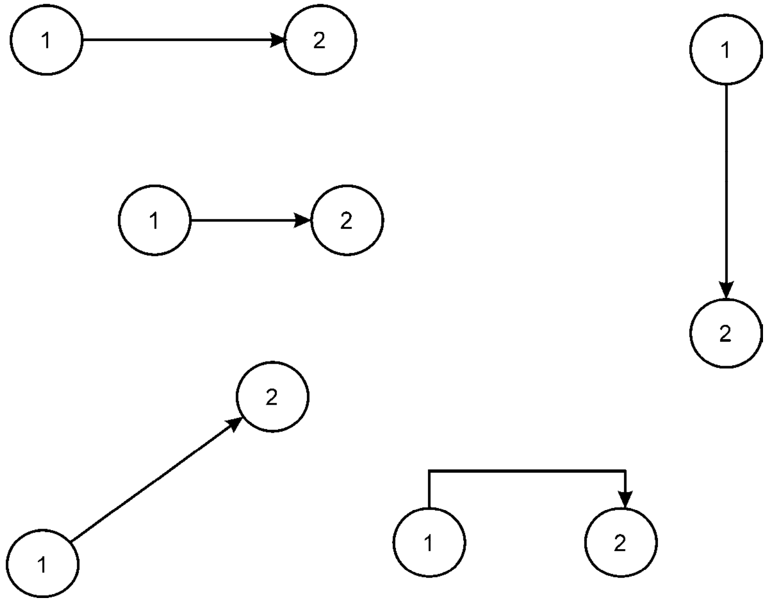


Figure 12

13.4.3 Fundamental properties governing the representation of events and activities: The representation of events and activities is governed by one simple *dependency rule* which requires that an activity which depends upon another activity is shown to emerge from the head event of the activity upon which it depends and that only dependent activities are drawn in this way. Thus, if activity B depends upon activity A, then the two activities are drawn in figure-13.

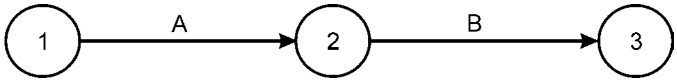


Figure 13

The dependency rule gives rise to two fundamental properties of events and activities:*

1. An event cannot occur until all activities leading to it are complete.
2. No activity can start until its tail event is reached.

The above two properties can be combined into a single one, namely that "no activity may start until all previous activities in the same chain are completed.

13.4.4 Logical sequencing and connection of activities: A project entails several activities. The arrows are arranged to show the plan of logical sequence in which the activities of the project are to be accomplished. The sequence is ascertained for each activity by answering the following three queries viz:

(i) Which activity or activities must be completed before the start of a particular activity?

(ii) Which activity or activities should follow this?

(iii) Which activities can be accomplished simultaneously?

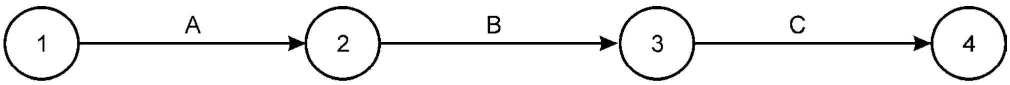
Take an example of a pipe line project. Three activities have been identified for this project namely trenching, laying down of pipes and welding of pipes. To complete this project logical sequence is to be understood. Welding cannot be done until pipes are laid down and pipes cannot be laid down until the pipes are trenched. The logical sequence is like that first trenching second laying and lastly welding activity should be carried out. The activities will be as follows:

A $\rightarrow$Trenching

B $\rightarrow$Laying

C $\rightarrow$Welding

The Network for the same is as follows



Network diagram of pipe line project

Figure 14

13.4.5 Errors in logical sequencing: Generally three types of errors in logical sequencing may arise while drawing a network diagram, particularly when it is a complicated one. These are known as *looping, dangling and redundancy.*

(1) **Looping:** Normally in a network, the arrow points from left to right. This convention is to be strictly adhered, as this would avoid illogical looping. Looping error is also known as Cycling error:

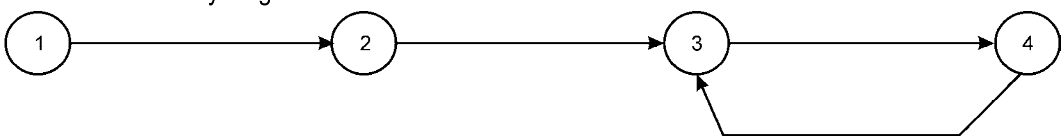


Figure 15

(2) **Dangling:** Activity which is not connected to any of the intermediate events or end event is called dangling activity. The situation represented by the following diagram is also at fault, since the activity represented by the dangling arrow 9-11 is undertaken with no result.

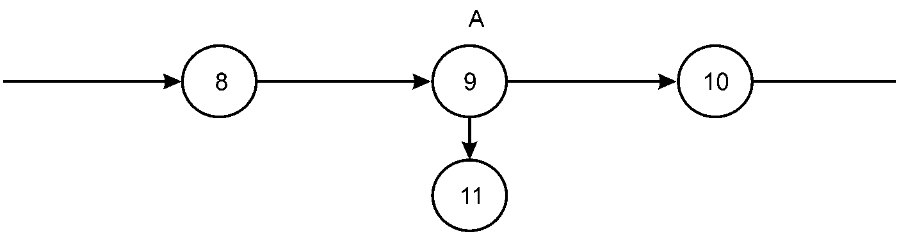


Figure 16

To overcome the problem arising due to dangling arrows, following rules may be adopted.

(i) All events, except the first and the last, must have at least one activity entering and one activity leaving them, and

(ii) All activities must start and finish with an event.

(3) **Redundancy**: When dummy activities are inserted in a network diagram unnecessarily, this type of error is called error of redundancy. It is shown in the following figure:

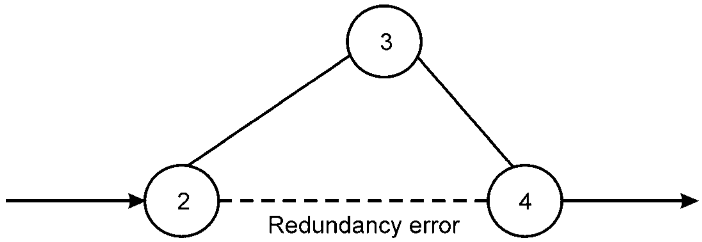


Figure 17

Figure-18 shows three cases for the following set of dependency relationships:

Activity C is dependent upon both A and B.

Activity D is dependent upon A alone.

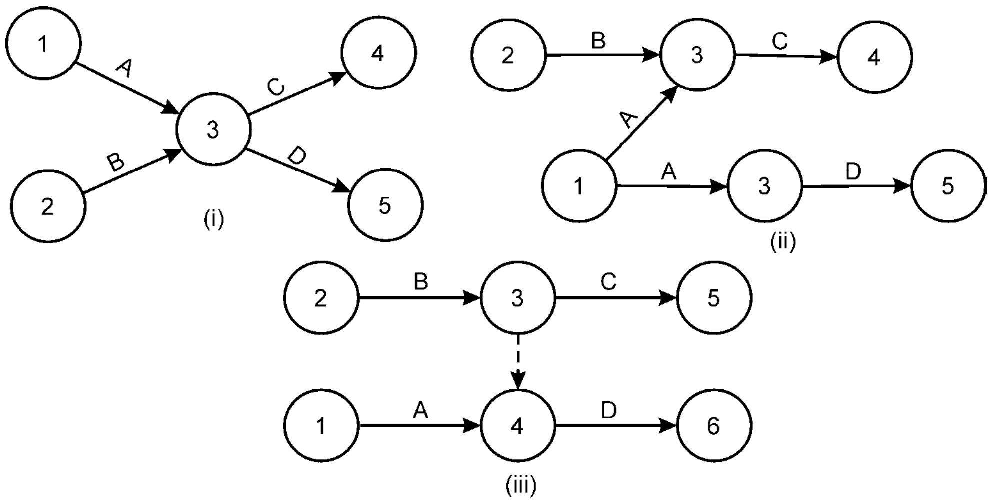


Figure 18

The first portrayal on (i) of figure-18 is clearly wrong since it shows D as dependent upon not only A but also B which is not desired. The other portrayal (ii) is also wrong since A is being shown twice and thus contravenes the fundamental axiom of network that there must be one arrow for each activity. The way out to this dilemma is the representation by means of the dummy activity. In the third portrayal of figure -18, C is dependent upon both A and B (via dummy) whereas D is dependent upon just A.

13.4.6 Numbering the events: The event numbers in a network should in some respect reflect their logical sequences. When a complicated network is drawn then the problem of assigning numbers to the events involved in the network arises. A rule devised by Delbert Ray

Fulkerson, involving the following steps may be followed to resolve the problem of numbering the events.

(i) An "initial" event is one which has arrow/arrows coming out of it and none of the arrow entering in it. In a network there will be only one such event. Call it "1".

(ii) Delete all arrows coming out from the event 1. This will give us at least one more "initial event".

(iii) Number these events as "2, 3...."

(iv) Delete all emerging arrows from these numbered events which will create new initial events. Then follow step (iii).

(v) Continue the above steps till last event is obtained which has no arrows coming out of it. Consider the numbering of events in the following figure.

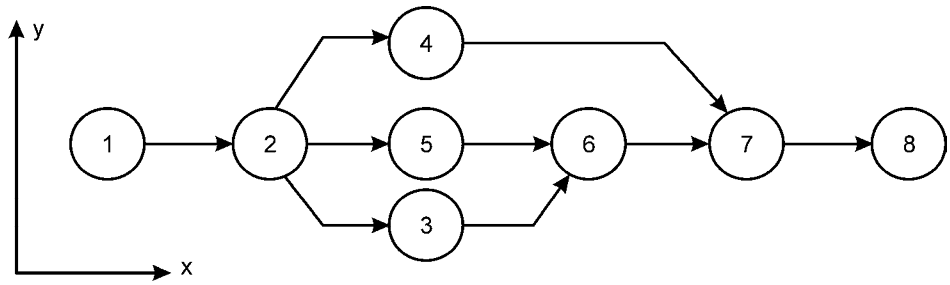


Figure 19

Here we proceed from left to right. The event with least x-co-ordinate is assigned the smallest integer, say 1. Other events are assigned progressively higher integers with regard to x-co-ordinate. If two or more events (4 and 5 above) have the same x-co-ordinate, the one towards arrow should have higher number.

Furthermore, it is not necessary, and in fact also not desirable to number the events consecutively. It would be a better scheme to number the events as 10, 20, 30, 40, 50, 60, 70, in the above diagram instead of 1,2,3,4,5,6,7. This affords insertion of more activities and events omitted by oversight or having become necessary in view of certain logic revisions.

It was mentioned earlier that it is desirable that all the activity arrows point from left to right. If the arrow is vertical it may point downwards or upwards.

For the sake of presentation it is to be recommended that activities emanating from one event or converging to another may make as great angles between themselves as possible.

A few more conventions are given below:

(i) Keep the arrow to the extreme right. (ii) As far as possible avoid drawing arrows that cross each other. Usually by suitable 'stretching' the network diagram it is possible to avoid this. (iii) Where, however, crossing is unavoidable, bridging may be done. This applies to dummies as well. Draw boldly a big network. Smaller ones are confusing. Use of pencil and rubber is recommended.

Exercise: Depict the following dependency relationships by means of network diagrams. The Alphabets stand for activities.

(i) A and B control F; B and C control G.

(ii) A and B control F; B controls G while C controls G and H.

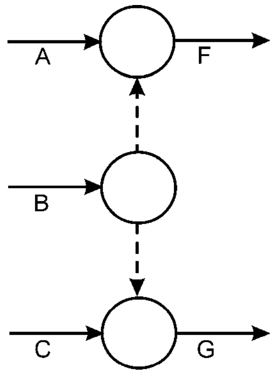
(iii) A controls F and G; B controls G; while C controls G and H.

(iv) A controls F and G; B and C control G with H depending upon C.

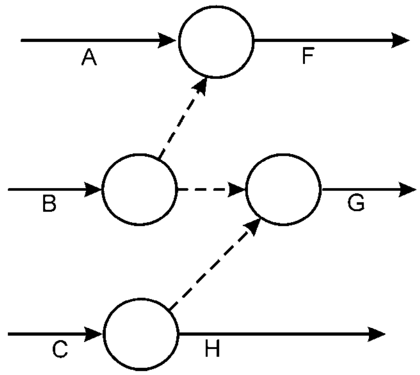
(v) F and G are controlled by A, G and H are controlled by B with H controlled by B and C.

(vi) A controls F, G and H; B controls G and H with H controlled by C.

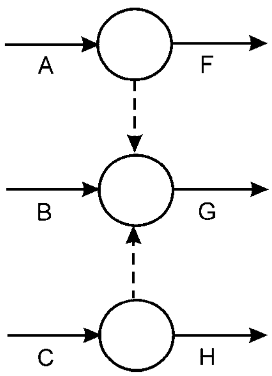
Answer. The required networks are given in figure-20



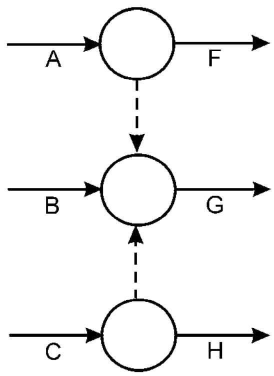
(I)



(ii)



(iii)



(iv)

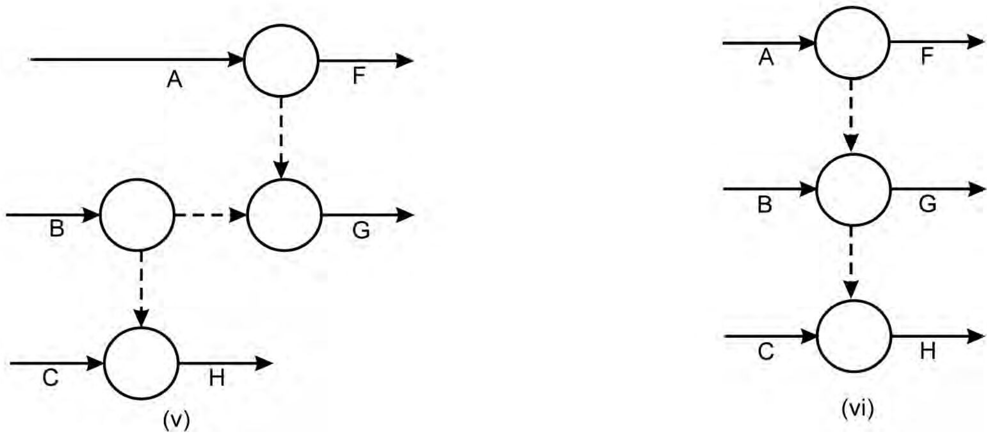


Figure 20

Exercise: Find out the superfluous (unnecessary) dummy activities in the networks below.

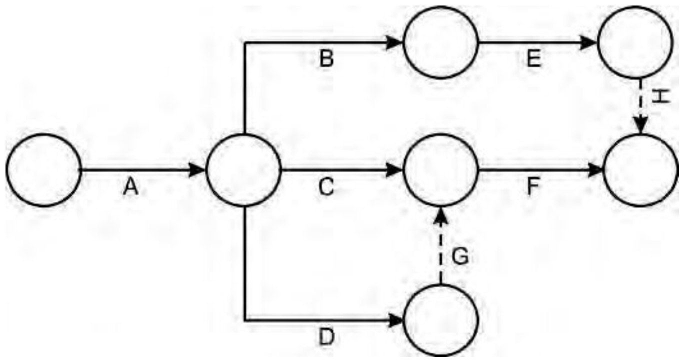


Figure 21

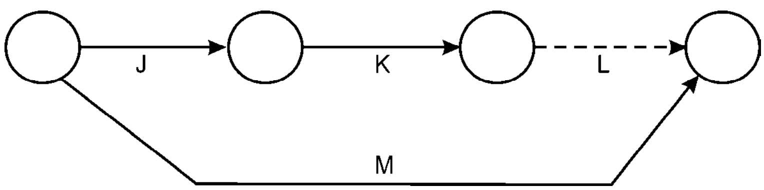


Figure 22

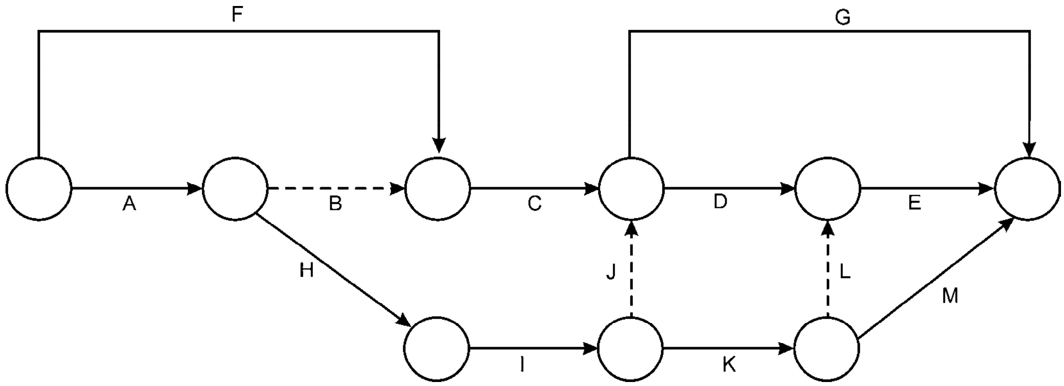


Figure 23

13.4.7 Basic steps involved in drawing a CPM/PERT network : Network is defined as a diagram representing the activities and events of a project, their sequence and inter-relationships.

The basic steps involved in drawing a network are:

(i) Breaking up of the entire project into smaller systems known as tasks.

(ii) For each task ascertain the activities and events to be performed.

(iii) For each activity determine the preceding and succeeding activities.

(iv) For each activity determine or estimate the time and other resources needed.

(v) Draw a network depicting the assembly of tasks into a project.

13.4.8 Network Construction

Illustration 1

The activities involved in the computer installation process are detailed below. You are required to draw the network.

	Activity	*Predecessor Activities*
A.	*Physical Preparation*	*None*
B.	*Organisational Planning*	*None*
C.	*Personnel Selection*	*B*
D.	*Equipment Installation*	*A*
E.	*Personnel Training*	*C*
F.	*Detailed Systems Design*	*C*
G.	*File Conversion*	*F*
H.	*Establish Standards and Controls*	*F*
I.	*Programme Preparation*	*H*
J.	*Programme Testing*	*I*
K.	*Parallel Operations*	*D, E, G, J.*
L.	*Finalise Systems Documentation*	*I*
M.	*Follow up.*	*K, L*

Solution:

The network is shown in figure 24. The construction has been carried out step by step which is explained below. The symbol '<' is to be read as "precedes" in these figures.

(i) Since A and B are preceded by no activity, they are shown emanating from the start event. We do not put arrows and the head events for A and B because they have been drawn as such only tentatively in the following step (s) their position may have to be changed.

(ii) Here C is shown as dependent upon B. Again we neither show arrow and nor the head event of C because its position is tentative.

(iii) D is shown as dependent upon A.

(iv) We show here C preceding E and F.

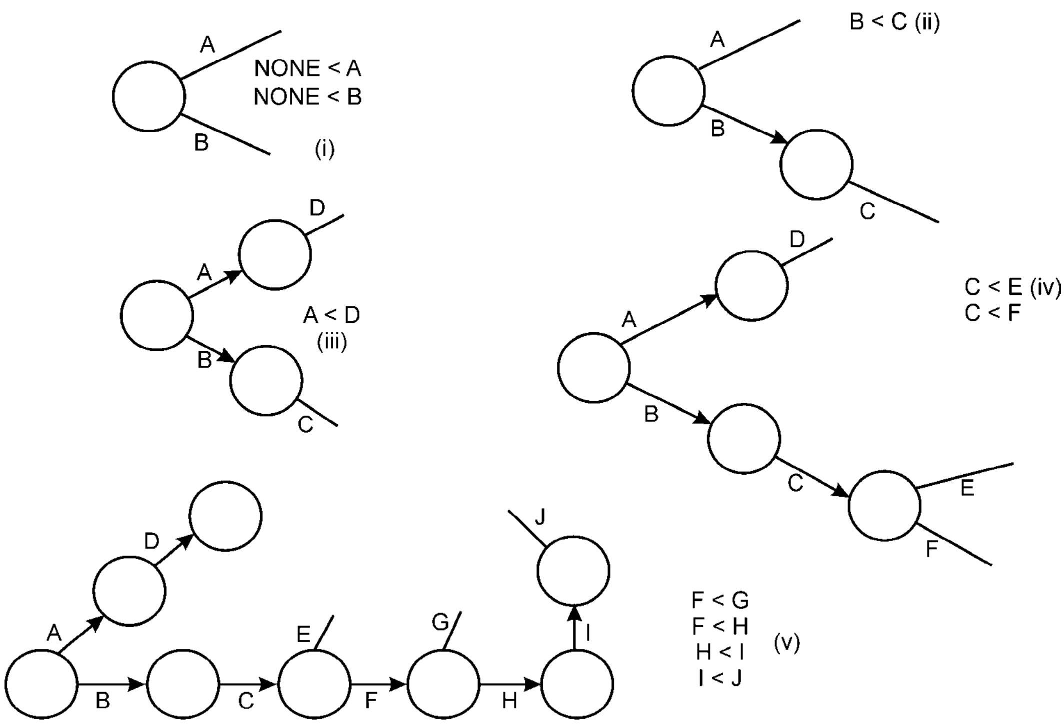
(v) F is shown preceding G and H, H precedes I and I precedes J.

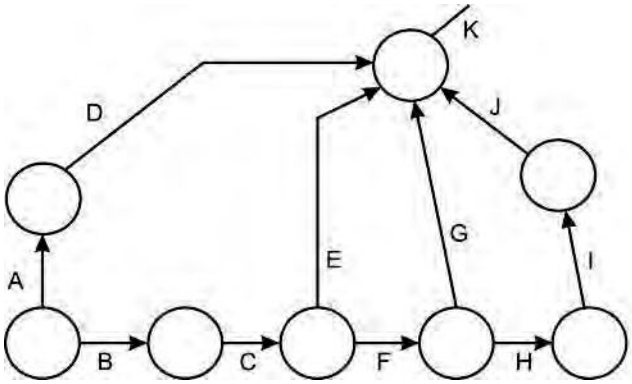
(vi) K is shown as dependent upon D, E, G and J, the latter four therefore, are made to converge on the same head event.

(vii) L is shown as dependent upon I and M as dependent on K and L. Note carefully that K and L are the terminal activities and, therefore, converge on the last event.

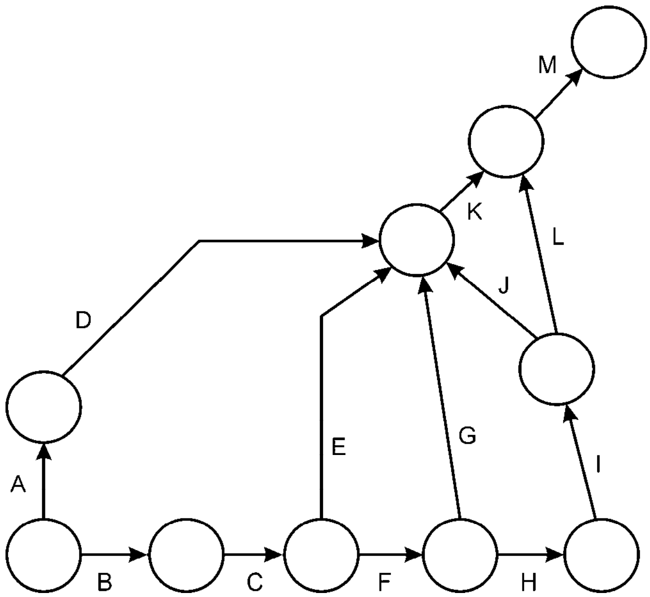
About this point the student is cautioned since the beginners do tend to show the last activities dangling and do not converge them to the common last event.

Of course, the network would be constructed in just one diagram (as shown in Figure-24). We drew so many for explanation only. However, for a stylish presentation it is desirable that this one diagram is suitably sketched i.e. faired up in another diagram. The student may number the events as an exercise.





D, E, G and J < K (vi)



I < L
L < M

(vii)

Figure 24

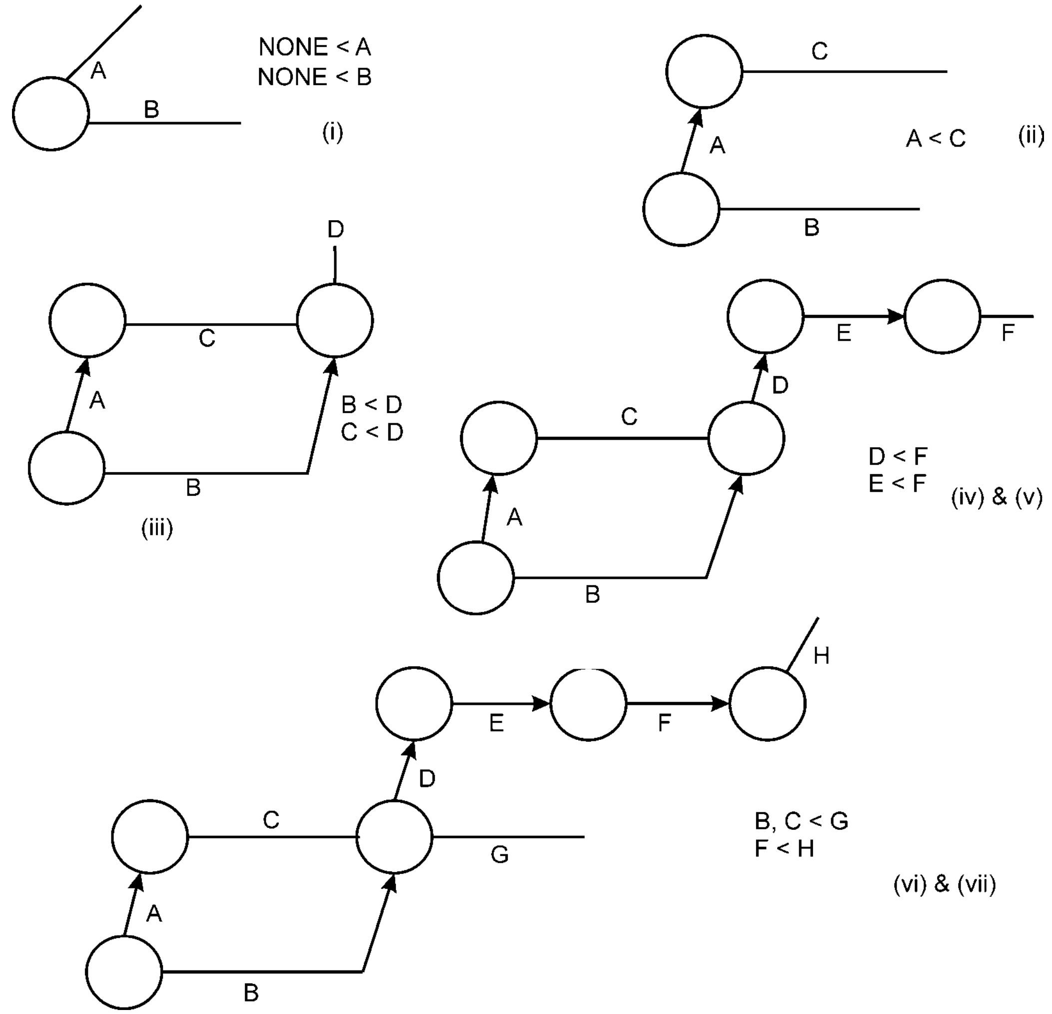
Illustration 2

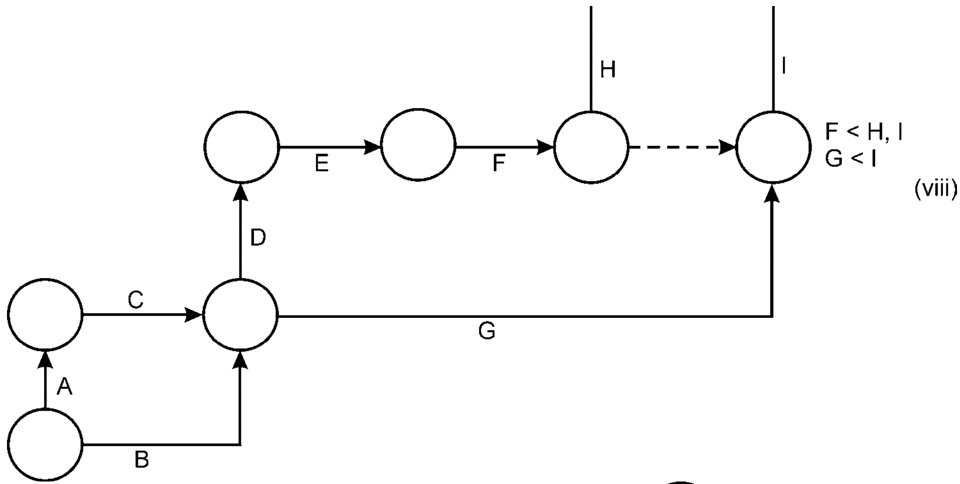
Activity	*Preceded by*
A.	*None*
B.	*None*
C.	*A*
D.	*B, C*
E.	*D*
F.	*E*
G.	*B, C*
H.	*F*

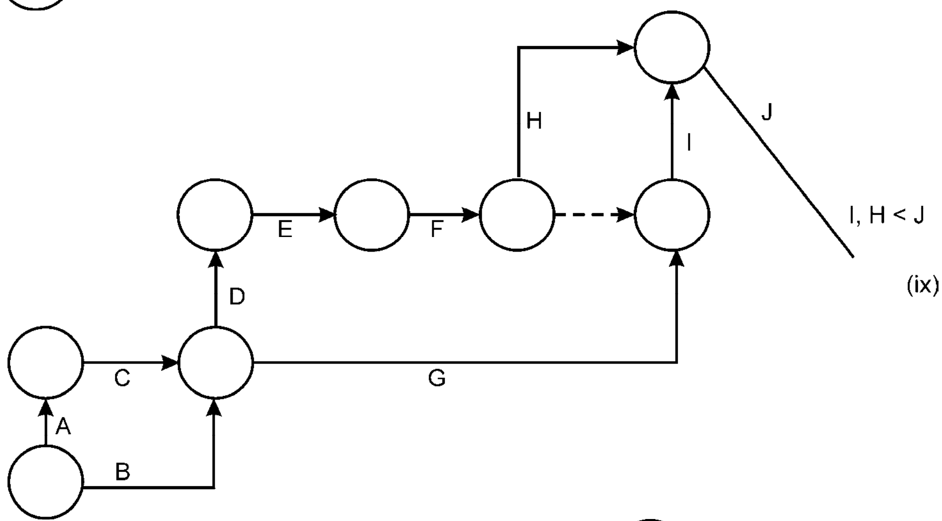
I.	*F, G*
J.	*H, I*
K.	*B*
L.	*F, G, K*

Solution:

The network is shown in figure-25 step by step with dependency relationships that are incorporated at each step which the student should, by now, be able to follow. Please carefully review the steps where dummies are incorporated.




H
I
E
F
F < H, I
G < I
(viii)
D
C
G
A
B


H
J
I
E
F
I, H < J
(ix)
D
C
G
A
B

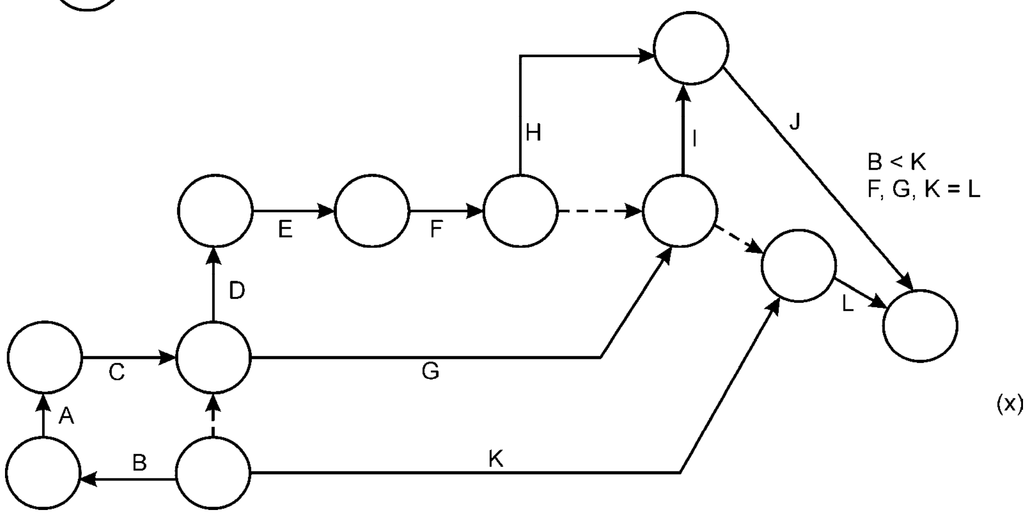

H
I
J
B < K
F, G, K = L
E
F
D
L
C
G
(x)
A
B
K

Figure 25

13.4.9 Concurrent activities: Activities may not always be discrete i.e. they may be done in part allowing the subsequent activities to commence before the preceding activity is fully completed. Activities of this kind are to be frequently encountered in batch production. If, for example, a batch of 50 spindles is to be processed on two machines obviously it is not necessary to process all the items of the batch on the first machine and then transfer these to the next machine. A few items processed on the first machine may be transferred to the second machine before completion of the entire batch on the first machine. Since this is a matter of great practical importance we shall dwell upon it at a greater length. Such simultaneous or concurrent activities are to be encountered in sewage work e.g,., trenching, laying pipe, welding pipe and back filling, all going on simultaneously with suitable lags on construction work.

Illustration 3

A batch of 4 axles is to be proceeded on the following three machines in this sequence:

Lathe (L), Milling (M) and Grinding (G). Instead of first working on these 4 axles on lathe and then on milling and finally on grinding in sequence, it is desired to process the first axle on the lathe and as and when it is processed, it is taken up on milling and the 2nd axle on the lathe, and so on. In other words, each of the three activities L,M and G have been quartered for the sake of concurrent operations. You are required to draw the network.

Solution

The dependency relationships are sorted out hereunder:

Quartered Activity	***Preceded by***
L_1	None
L_2	L_1
L_3	L_2
L_4	L_3
M_1	L_1
M_2	L_2, M_1
M_3	L_3, M_2
M_4	L_4, M_3
G_1	M_1
G_2	M_2, G_1
G_3	M_3, G_2
G_4	M_4,G_3

The network is now constructed below. (The student is urged to draw his own network. Mere understanding of our solution is not enough.)

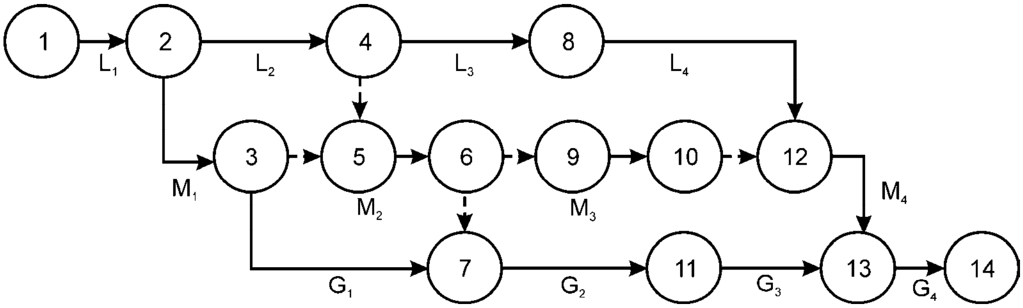


Figure 26

(**N.B.:** The concurrent activities so drawn are known: as 'ladders' in the network jargon.)

Illustration 4

Draw a network diagram for the following data.

Task	*Immediate Predecessor*
A	—
B	—
C	*B*
D	*B*
E	*B*
F	*E*
G	*A, D,C.*

Solution

The required network is given below.

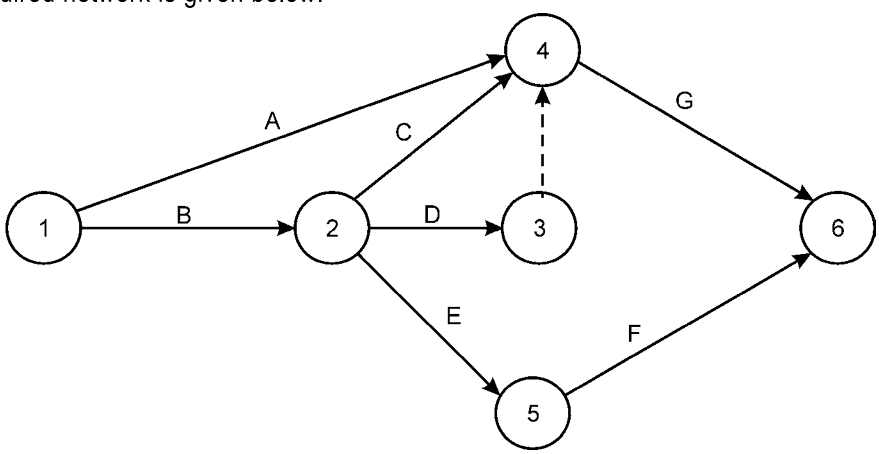


Figure 27

Illustration–*5*

Draw a network diagram for the following data.

Activity	*Preceding activities*
A	—
B	*A*
C	*A*
D	*B*
E	*A*
F	*B, E*
G	*C*
H	*D, F*
I	*G*
J	*H, I*

Solution: The required network is given below:

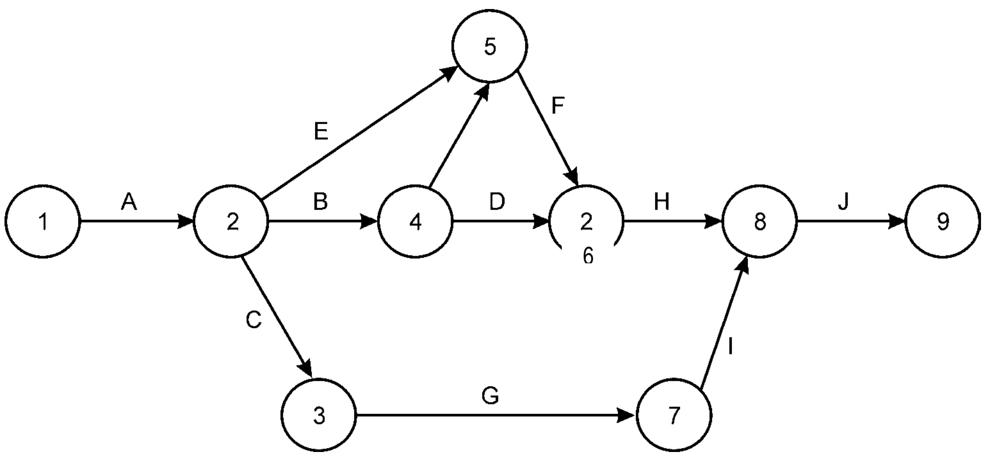


Figure 28

13.5 Critical Path Analysis

You have already been familiarised with the logic of the critical path analysis by way of introduction of this chapter.

The purpose of the analysis is two-fold: (i) **to find the critical path, i.e. the sequence of activities with the longest duration.** Once it is found it is marked in bold sequence of arrows on the network. For a simple network as of figure -29 the various sequences can be enumerated and the durations of activities encompassed by them simply added, to find the critical sequence. As stated earlier, one could indeed end up with more than one critical sequence; and (ii) **to find the float associated with each non-critical activity.**

Systematic analysis: The enumeration method would be too cumbersome computationally for any real life project, Even the computer would be hard pressed to proceed this way. There is a systematic way that cuts short the analysis time to manageable proportions.

It is accomplished by performing the following steps:

1. Calculate the time schedule for each activity. This represents the time by which an activity must begin and the time before which it must be completed. The time schedule data for each activity includes, the earliest start, earliest finish, the latest start, latest finish, and finally the float, which is the spare time associated with an activity.
2. Calculate the time schedule for the completion of entire project. This represents the scheduled date for the completion of the entire project and the probability of completing the project on or before the deadline.
3. Identify the critical activities. These activities are the ones which must be started and completed on schedule, or else the project is likely to be delayed.
4. Determine the critical path for the network. This path represents the critical activities which must be closely followed in order to complete the project on time.

13.5.1 Scheduling computations: After preparing the network diagram, we wish to know how long it will take to complete the project and also to identify the activities in the network that are to be placed under strict control. The basic scheduling computations involve a **forward** and then a **backward pass** through the network. The process of tracing the network from START to END is called forward pass, and from END to START is called backward pass. Based on a specified occurrence time for the initial network event, the forward pass computations give the earliest start and finish times for each activity. By the specification of the latest allowable occurrence time for the terminal network event, the backward pass computation will give the latest allowable start and finish times for each activity.

13.5.2 Forward pass computations: As stated above, the purpose of the forward pass is to compute the earliest start (EST) and finish time (EFT) for each activity. The EST time indicates the earliest time that a given activity can be scheduled. Earliest finish time for an activity indicates the time by which the activity can be completed, at the earliest. To compute these time estimates, we will first of all compute the earliest allowable occurrence time for various events of the network.

It is a convention to keep the earliest allowable occurrence time of the START event as zero.

To understand, how this time estimate for other events is computed, let us consider the following network diagram.

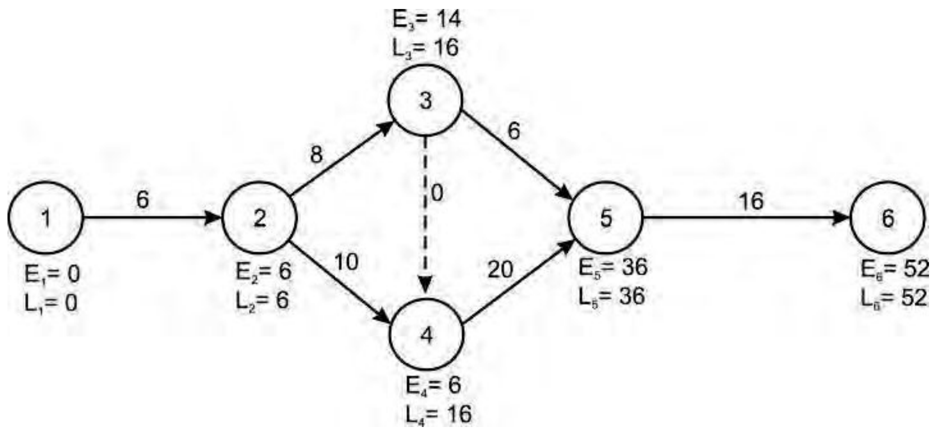


Figure 29

Earliest allowable occurrence time of an event or Earliest Event Time:

In the network shown above, event 1 stands for the beginning of the activity 1 – 2 and we can say that it occurs at the time zero i.e. $E_1 = 0$. Event 2 stands for the finish of the activity 1–2 thus event 2 can occur at the earliest time E_2 which is computed as

$$E_2 = 0 + D_{12} = 0 + 6 = 6$$

Where, D_{12} stands for the duration of activity 1 – 2

Event 3 stands for finish of the activity 2–3 and its earliest time is

$$E_3 = E_2 + D_{23} = 6 + 8 = 14$$

The event 4 can occur either at the end of the activity 3 – 4 or at the finish of activity 2 – 4. In this case, there will be two time estimates as follows:

$$E_4 = E_3 + D_{34} = 14 + 0 = 14$$

$$E_4 = E_2 + D_{24} = 6 + 10 = 16$$

In case two or more time estimates exist for a particular event, then the time estimate with maximum value is retained as the earliest event time and other values are discarded. This maximum value represents the completion of all the activities ending at the event under consideration. In the above example, the earliest event time for event 4 will be 16.

A general rule can also be given here for determining the earliest event time as below:

$$E_j = \mathrm{M}ax\,(E_i + D_{ij})$$

Where, E_j is the earliest time for event j, E_i is the earliest time for event *i*, and D_{ij} is the duration of the activity *i-j*.

❖ **Earliest start and earliest finish times of an activity**

After computing the earliest event time of various events, one can easily compute the earliest start and finish times of all the activities on the network. **The earliest start time of an activity**

is given by the earliest allowable occurrence time of the tail event of that activity. Thus, in our example, the earliest start time of the activity 1–2 will be given by the earliest time of the event 1 i.e. it will be 0. The earliest start time for the activities 2 – 3 and 2 – 4 will be given by the earliest time of event 2 which is equal to 6. The earliest time for the activities 3–4 and 3–5 will be 14 which is the earliest time for the event 3 and so on.

The earliest finish time of an activity will be simply equal to the earliest start time of the activity plus the duration of that activity. Hence, in our example, earliest finish time of activity 1–2 will be 0+6 = 6, for activity 2–3, it will be 6+8 = 14 and for activity 2–4, it will be 6+10 = 16 and so on.

The complete computations for all the activities are shown in columns (3) and (4) of table -1 given on the next page.

13.5.3 Backward pass computations: The purpose of the backward pass is to compute the *latest start (LST) and finish times (LFT)* for each activity. These computations are precisely a "mirror image" of the forward pass computations. The term "latest allowable occurrence time of an event" (denoted by L_i) is used in the sense that the project terminal event must occur on or before some arbitrary scheduled time. **Thus, the backward pass computations are started rolling back by arbitrarily specifying the latest allowable occurrence time for the project terminal event.** If no schedule date for the completion of the project is specified, then the convention of setting the latest allowable time for the terminal event equal to its earliest time, determined in the forward pass, is usually followed, i.e. L = E* for the terminal event of the project. This convention is called the **zero slack convention. Following this, one can also interpret the latest allowable activity finish time (LFT) as the time to which the completion of an activity can be delayed without directly causing any increase in the total time to complete the project.**

To explain the computation, let us again consider the network diagram in figure-29. The terminal event is 6 so we set $L_6 = E_6 = 52$ and we start rolling back. The latest allowable occurrence time for the events 5 and 4 are $L_5 = 52 - 16 = 36$ and $L_4 = 36–20 = 16$ respectively. It may be noted here that we can roll back to event 3 via activity 3-5 as well as activity 4–3. So there are two latest allowable occurrence times for the event 3 as given below;

$$L_3 = L_4 - D_{34} = 16 - 0 = 16$$
$$L_3 = L_5 - D_{35} = 36 - 6 = 30$$

We retain the minimum value as the latest occurrence time for the event 3 and ignore other values. Therefore, the latest allowable occurrence time for the event 3 is 16. Similarly

$$L_2 = L_3 - D_{23} = 16 - 8 = 8$$

and $$L_2 = L_4 - D_{24} = 16 - 10 = 6$$

The latest occurrence time for the event 2 is thus 6 and the latest occurrence time for the event 1 is equal to its earliest time i.e. zero.

In general, the latest allowable occurrence time of an event can be calculated by selecting an appropriate formula among the following two:

$L_i = L_j - D_{ij}$

or $L_i = \text{minimum } (L_j - D_{ij})$

The second formula is used for the event having two or more latest allowable occurrence time estimates.

❖ **Latest start and latest finish times of an activity**

After computing latest allowable occurrence time for various events, one can compute the latest start and latest finish times of an activity. **The latest finish time of an activity is equal to the latest allowable occurrence time of the head event of that activity.**

i.e. LFT (i – j) = L_j

The latest start time of an activity is equal to its latest finish time minus its duration.

i.e LST (i – j) = LFT (i – j) – D_{ij}

These computations are shown in column (4) and column (6) of table-1 given below:

Table 1

		START		FINISH	
Activity	**Duration**	**Earliest time**	**Latest time**	**Earliest time**	**Latest Time**
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
1–2	6	0	6	0	6
2–3	8	6	14	8	16
2–4	10	6	16	6	16
3–4	0	14	14	16	16
3–5	6	14	20	30	36
4–5	20	16	36	26	36
5–6	16	36	52	36	52

13.5.4 The critical path determination: After having computed various time estimates, we are now interested in finding the critical path of the network. A network will consist of a number of paths. A path is a continuous series of activities through the network that leads from the initial event (or node) of the network to its terminal event. For finding the critical path, we list out all possible paths through a network along with their duration. In the network under consideration, various paths have been listed below.

Path	Length (in days)
1–2–3–5–6	36
1–2–4–5–6	52
1–2–3–4–5–6	50

❖ **Critical path:** A path in a project network is called critical if it is the longest path. The activities lying on the critical path are called the critical activities.

In the above example, the path 1–2–4–5–6 with the longest duration of 52 days is the critical path and the activities 1–2, 2–4, 4–5 and 5–6 are the critical activities.

13.5.5 Calculation of Floats: It may be observed that for every critical activity in a network, the earliest start and latest start time are the same. This is so since the critical activities cannot be scheduled later than their earliest schedule time without delaying the total project duration, they do not have any flexibility in scheduling. However, non-critical activities do have some flexibility i.e. these activities can be delayed for some time without affecting the project duration. This flexibility is termed as *slack* in case of an event and as *float* in case of an activity.

Some people do not make any distinction between a slack and a float.

❖ **Slack time for an event**

The slack time or slack of an event in a network is the difference between the latest event time and the earliest event time. Mathematically it may be calculated using the formula $L_i - E_i$. Where, L_i is the latest allowable occurrence time and E_i is the earliest allowable occurrence time of an event i.

❖ **Total float of an activity:** The total activity float is equal to the difference between the earliest and latest allowable start or finish times for the activity in question. Thus, for an activity (i-j), the total float is given by

$$TF_{ij} = LST - EST \text{ or } TF_{ij} = LFT - EFT$$

In other words, it is the difference between the maximum time available for the activity and the actual time it takes to complete. Thus, total float indicates the amount of time by which the actual completion of an activity can exceed its earliest expected completion time without causing any delay in the project duration.

❖ **Free Float:** It is defined as that portion of the total float within which an activity can be manipulated without affecting the float of the succeeding activities. *It can be determined by subtracting the head event slack from the total float of an activity.*

i.e $$FF_{ij} = TF_{ij} - (\text{slack of event j})$$

The free float indicates the value by which an activity in question can be delayed beyond the earliest starting point without affecting the earliest start, and therefore, the total float of the activities following it.

❖ **Independent float:** It is defined as that portion of the total float within which an activity can be delayed for start without affecting float of the preceding activities. *It is computed by subtracting the tail event slack from the free float of an activity.*

$$IF_{ij} = FF_{ij} - (\text{slack of event i})$$

The independent float is always either equal to or less than the free float of an activity. If a negative value is obtained, the independent float is taken to be zero.

❖ **Interfering float:** Utilization of the float of an activity can affect the float of subsequent activities in the network. Thus, interfering float can be defined as that part of the total float which causes a reduction in the float of the successor activities. In other words, *it can be defined as the difference between the latest finish time of the activity under consideration and the earliest start time of the following activity, or zero, whichever is larger.* Thus, interfering float refers to that portion of the activity float which cannot be consumed without affecting adversely the float of the subsequent activity or activities.

Illustration 6

Activity	Duration
1–2	4 days
1–3	12 days
1–4	10 days
2–4	8 days
2–5	6 days
3–6	8 days
4–6	10 days
5–7	10 days
6–7	0 days
6–8	8 days
7–8	10 days
8–9	6 days

With the help of the activities given above draw a network. Determine its critical path, earliest start time, earliest finish time, latest start time, latest finish time, total float, free float and independent float.

Solution

The network based on the activities given in the example is as follows:

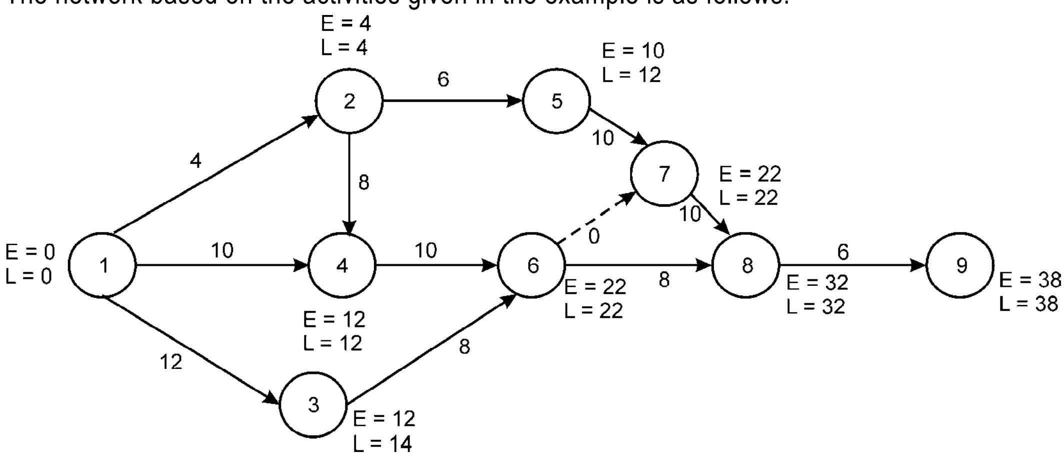


Figure - 30

The time estimates and floats are calculated as below:

Job or activity	Duration of an activity D_{ij}	EST (= E_i)	EFT (E_i+D_{ij})	LST (L_j–D_{ij})	LFT (= L_j)	Slack of		Total float (LST-EST)	Free float (Total Float -Slack of head event)	Independent (Free float-slack of tail event)
						tail event (L_i–E_i)	head event (L_j–E_j)			
1-2	4	0	4	0	4	0	0	0	0	0
1-3	12	0	12	2	14	0	2	2	0	0
1-4	10	0	10	2	12	0	0	2	2	2
2-4	8	4	12	4	12	0	0	0	0	0
2-5	6	4	10	6	12	0	2	2	0	0
3-6	8	12	20	14	22	2	0	2	2	0
4-6	10	12	22	12	22	0	0	0	0	0
5-7	10	10	20	12	22	2	0	2	2	0
6-7	0	22	22	22	22	0	0	0	0	0
6-8	8	22	30	24	32	0	0	2	2	2
7-8	10	22	32	22	32	0	0	0	0	0
8-9	6	32	38	32	38	0	0	0	0	0

Critial path is represented by 1-2-4-6-7-8-9. The project duration is 38 days.

(Note: In this table we have given detailed calculations merely for explanation.)

Illustration 7

Analyse the network below for the critical path and for different floats

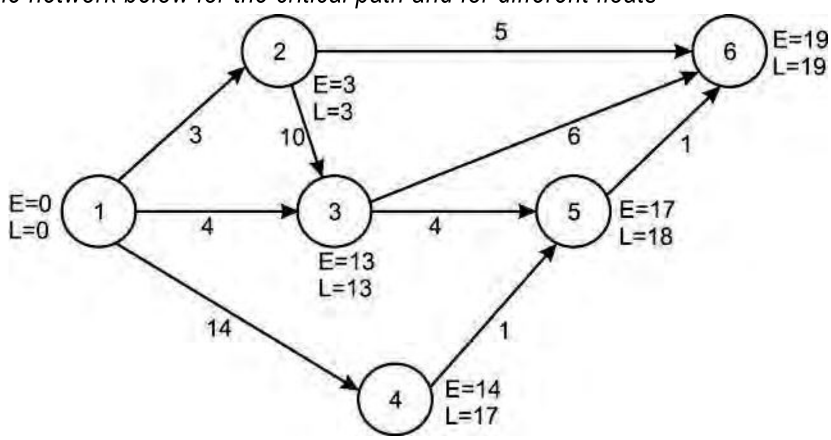


Figure 31

Solution

Network Analysis Table

Activity	Duration	Start		Finish		Float			
		E	L	E	L	Total	Free	Ind	Interfering
1-2	3	0	0	3	3	0	0	0	0
1-3	4	0	9	4	13	9	9	9	0
1-4	14	0	3	14	17	3	0	0	3
2-3	10	3	3	13	13	0	0	0	0
2-6	5	3	14	8	19	11	11	11	0
3-5	4	13	14	17	18	1	0	0	1
3-6	6	13	13	19	19	0	0	0	0
4-5	1	14	17	15	18	3	2	0	1
5-6	1	17	18	18	19	1	1	0	0

(N.B.: Activities must always be arranged in the i-j sequence.)

Explanation: Consider activity 1 – 3. The slack of head event 3 is 13 – 13 = 0. Therefore, free float = 9 – 0 = 9. Likewise slack of tail event 1 is 0 – 0 = 0. Therefore independent float = 9.

Illustration 8

Analyse the network below for the critical path and calculate total float, free float and independent float for each activity.

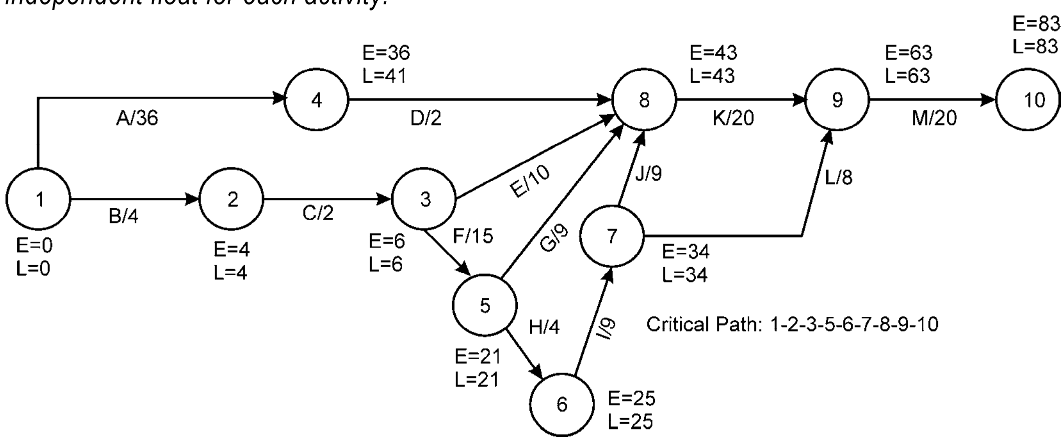


Figure 32

Solution

Network Analysis Table

i-j	D	Start		Finish		Floats		
		E	L	E	L	T	F	I
1- 2	4	0	0	4	4	0	0	0
1-4	36	0	5	36	41	5	0	0
2-3	2	4	4	6	6	0	0	0
3-5	15	6	6	21	21	0	0	0
3-8	10	6	33	16	43	27	27	27
4-8	2	36	41	38	43	5	5	0
5-6	4	21	21	25	25	0	0	0
5-8	9	21	34	30	43	13	13	13
6-7	9	25	25	34	34	0	0	0
7-8	9	34	34	43	43	0	0	0
7-9	8	34	55	42	63	21	21	21
8-9	20	43	43	63	63	0	0	0
9-10	20	63	63	83	83	0	0	0

Critical Path: BCFHIJKM = 83 weeks.

13.6 Distinction Between PERT & CPM

PERT	CPM
(i) PERT stands for Program Evaluation Review Technique	(i) CPM stands for Critical Path Method
(ii) It is a Probabilistic tool using three estimates of duration	(ii) It is a deterministic tool, uses only single estimate of duration
(iii) It is a basically a tool for planning	(iii) CPM can be used to control both time and cash.
(iv) PERT is more suitable for R & D related projects.	(iv) CPM is best suited for routine projects/ works where time and cost and accurately.
(v) PERT is event oriented	(v) CPM is activity oriented.

Summary

- Two most commonly used network Models
  - ✓ Critical Path Method (CPM)
  - ✓ Programme Evaluation Review Technique (PERT)
- Construction of Network diagrams
- Calculation of Critical Path and Project Completion time together with Floats, earliest start and finishing times
- Application of technique to crash cost problems
- Critical path should be longest route through the network
- Resource schedule problems and use of bar charts
- Identifying the project duration
- Cheapest critical activities to crash and not reduce project completion time by reducing non-critical activities

14

Program Evaluation and Review Technique

LEARNING OBJECTIVES

After studying this unit you will be able to :

- Work out how network analysis helps managers to plan when to start various tasks to monitor actual progress and to know when control action is needed to prevent delay in completion of the project
- Draw diagram and chart representing inter-relationships between various elements of the project and finally establishing critical path
- Identify dummy activities
- Identify earliest event times and latest event times for each event
- Identify total float, free float and independent float
- Crash times, costs and consumption of resources
- Help in decision making about project times and resource allocation
- Calculate the probability of completing the network in a given time when multiple time estimates of activity duration are available

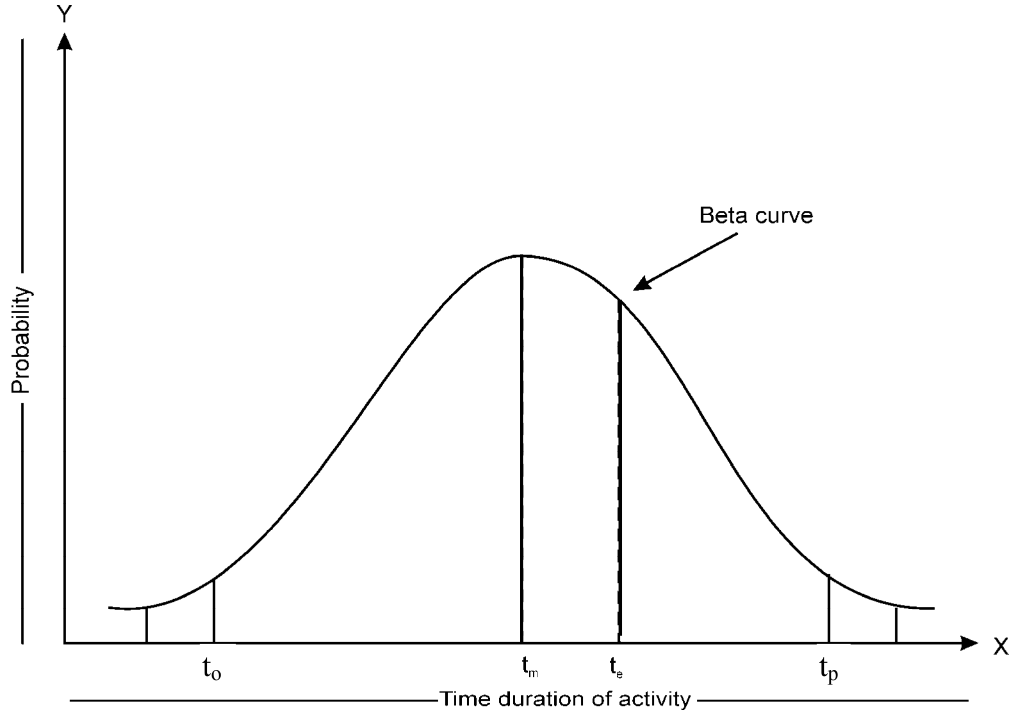
14.1 Introduction

Hitherto our emphasis has been on CPM which is applicable where activity durations can be estimated from experience, past historical records and work study techniques fairly precisely. *CPM is incapable of handling uncertainty in timing* which is a rule rather than an expectation for innovational projects such as introducing a new product or oil exploration project. PERT (Program Evaluation and Review Technique) is more relevant for handing such projects which have a great deal of uncertainty associated with the activity durations. To take these uncertainty into account, three kinds of times estimates are generally obtained. These are:

(a) *The Optimistic Times Estimate:* This is the estimate of the **shortest possible** time in which an activity can be completed under ideal conditions. For this estimate, no provision for delays or setbacks is made. We shall denote this estimate by t_o.

(b) *The Pessimistic Time Estimate:* This is the **maximum possible time** which an activity could take to accomplish the job. If everything went wrong and abnormal situations prevailed, this would be the time estimate. It is denoted by t_p.

(c) *The Most Likely Time Estimate :* This is a time estimate of an activity which lies between the optimistic and the pessimistic time estimates. It assumes that things go in a normal way with few setbacks. It is represents by t_m. Statistically, it is the model value if duration of the activity.



t_o = optimistic time
t_m = most likely time
t_e = expected time
t_p = pessimistic time

Figure 1

Estimate of Probability: Due to variability in the activity duration, the total project may not be completed exactly in time. Thus, it is necessary to calculate the probability of actual meeting the schedule time of the project as well as activities.

Probability of completing the project by schedule time (T_s) is given by

$$Z = \frac{T_s - T_e}{\sigma_e}$$

Te represents the duration on the critical path. T_e can be calculated by adding the expected time of each activity lying on the critical path. σ_e represents standard deviation of the critical path. Variance of the critical path can be getting by adding variances of critical activities. σ_e is the square root of variance of the critical path.

Beta distribution is a assumed for these "guess estimates" and PERT analysts have found that beta-distribution curve happens to give fairly satisfactory results for most of the activities. For a distribution of this type, the *standard deviation* is approximately one sixth of the range, i.e.

$$S_t = \frac{t_p - t_o}{6}$$

The variance, therefore; is

$$S_t^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

Expected time: The expected time (t_e) is the average time taken for the completion of the job. By using beta-distribution, the expected time can be obtained by following formula.

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Illustration 1

If the critical path of a project is 20 months along with a standard duration of 4 months, what is the probability that the project will be completed within:

(a) 20 months (b) 18 months (c) 24 months?

Solution

(a) $Z = \frac{20 - 20}{4} = 0$; Probability = 0.50

(b) $Z = \frac{18 - 20}{4} = -0.50$; Probability = 0.31

(c) $Z = \frac{24 - 20}{4} = 1$; Probability = 0.84

Illustration 2

PERT calculations yield a project length of 60 weeks with variance of 9. Within how many weeks would you expect the project to be completed with probability of 0.99?

Solution

$T_e = 60$ S.D. $= \sqrt{9} = 3$

$Z = \frac{T_s - T_e}{\sigma} = \frac{T_s - 60}{3} = 2.3$

$60 + 3 \times 2.3 = 67$ weeks

Note: Prob. 0.99 means Z = 2.3

Illustration 3

A small project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending (j) node numbers.

(a) Draw the project network and identify all the paths through it.

(b) *Find the expected duration and variance for each activity. What is the expected project length?*

(c) *Calculate the variance and standard deviation of project length. What is the probability that the project will be completed?*

(i) *At least 3 weeks earlier than expected?*

(ii) *No more than 3 weeks later than expected?*

(d) *If the project due date is 18 weeks, what is the probability of not meeting the due date?*

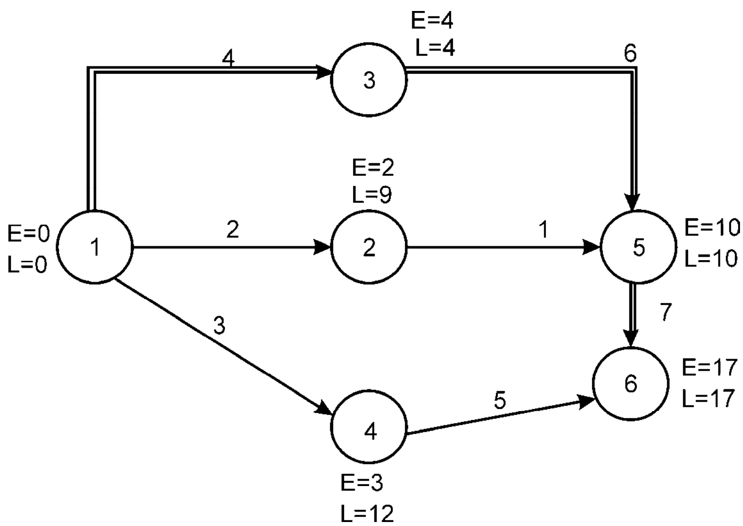
(e) *What due date has about 90% chance of being met?*

	Activity Estimated duration in Weeks		
i–j	***Optimistic***	***Most Likely***	***Pessimistic***
1–2	*1*	*1*	*7*
1–3	*1*	*4*	*7*
1–4	*2*	*2*	*8*
2–5	*1*	*1*	*1*
3–5	*2*	*5*	*14*
4–6	*2*	*5*	*8*
5–6	*3*	*6*	*15*

Solution

(a) The network is drawn in Fig. 2. The various paths are as follows.

1–2–5–6, 1–3–5–6, 1–4–6



Critical Path 1–3–5–6

Figure 2

(b) Expected duration and variances for various activities are computed below.

	T_e		V_t	
1–2;	$\frac{1+1\times4+7}{6}$	= 2;	$\left[\frac{7-1}{6}\right]^2$	$=1$
1–3;	$\frac{1+16+7}{6}$	= 4;	$\left[\frac{7-1}{6}\right]^2$	$=1$
1–4;	$\frac{2+8+8}{6}$	=3;	$\left[\frac{8-2}{6}\right]^2$	$=1$
2–5;	$\frac{1+4+1}{6}$	=1;	$\left[\frac{1-1}{6}\right]^2$	$=0$
3–5;	$\frac{2+20+14}{6}$	=6;	$\left[\frac{14-2}{6}\right]^2$	$=4$
4–6;	$\frac{2+20+8}{6}$	=5;	$\left[\frac{8-2}{6}\right]^2$	$=1$
5–6;	$\frac{3+24+15}{6}$	=7;	$\left[\frac{15-3}{6}\right]^2$	$=4$

(c) Expected project Length = 17 weeks

Variance of the critical path 1–3–5–6 = 1+4+4 = 9

$\therefore$ Standard Deviation = $\sqrt{9}$ = 3

(i) $Z = \frac{14-17}{3} = -1$; Probability = 0.159

(ii) $Z = \frac{20-17}{3} = +1$; Probability = 0.841

(d) $T_e = 17$; $T_1 = 18$; $Z = \frac{18-17}{3} = 0.333$.

Therefore, probability of meeting the due date = 0.63.

And probability of not meeting the date = 0.37 (1–0.63)

(e) At 90% Probability, Z = 1.3 approximately.

$\therefore 1.3 = \frac{T_s - 17}{3}$ or T_S = 20.9 weeks approximately.

☞ Critical Path is highlighted through double line arrows.

Illustration 4

Find event variances in the network of previous illustration.

Solution

Event variance for both the T_E and T_L of each event are derived below. The computational procedure should be self evident. We shall put to use the variances of the various activities derive in part (b) of solution to example above.

Earliest Time, T_E (D=Duration)

Event	*Longest path leading to it*	**Variance*
1	Nil (D=0)	0
2	1–2 (D=2)	1
3	1–3 (D=4)	1
4	1–4 (D=3)	1
5	1–3, 3–5 (D = 4 + 6 = 10)	1 + 4 = 5
6	1–3, 3–5, 5–6 (D=17)	1 + 4 + 4 = 9

Suppose we wish to find the probability of reaching event 5 in 9 days. This can be computed as below:

$$Z = \frac{9-10}{\sqrt{5}} = \frac{(-)1}{\sqrt{5}} = \frac{-\sqrt{5}}{5} = \frac{(-)\ 2.236}{5} = -0.4472$$

Probability of reaching event 5 in 9 days is equal to 0.5 – .1723 = .3275. Likewise we can determine probabilities of reaching other events.

Longest Time T_L

Event	*Longest path from it to last event 6*	*Variance*
1	1–3, 3–5, 5–6	9
2	2–5, 5–6	4
3	3–5, 5–6	8
4	4–6	1
5	5–6	4
6	Nil	0

Illustration 5

Shown below is a PERT network and a related set of activity times:

i–j	A	B	C	D	E	F	G	H	I	J	K	L
t_o	10	12	8	4	0	12	6	9	4	0	5	9
t_m	13	15	11	7	0	18	12	12	6	0	8	12
t_p	22	18	20	16	0	36	18	27	8	0	11	33

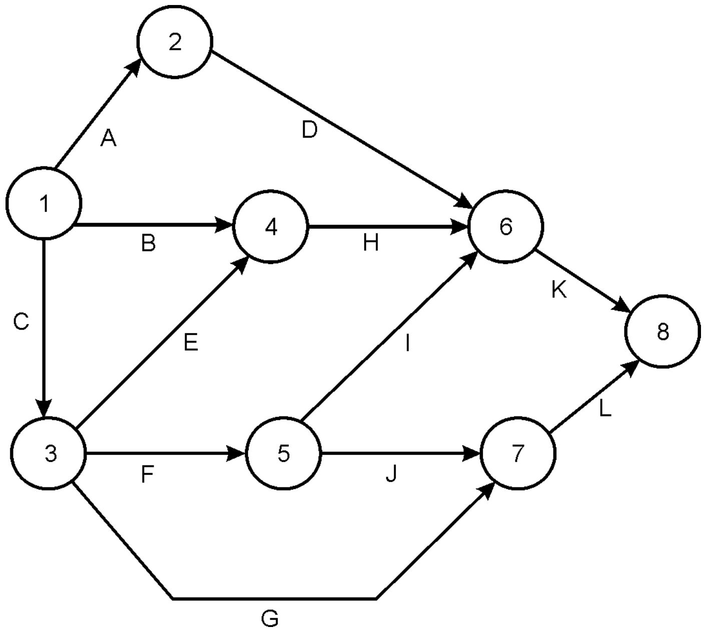


Figure 3

Required

(a) Determine the expected completion time of each activtiy.

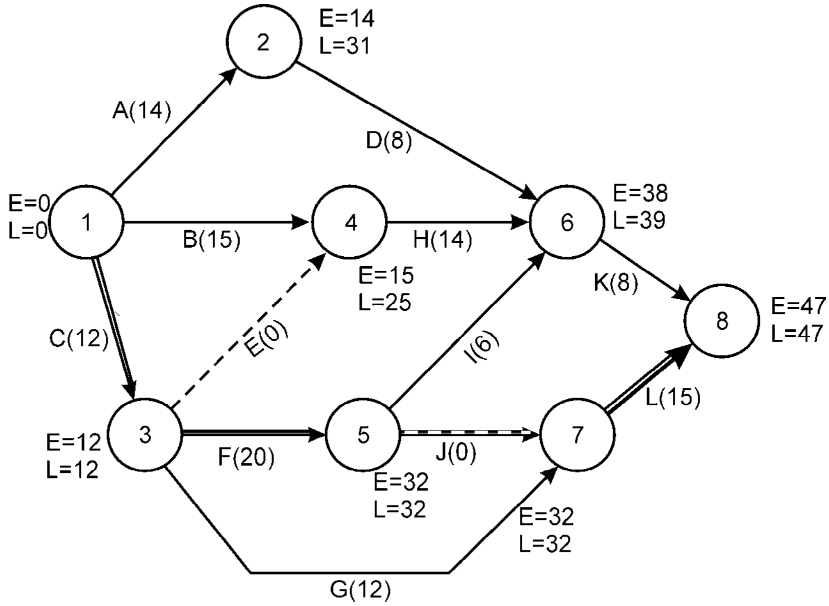
(b) Determine the earliest expected completion time, the latest expected completion time and float of each activity.

(c) What is the total project completion time, and what are activities on the critical path?

(d) Determine S.D. of expected completion time for only those activities on the critical path.

(e) Determine the probability that the project will be completed within 41 weeks.

Solution



1–3–5–7–8

Figure 4

$$T_e = \frac{t_p + 4t_m + t_0}{6} \; ; \; V_t = \left(\frac{t_p - t_0}{6}\right)^2$$

	Time estimates				Start		Finish		LST – EST		
i–j	**t_o**	**$4t_m$**	**t_p**	**t_e**	**EST**	**LST**	**EFT**	**LFT**	**Float**	**V_t**	**S.D**
1–2	10	52	22	14	0	17	14	31	17		
1–3	8	44	20	12	0	0	12	12	0	4	2
1–4	12	60	18	15	0	10	15	25	10		
2–6	4	28	16	8	14	31	22	39	17		
3–4	0	0	0	0	12	25	12	25	13		
3–5	12	72	36	20	12	12	32	32	0	16	4
3–7	6	48	18	12	12	20	24	32	8		
4–6	9	48	27	14	15	25	29	39	10		
5–6	4	24	8	6	32	33	38	39	1		
5–7	0	0	0	0	32	32	32	32	0	0	0
6–8	5	32	11	8	38	39	46	47	1		
7–8	9	48	33	15	32	32	47	47	0	16	4
									$\Sigma V_t = 36$		

Hence S.D. of the critical path = $\sqrt{36}$ = 6

Probability of completion critical path in 41 weeks is computed below:

$$Z = \frac{41 - 47}{6} = -1$$

$\therefore$ Probability = 0.159 (Answer).

14.2 Updating the Network

The progress of various activities in a project network is measured periodically. Normally, either most of the activities are ahead or behind the schedule. It is therefore, necessary to update or redraw the network periodically to know the exact position of completion of each activity of the project. The task of updating the network may be carried out once in a month. Sometimes the updating of the network may provide useful information to such an extent that it may demand the revision of even those very activities which have not started. Even the logic may also change i.e. some of the existing activities may have to be dropped and new activities may be added up. In brief the network should be amended accordingly in the light of new developments.

It is also not unlikely that the total physical quantum of work accomplished at a point of time may exceed what was planned but the progress against the critical path alone may be slower than the scheduled pace. To understand how the task of updating is carried out, consider the following example:

Illustration 6

After 15 days of working the following progress is noted for the network of an erection job.

(a) Activity 1-2, 1-3 and 1-4 completed as per original schedule.

(b) Activity 2-4 is in progress and will be completed in 3 more days.

(c) Activity 3-6 is in progress and will need 18 days more for completion.

(d) Activity 6-7 appears to present some problem and its new estimated time of completion is 12 days.

(e) Activity 6-8 can be completed in 5 days instead of originally planned for 7 days.

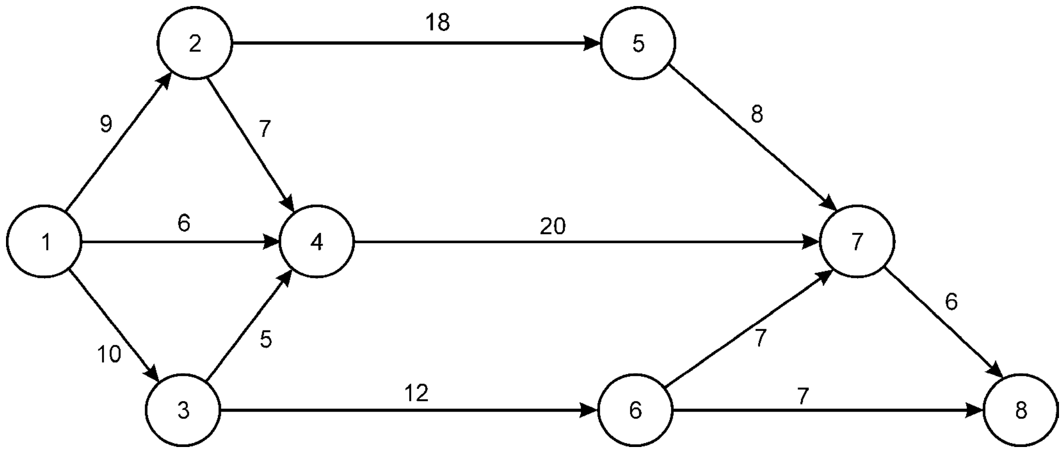


Figure 5

You are required to:

(i) Update the above diagram after 15 days of the start of work based on the assumptions given above.

(ii) Write down the critical path with total project duration.

Solution

Path	Duration
1–2–5–7–8	9+18+8+6 = 41
1–2–4–7–8	9+7+20+6 = 42
1–4–7–8	6+20+6 = 32
1–3–4–7–8	10+5+20+6 = 41
1–3–6–7–8	10+12+7+6 = 35
1–3–6–8	10+12+7 = 29

Critical path 1–2–4–7–8 = 42 days

The new formulation of the problem is as follows:

(i) Activities 1-2, 1-3 and 1- 4 need 9, 10 and 6 days respectively as per original programme.

(ii) Activity 2-4 needs 15 + 3-9 = 9 days instead of original programme of 7 days.

(iii) Activity 3-6 needs 15 + 18 -10 = 23 days.

(iv) Activity 3-4 needs 5 days.

(v) Activities 2-5, 4-7 and 5-7 need 18, 20 and 8 days respectively.

(vi) Activity 6-7 needs 12 days as no work was scheduled to be started for this activity on 15th day.

(vii) Activities 6-8 and 7-8 need 5 and 6 days respectively.

Revised

Path	Duration
1–2–5–7–8	9+18+8+6 = 41
1–2–4–7–8	9+9+20+6 = 44
1–4–7–8	6+20+6 = 32
1–3–4–7–8	10+5+20+6 = 41
1–3–6–7–8	10+23+12+6 = 51
1–3–6–8	10+23+5 = 38

Critical path 1–3–6–7–8 = 51 days

The new diagram based on the above listed activities will be as follows:

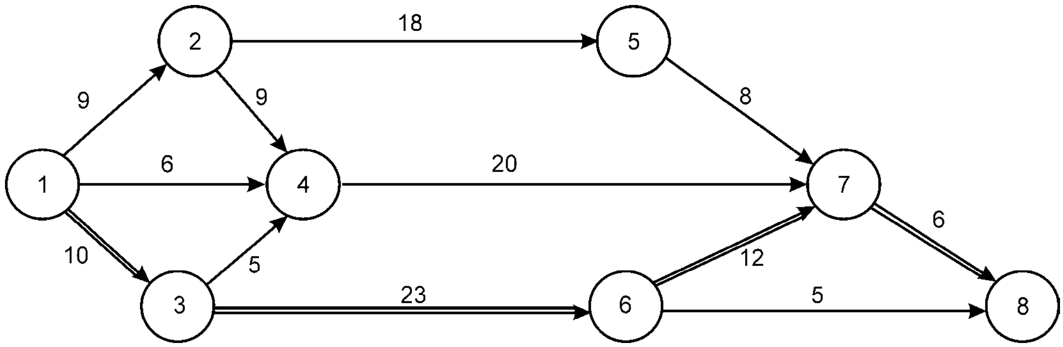


Figure 6

14.3 Project Crashing

In the discussion on PERT, we saw how the probability of completion of a project can be computed for a specified duration. There are usually compelling reasons to complete the project earlier than the originally estimated duration of the critical path computed on the basis of normal activity times, by employing extra resources. An example would be introduction of a new project. The motive in hastening the project might be to ensure that the competitors do not steal a march. In the present section we will deal with those situations which will speak of the effect of increase or decrease in the total duration for the completion of a project and are closely associated with cost considerations. In such cases when the time duration is reduced, the project cost increases, but in some exceptional cases project cost is reduced as well. The reduction in cost occurs in the case of those projects which make use of a certain type of resources, for example, a machine and whose time is more valuable than the operator's time. Before we take up an example of project cost control, it is better to understand well the following preliminaries and their definitions.

Activity Cost : It is defined as the cost of performing and completing a particular activity or task.

Crash Cost Cc	:	This is the direct cost that is anticipated in completing an activity within the crash time.
Crash time, Ct	:	This is the minimum time required to complete an activity.
Normal Cost, Nc	:	This is the lowest possible direct cost required to complete an activity.
Normal time, Nt	:	This is the minimum time required to complete an activity at normal cost.

Activity cost slope: The cost slope indicates the additional cost incurred per unit of time saved in reducing the duration of an activity. It can be understood more clearly by considering the figure 7.

Let OA represent the normal time duration for completing a job and OC the normal cost involved to complete the job. Assume that the management wishes to reduce the time of completing the job to OB from normal time OA. Therefore under such a situation the cost of the project increases and it goes up to say OD (Crash Cost). This only amounts to saving that by reducing the time period by BA the cost has increased by the amount CD. The rate of increase in the cost of activity per unit with a decrease in time is known as cost slope and is described as below.

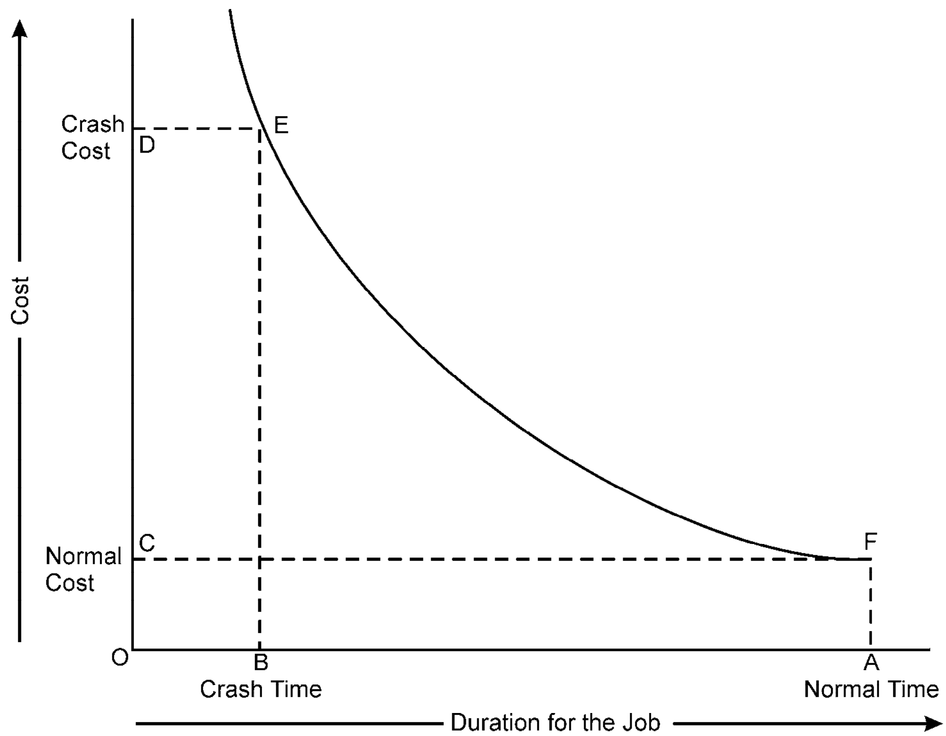


Figure 7

$$\text{Activity Cost Slope} = \frac{CD}{AB} = \frac{OD-OC}{OA-OB}$$

$$= \frac{\text{Crash Cost} - \text{Normal Cost}}{\text{Normal Time} - \text{Crash Time}}$$

Optimum duration: The total project cost is the sum of the direct and the indirect costs. In case the direct cost varies with the project duration time, the total project cost would have the shape indicated in the following figure:

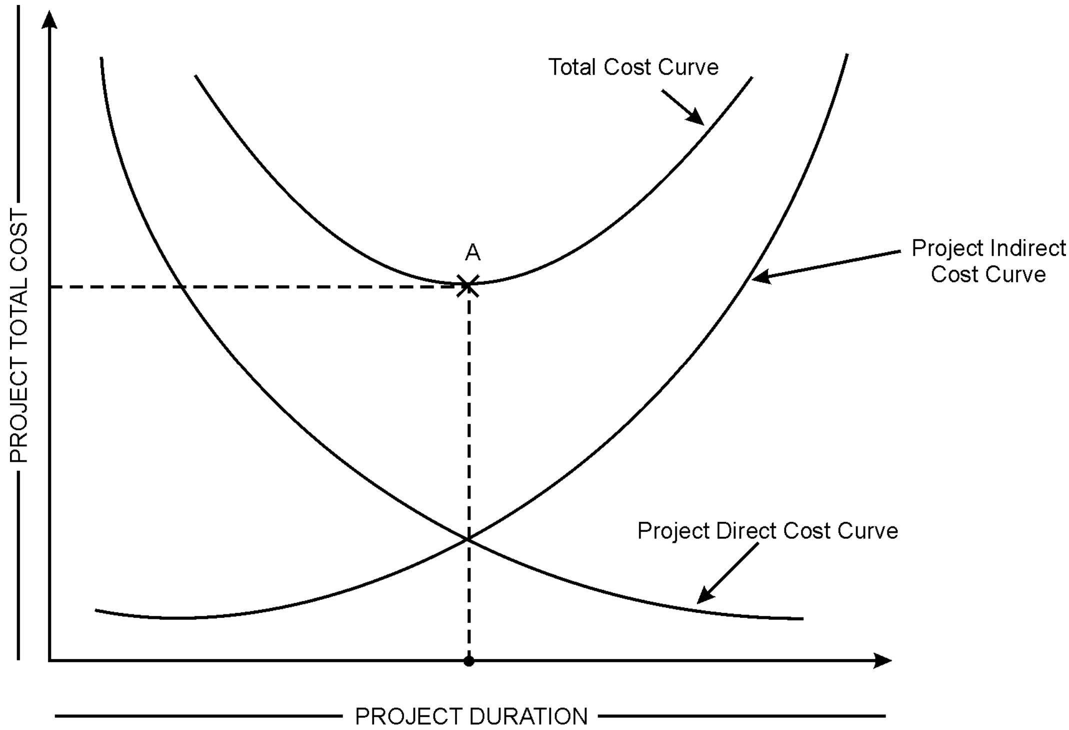


Figure 8

At the point A, the cost will be minimum. The time corresponding, to point A is called the optimum duration and the cost as optimum cost for the project.

The example below which the student should go through carefully is intended to explain cost implications of hastening a project.

Illustration 7

The following data pertains to the network drawn in figure 9 (A) given on the next page. It is desired to compress the project to the least possible duration day by day and estimate the extra cost.

i–j	T_n	T_e	*Cost slope*
1–2	*3*	*2*	*700*
1–3	*7*	*4*	*200*
2–3	*5*	*3*	*100*
2–4	*8*	*6*	*200*
3–4	*4*	*2*	*400*

Solution

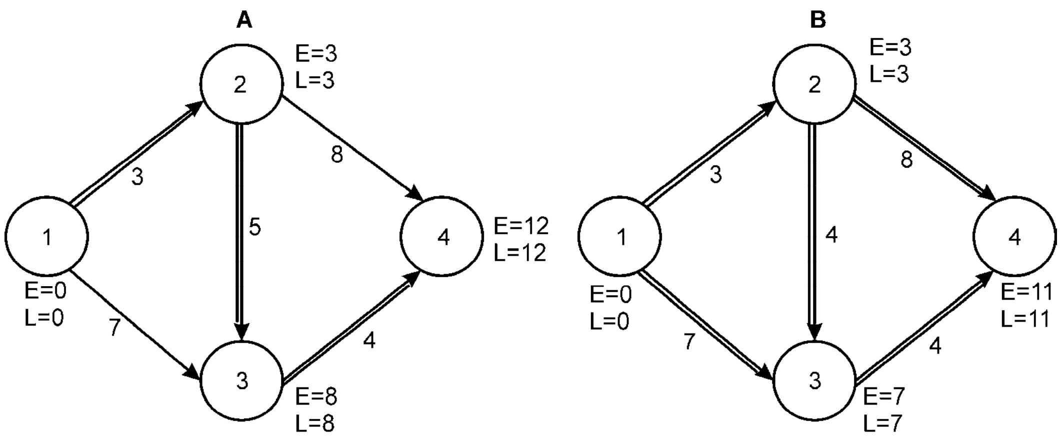
The critical path is 1-2-3-4 in figure 9 (A). It can be, seen that the critical path 1-2-3-4 is longer than either of the paths; 1-3-4 and 1-2-4 by one day. Therefore, the project can be compressed by one day along the critical path; 2-3 having the least cost slope is therefore crashed by a day.

The revised network is depicted in figure 9 (B) where all activities have become critical. The following choices of compression exist now. Each set of activities is so chosen that it *reduces all the paths by a day*.

Crash each activity in one of the following sets by a day.

1–2	2–4	1–3
1–3	3–4	2–3
		2–4
Cost = 900	Cost = 600	Cost = 500

The last set of crashing 1-3, 2-3 and 2-4 is the least expensive and these activities are crashed accordingly.



DURATION = 12
REDUCE 2-3 BY 1 DAY

DURATION = 11
REDUCE 2-3, 1-3 & 2-4 BY
1 DAY EACH

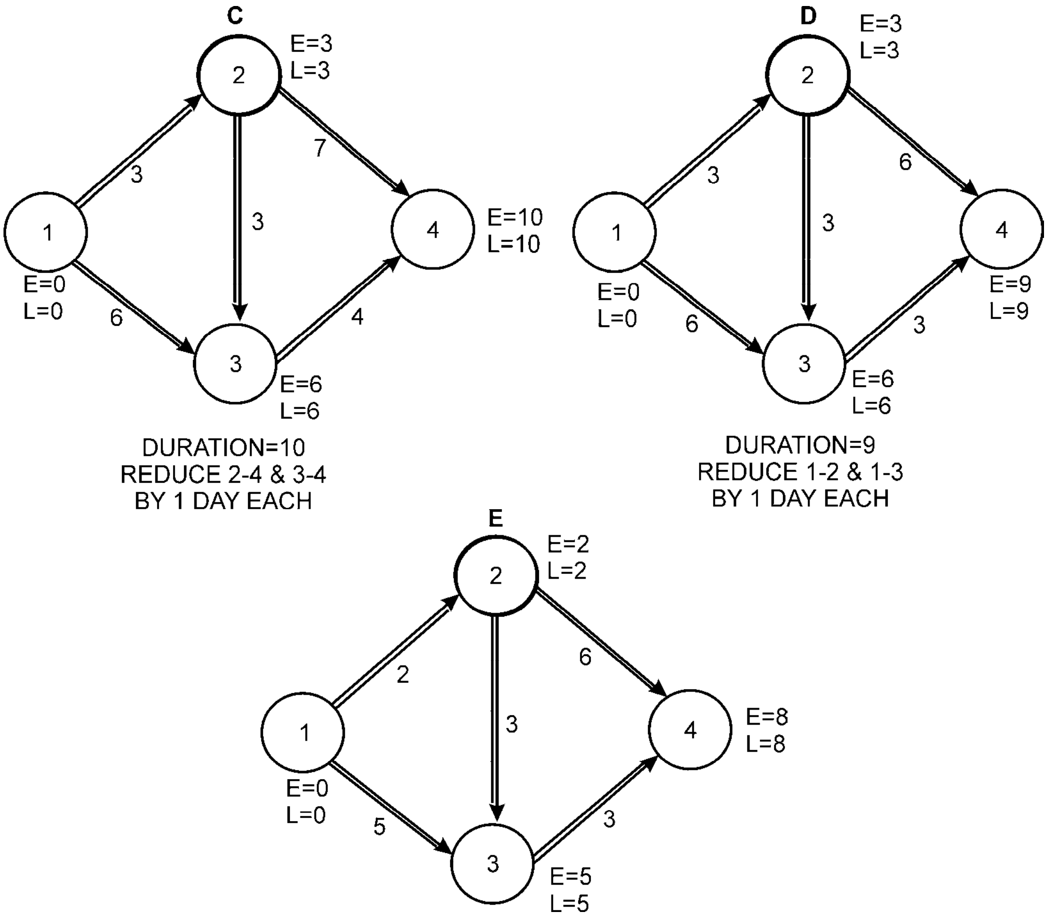


Figure 9 [(A) to (E)]

Extra cost	= 200
	100
	200
	₹ 500

This is shown in figure 9 (C). Activity 2-3 having been crashed to the limit is dropped out from further consideration. In 9 (C). the following choices of crashing each activity a day exist:

	1–3	2–4
	1–2	3–4
Extra cost	₹ 900	₹ 600

Extra cost the last is selected (Extra cost = ₹ 600)

The revised network is shown in figure 9 (D) where 2-4 joins 2-3 in that it is also crashed to limit. The only possibility of compressing the network in figure 9 (D) is to crash 1-2 and 1-3 by a day each. This is done and the final network is shown in figure 9 (E).

Extra cost = ₹ 900

Although 1-3 and 3-4 have not reached their crashing limits in figure 9 (E) there is no use also to crash these since this would not compress the project which can be compressed to 8 days only.

Total extra cost = ₹ 100 + ₹ 500 + ₹ 600 + ₹ 900 =₹ 2,100.

However, If just the least duration plan was required one could go about the problem in a much simpler way as follows. Draw the network with t_e's. This is done in the figure 9 (F).

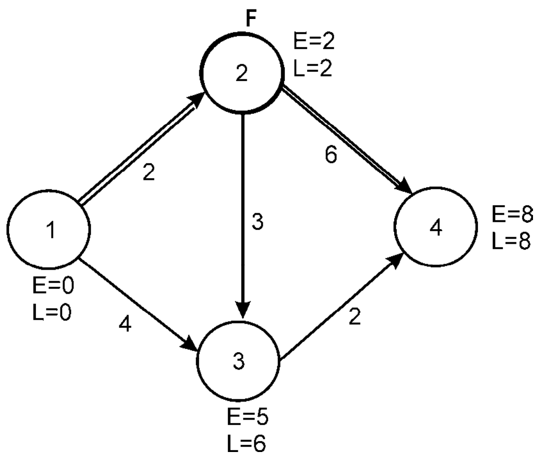


Figure 9(F)

The network is analysed with 1-2-4 as the critical path of 8 day's duration. The other paths have to be contracted to 8 day's duration. This can be done in one of the following ways:

(i) Increase 1-3 and 3-4 by a day each with cost reduction of ₹ 200 + ₹ 400 = ₹ 600.

(ii) Increase 2-3 and 1-3 by a day and 2 days respectively with cost reduction of ₹ 100 + ₹ 400 = ₹ 500.

Obviously the 1st course is to be preferred and the network. if now revised. would be identical with the one of figure 9 (E).

However, management would be interested in the least total cost duration rather than the least possible duration.

Now, suppose there is an indirect cost of ₹ 800 per day. What would then be the least cost project duration for the example on hand ? The various cost data are tabulated below:

Duration	Direct (Crashing) cost	Indirect cost	Total cost
12	0	9,600	9,600
11	100	8,800	8,900
10	600	8,000	8,600
9	1,200	7,200	8,400
8	2,100	6,400	8,500

Thus 9 days is the least total cost duration and rationally the management should go in for this unless high opportunity losses compel them to select a lower duration project plan.

Illustration 8

Consider the following network and the table for a particular project which consists of 7 activities.

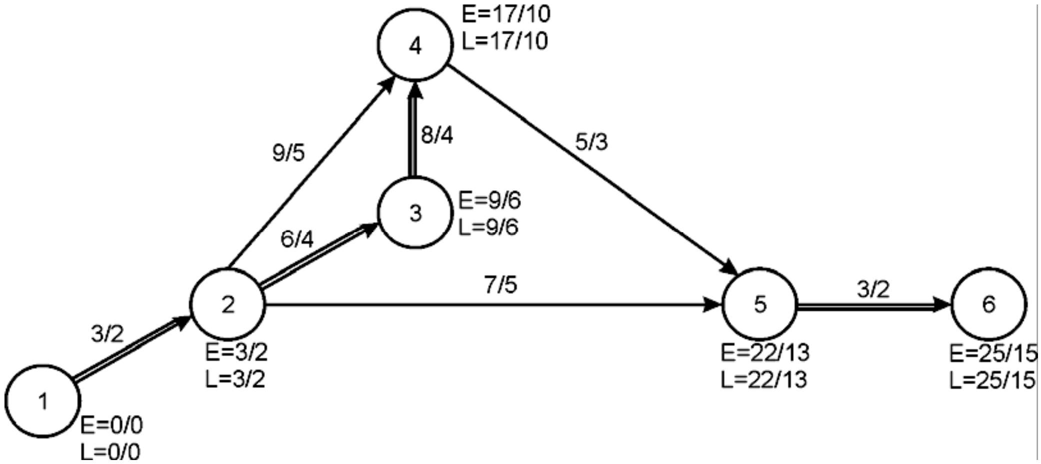


Figure 10

(The number indicated along the activity arrows are the normal duration.)

Activity	*Normal*		*Crash*		*Cost slopes*		
	Times (days)	*Cost ₹*	*Time (days)*	*Cost ₹*	*ΔT*	*ΔC*	*ΔC/ΔT*
1–2	*3*	*360*	*2*	*400*	*1*	*40*	*40*
2–3	*6*	*1,440*	*4*	*1,620*	*2*	*180*	*90*
2–4	*9*	*2,160*	*5*	*2,380*	*4*	*220*	*55*
2–5	*7*	*1,120*	*5*	*1,600*	*2*	*480*	*240*
3–4	*8*	*400*	*4*	*800*	*4*	*400*	*100*
4–5	*5*	*1,600*	*3*	*1,770*	*2*	*170*	*85*
5–6	*3*	*480*	*2*	*760*	*1*	*280*	*280*
		7,560		*9,330*			

The indirect cost is ₹ 160/- per day. Determine optimum duration in the above case.

Solution

The normal duration of the project is obtained from the critical path and not by merely summing up the normal duration of all the activities. The overall normal duration in the above project is 25 days and the total direct cost is ₹ 7,560/-. If due to some reason it is felt that the project is to be hastened, the question then to be answered is: "what is the minimum time required for the completion of the project?"

According to the crash time given in the table; the critical path based on these crash time estimates still appears as 1-2-3-4-5-6 and the crash duration is 15 days. Hence, the project may take 15 to 25 days depending upon the money the management is prepared to spend. If the management decides to complete the project in 15 days, then the direct cost will be ₹ 9330/-. The figure has been arrived by speeding up or crashing all the activities as mentioned in the, given table. But there are several non-critical activities in the project, which need not be crashed to reduce the project duration. Speeding up of such non critical activities involves extra amount. As any extra amount spent on these activities is not going to reduce the project duration, therefore it is better not to reduce the duration of these non-critical activities. Duration of these non-critical activities should be reduced only when they become critical activities, during the process of reducing the project duration time.

The two objectives behind the reduction of project time are:

(1) To complete the project before a certain target date.

(2) To reduce the overall cost of the project. This objective can only be achieved when the indirect cost per day is greater than some of the cost slopes as given in the table. In the exercise under consideration, the five activities 1-2, 2-3, 2-4, 3-4 and 4-5 have cost slopes lower than the indirect cost. In such a situation, the management would be very much interested in cutting down the project time, thereby decreasing the total indirect cost. In order to solve this example we proceed as follows:

First step is to identify those activities along the critical path whose cost slopes are less than the indirect cost. In our network such activities are 1-2,2-3,3-4 and 4-5. We take these activities in the order of increasing cost slopes. Activity 1-2 has a slope of ₹ 40/per day. This activity can be cut down by 1 day at a cost of ₹ 40/-. The next activity in the order of cheapness is 4-5. This can be cut down by 2 days at a cost of ₹ 170. Next we may take the activity 2-3 whose cost slope is ₹ 90/-. This activity can be contracted by 2 days at a cost of ₹ 180. So far three activities viz. 1-2,4-5 and 2-3 have been contracted by a total of 5 days at an overall cost of ₹ 390/-.

From the given table, we notice that the next activity in the order of priority is 3-4 with a cost slope ₹ 100/- per day. This activity can be cut down by 4 days. But a look on the network after the performance of above reduction in time reveals that activity 2-4 has float of only 3 days.

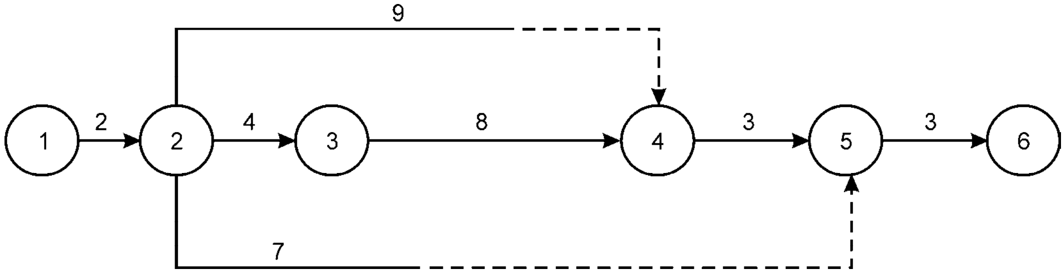


Figure 11

Hence we cannot cut down the duration for the activity 3-4 by four days without affecting the activity 2-4. Therefore one should cut down the duration on activity 3-4 by 3 days at a cost of

₹ 300/- and now by this process a new sub-critical path viz. 1-2-4-5-6 may come into existence.

According to the given Table, activity 3-4 can be further contracted by 1 day. But this cannot be achieved without contracting activity 2-4. Therefore, the cost slope must include both of these activities i.e. activities 2-4 and 3-4. The combined cost slope of these : two activities is ₹ 55 + ₹ 100 = ₹ 155. The project duration has now been reduced to 16 days, as below:

Activity	Duation reduced by (Days)	Cost slope (₹)	Cost of contraction (₹)
1–2	1	40/-	40/-
4–5	2	85/-	170/-
2–3	2	90/-	180/-
3–4	3	100/-	300/-
3–4,2–4	1	155/-	155/-
Total	9		845/-

During the above process a total reduction of 9 days has been achieved at an extra cost of ₹ 845/-. Also the indirect cost @ ₹ 160/- per day for 9 days resulting in ₹ 1,440/- has been saved.

In other words, the project duration has been reduced from 25 days to 16 days. As a result of this duration of project its direct cost has now become ₹ 8,405/- and ₹ 2,560/- as its indirect cost. The overall project cost with the new time schedule i.e. 16 days is ₹ 8,405 + ₹ 2,560 = ₹ 10,965/- as against the overall cost of ₹ 11,560, it thus results in a net saving of ₹ 595/-.

Illustration 9

The Arcot Machinery Company has been offered a contract to build and deliver nine extruding presses to the Home Botting Company. The contract price negotiated is contingent upon meeting a specified delivery time, with a bonus offered for early delivery. The marketing department has established the following cost and time information:

Activity	*Normal time (weeks)*			*Normal Cost*	*Crash Time (Weeks)*	*Crash Cost*
	a	*b*	*m*	*(₹)*		*(₹)*
1–2	*1*	*5*	*3*	*5,000*	*1*	*9,000*
2–3	*1*	*7*	*4*	*8,000*	*3*	*14,000*
2–4	*1*	*5*	*3*	*4,000*	*2*	*6,000*
2–5	*5*	*11*	*8*	*5,000*	*7*	*6,000*
3–6	*2*	*6*	*4*	*3,000*	*2*	*5,000*
4–6	*5*	*7*	*6*	*2,000*	*4*	*3,600*
5–7	*4*	*6*	*5*	*10,000*	*4*	*14,000*
6–7	*1*	*5*	*3*	*7,000*	*1*	*10,600*

Normal delivery time is 16 weeks for a contract price of ₹ 62,000.

On the basis of the calculated profitability for each delivery time specified in the following table, what delivery schedule do you recommend that the company may implement?

Contract Delivery Time (weeks)	*Contract Amount (₹)*
15	*62,500*
14	*65,000*
13	*70,000*
12	*72,500*

(Here a = t_o : optimistic time, b=t_p; pessimistic time, m=t_m; most likely time.)

Solution

Let us first calculate the expect duration of each activity.

Activity	Normal Time (weeks)			Expected duration (weeks)	Normal Cost (₹)	Crash Time (weeks)	Crash Cost (₹)	Cost Slope (₹)
	a	b	m					
1-2	1	5	3	3	5,000	1	9,000	2,000
2-3	1	7	4	4	8,000	3	14,000	6,000
2-4	1	5	3	3	4,000	2	6,000	2,000
2-5	5	11	8	8	5,000	7	6,000	1,000
3-6	2	6	4	4	3,000	2	5,000	1,000
4-6	5	7	6	6	2,000	4	3,600	800
5-7	4	6	5	5	10,000	4	14,000	4,000
6-7	1	5	3	3	7,000	1	10,600	1,800
					44,000			

The network for the given problem is drawn below:

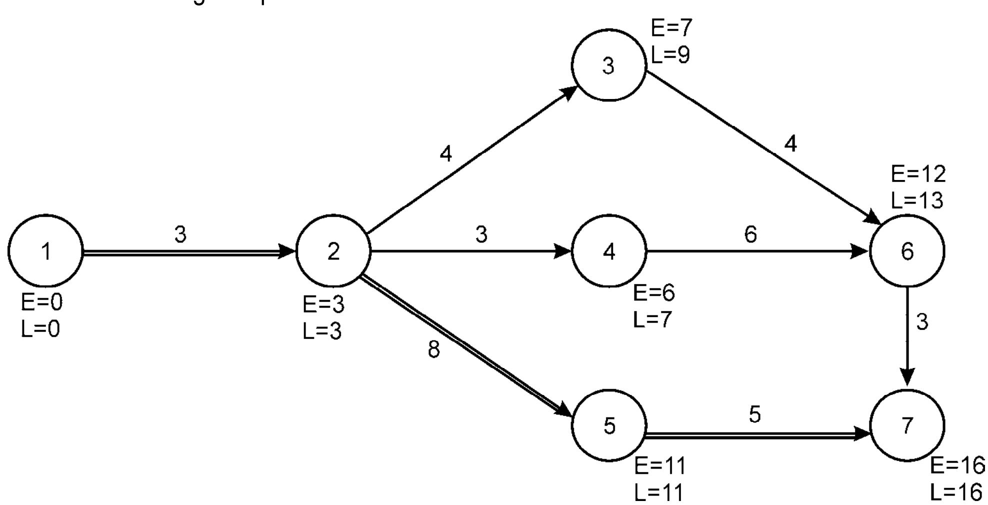


Figure 12

The critical path is 1-2-5-7 with total duration 16 weeks. Cost of all activities is ₹44,000. Contract price is ₹ 62,000 for normal delivery time of 16 weeks. Hence the profit is of ₹ 18,000.

For calculating the most profitable delivery schedule, let us start crashing the activities on the critical path. Cost slopes for various activities are given in the above table.

Step 1: The critical activity 2-5 has the least cost slope of ₹ 1,000, so we crash this activity by 1 day. Crashing cost is ₹ 1,000. The contract amount for delivery time of 15 weeks is ₹ 62,500

Profit = ₹ 62,500 - (₹ 44,000 + ₹ 1,000)

= ₹ 62,500 -₹ 45,000 = ₹ 17,500

Step 2: Now there are two critical paths viz. 1-2-4-6-7 and 1-2-5-7 with duration of 15 weeks. So we crash activity 1-2 by 1 day at crash cost of ₹ 2,000. Project duration is 14 weeks.

Profit = ₹ 65,000 - ₹ (45,000 + 2,000) = ₹ 18,000.

Step 3: We again crash activity 1-2 by 1 day. So the project duration is 13 weeks

Profit = ₹ 70,000 - (₹ 47,000 + ₹ 2,000)

= ₹ 70,000 - ₹ 49,000 = ₹ 21,000

Step 4: Now we crash activity 4-6 for path 1-2-4-6-7 at crash cost of ₹ 800 and activity 5-7 for path 1-2-5-7 at crash cost of ₹ 4000. Project duration is now 12 weeks.

Profit = ₹ 72,500 - (₹ 49,000 + ₹ 4000 + ₹ 800)

= ₹ 72,500 - ₹ 53,800 = ₹ 18,700

No further crashing is possible.

From step 3, it can be seen that the profit is maximum when the project duration is 13 weeks. Hence, the company should implement the delivery schedule of 13 weeks at a contract amount of ₹ 70,000 to gain maximum profit of ₹ 21,000

14.4 Resource Smoothing

It is a network technique used for smoothening peak resource requirement during different periods of the project network. Under this technique the total project duration is maintained at the minimum level. For example, if the duration of a project is 15 days, then the project duration is maintained, but the resources required for completing different activities of a project are smoothened by utilising floats available on non critical activities. These non critical activities having floats are rescheduled or shifted so that a uniform demand on resources is achieved. In other words, the constraint in the case of resource smoothing operation would be on the project duration time. Resource smoothing is a useful technique or business managers to estimate the total resource requirements for various project activities.

In resources smoothing, the time-scaled diagram of various activities and their floats (if any), along with resource requirements are used. The periods of maximum demand for resources are identified and non critical activities during these periods are staggered by rescheduling them according to their floats for balancing the resource requirements.

Before explaining the process, let us first illustrate the concept of ' Time Scaled Diagram' which we will be using in the subsequent problems:

Time Scaled Diagrams: In the network diagrams which we have considered, it has been stressed that the length of the individual arrows has no relation to the duration of the activity which each arrow represented. It is of course possible to draw the arrows to a time scale, and this can be a very useful method of presentation for small networks.

The figure 11 shows the network diagram drawn to a horizontal time scale. The critical path has been arranged as a straight line with non-critical events above or below it. Solid lines represent activities, dotted horizontal lines represent float, and dotted vertical lines represent dummies. The presentation has obvious advantages. The events are entered in the time scale version at the earliest start time. (The latest start times can also be used as an alternative).

Illustration 10

Consider a project consisting of thirteen activities having duration and resource requirements shown below:

Activity	*Duration (days)*	*Labourers required*
A	2	2L
B	4	2C,2L
C	4	4C
D	3	2L
E	4	6C
F	7	2L
G	2	4C
H	5	4C,2L
I	2	2C
J	5	2C
K	2	2L
L	3	4L
M	2	4L

Here L stands for labourers and C stands for carpenters, we shall analyse this project from the point of view of resources to bring out the necessary steps involved in the analysis and smoothing of resources.

From the values given in table above E and L are calculated and given in the network (Fig. 13)

The critical path for this network is 1–2–4–6–8–9–10–11, in figure 14 is shown the time-scaled version of the same network. The critical path is shown along the horizontal line. The last two rows (also referred to as resource accumulation table) in figure 14 give the number of labourers and carpenters required each day. It can be seen that the demand on the resources is not even. On the 7th and 8th days the demand for carpenters is as high as 14, whereas on the 11th, 12th and 13th days it is two only. If the carpenters and labourers are to be hired for the entire project duration of 22 days, then during most of the days they will be idle and the company will have to hire at least 14 carpenters and 4 labourers.

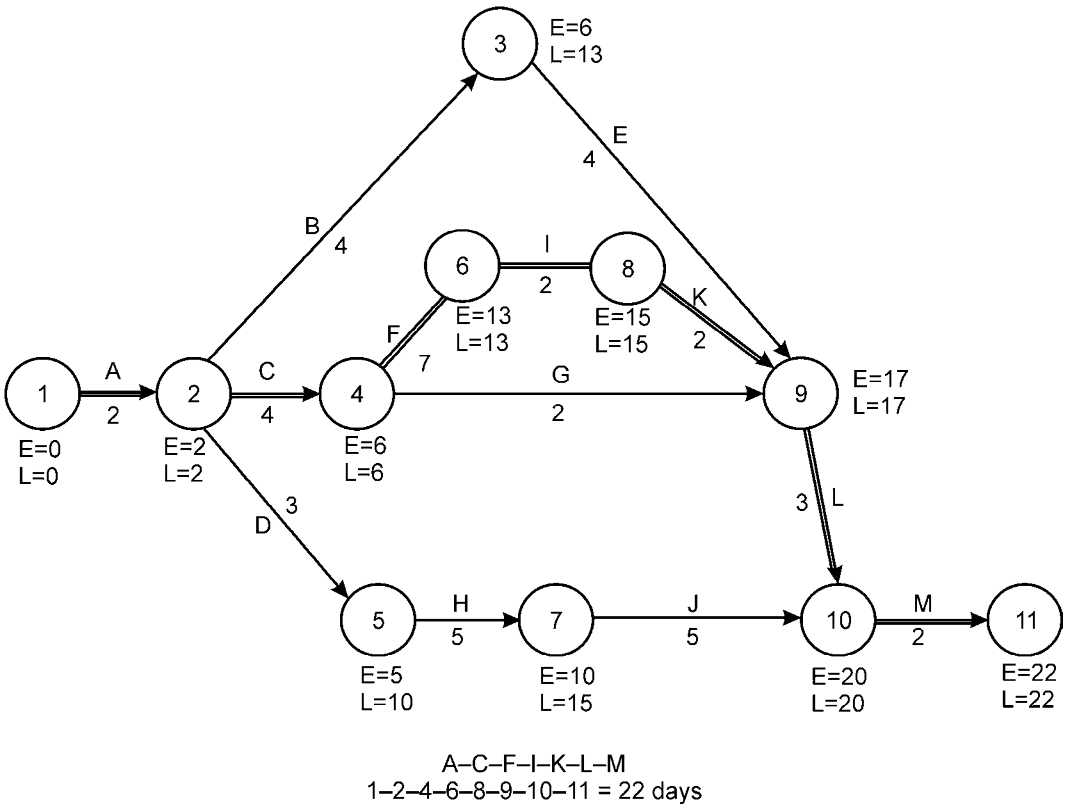


Figure 13

Activity	Duration	EST	EFT =EST +D	LST= LFT–D	LFT	Slack of Tail Event	Slack of Head Event	Total Float =LST –EST	Free Float = Total Float - Slack of Head Event	Independent Float=Free Float–Slack of Tail Event
A(1–2)	2	0	2	0	2	0	0	0	0	0
B(2–3)	4	2	6	9	13	0	7	7	0	0
C(2–4)	4	2	6	2	6	0	0	0	0	0
D(2–5)	3	2	5	7	10	0	5	5	0	0
E(3–9)	4	6	10	13	17	7	0	7	7	0
F(4–6)	7	6	13	6	13	0	0	0	0	0
G(4–9)	2	6	8	15	17	0	0	9	9	9
H(5–7)	5	5	10	10	15	5	5	5	0	0*
I(6–8)	2	13	15	13	15	0	0	0	0	0
J(7–10)	5	10	15	15	20	5	0	5	5	0
K(8–9)	2	15	17	15	17	0	0	0	0	0
L(9–10)	3	17	20	17	20	0	0	0	0	0
M(10–11)	2	20	22	20	22	0	0	0	0	0

* If negative value is obtained, the independent float is taken to be zero.

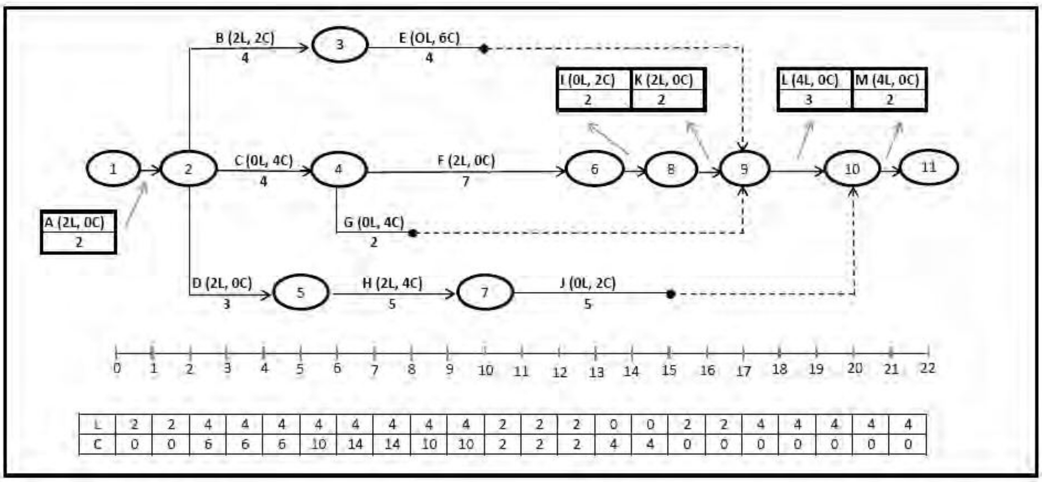
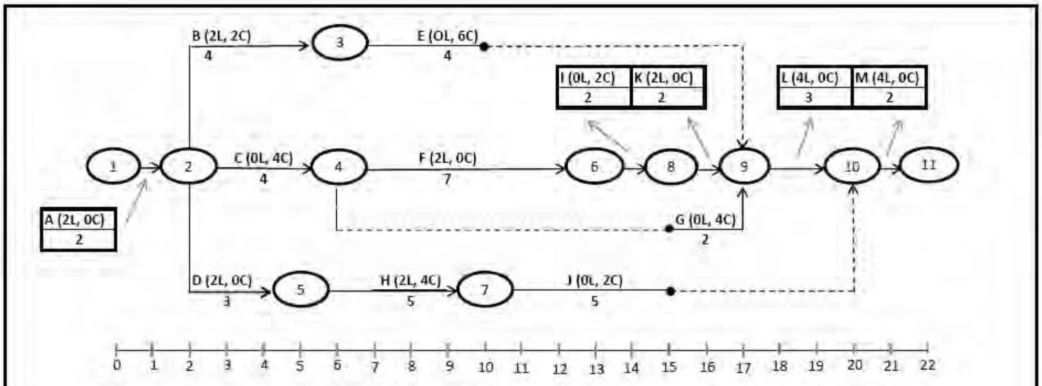


Figure 14

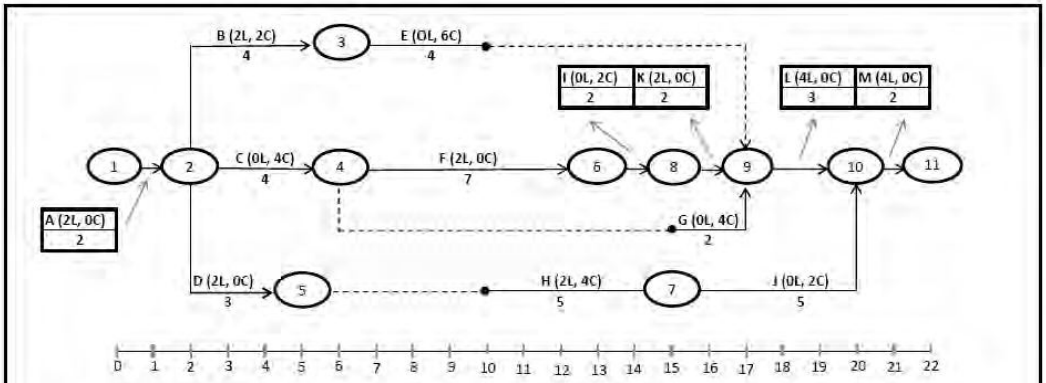
We will attempt to re-schedule our activities in-such a way as to utilize the resources in a fairly uniform manner. As mentioned above the maximum demand on the resources occurs on the 6th, 7th, 8th, 9th and 10th days. The activities on these days will have to be shifted depending upon their floats such that the demand comes down. As can be seen from the above time-scaled version, activity 4-9 has maximum float, therefore we will try to shift activity 4-9 so that is starts on the 16th day instead of the 7th day. This reduces the demand on the carpenters from 14 to 10 on the 7th and 8th days so that the maximum demand for the carpenters on any day is now 10 and not 14. The modified resource accumulation table and the time-scaled version of the project is shown in figure 15.



L	2	2	4	4	4	4	4	4	4	4	2	2	2	0	0	2	2	4	4	4	4	4
C	0	0	6	6	6	10	10	10	10	10	2	2	2	4	4	4	4	0	0	0	0	0

Figure 15

It is evident from the above figure that the maximum demand now is for 10 carpenters on the 6th, 7th, 8th, 9th and 10th days. We will now try to explore that possibility of further smoothing the resources which is possible because activities 2–5, 5–7 and 7–10 have a total float of 5 days. The resultant time-scaled network and the resource accumulation table are given below in fig. 16.



L	2	2	4	4	4	2	2	2	2	2	4	4	4	2	2	2	2	4	4	4	4	4
C	0	0	6	6	6	6	6	6	6	6	4	4	4	6	6	6	6	2	2	2	0	0

Figure 16

As can be seen from the above figure the requirement for labourers is 4 and the requirement for carpenters reduces to 6 as against 14 carpenters originally estimated. Hence, by judiciously utilizing the float, we can smooth the demand on the resources.

14.5 Resource Levelling

It is also a network technique which is used for reducing the requirement of a particular resource due to its paucity. The process of resource levelling utilizes the large floats available on non-critical activities of the project and thus cuts down the demand on the resource. In resource levelling, the maximum demand of a resource should not exceed the available limit at any point of time. In order to achieve this, non critical activities are rescheduled by utilising their floats. *Sometimes, the use of resource levelling may lead to enlonging the completion time of the project*. In other words, in resource levelling, constraint is on the limit of the resource availability.

14.6 Miscellaneous Illustrations

Illustration 11

A project consists of eight activities with the following relevant information:

Activity	*Immediate Predecessor*	*Estimated Duration (Days)*		
		Optimistic	*Most Likely*	*Pessimistic*
A	–	*1*	*1*	*7*
B	–	*1*	*4*	*7*
C	–	*2*	*2*	*8*
D	*A*	*1*	*1*	*1*
E	*B*	*2*	*5*	*14*
F	*C*	*2*	*5*	*8*
G	*D,E*	*3*	*6*	*15*
H	*F,G*	*1*	*2*	*3*

(i) *Draw the PERT network and find out the expected project completion time.*

(ii) *What duration will have 95% confidence for project completion?*

(iii) *If the average duration for activity F increases to 14 days, what will be its effect on the expected project completion time which will have 95% confidence?*

Solution

(i) The required network is drawn below:

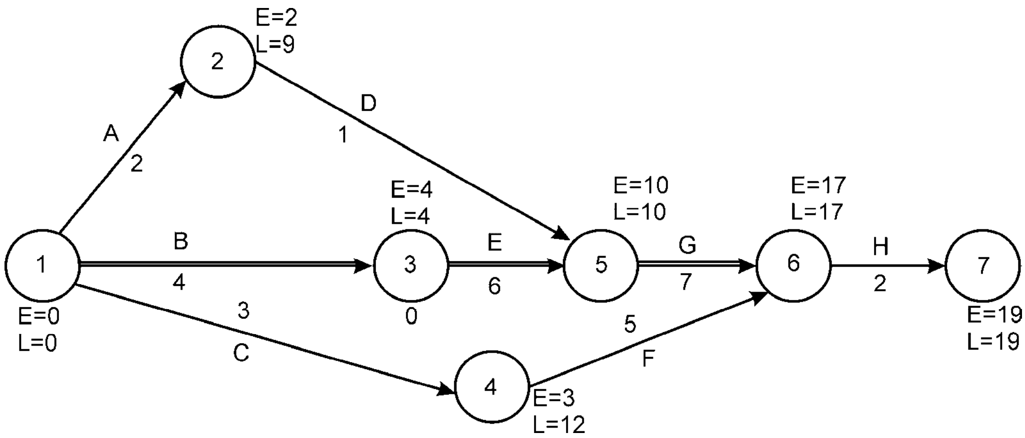


Figure 17

The expected time for each activity shown in the network above is calculated in the following table:

Activity		Estimated Duration (Days)			Expected Duration $\left(\frac{a+4m+b}{6}\right)$	Variance $\left(\frac{b-a}{6}\right)^2$
		Optimistic a	Most likely m	Pessimistic b		
A	1–2	1	1	7	2	1
B	1–3	1	4	7	4	1
C	1–4	2	2	8	3	1
D	2–5	1	1	1	1	0
E	3–5	2	5	14	6	4
F	4–6	2	5	8	5	1
G	5–6	3	6	15	7	4
H	6–7	1	2	3	2	1/9

The critical path is given by 1-3-5-6-7 or B-E-G-H and the expected project completion time is 19 days.

(ii) The variance for critical path is 1 + 4 + 4 + 1/9 = 82/9

Standard deviation of critical path = $\sigma_1 = \sqrt{\frac{82}{9}}$ = 3.02 (approx.)

To calculate the project duration which will have 95% chances of its completion, we utilise the value of Z corresponding to 95% confidence which is 1.645.

Thus, $\frac{X-19}{3.02} = 1.645$,

or X = 1.645 × 3.02 + 19 = 23.97 days = 24 days

Hence, 24 days of project completion time will have 95% probability of its completion.

(iii) If the average duration for activity F increases to 14 days, then the path 1-4-6-7 i.e., C-F-H will also become critical path with expected project completion time of 19 days. Now, activities C and F are also critical activities.

Variance of Critical path C – F – H = 1 + 1 + 1/9 = 19/9

Variance of Critical path B – E – G – H = 1 + 4 + 4 + 1/9 = 82/9

$\sigma = \sqrt{\frac{82^*}{9}}$ = 3.02 (appx)

* If there are two equal longest path, Higher of the two would be picked up.

We now wish to calculate the expected project completion time that will have 95% confidence level,

P (Z < 1.645) = 0.95

$$\frac{X-\mu}{\sigma} = \frac{X-19}{3.02} = 1.645$$

or X = 23.97 days.

Hence the project duration of 23.97 days will have 95% confidence of completion.

Illustration 12

The following information is given below:

Activity	***(1–2)***	***(2–3)***	***(2–4)***	***(3–5)***	***(4–6)***	***(5–6)***	***(5–7)***	***(6–7)***
Pessimistic time (in weeks)	*3*	*9*	*6*	*8*	*8*	*0*	*5*	*8*
Most likely time (in weeks)	*3*	*6*	*4*	*6*	*6*	*0*	*4*	*5*
Optmistic time (in weeks)	*3*	*3*	*2*	*4*	*4*	*0*	*3*	*2*

Draw the network diagram for the above.

Calculate:

(i) Variance to each activity.

(ii) Critical path and expected project length.

(iii) The probability that the project will be completed in 23 weeks is drawn below:

If there are two equal longest path higher of the two variance would be picked up.

Solution

The required network is drawn below:

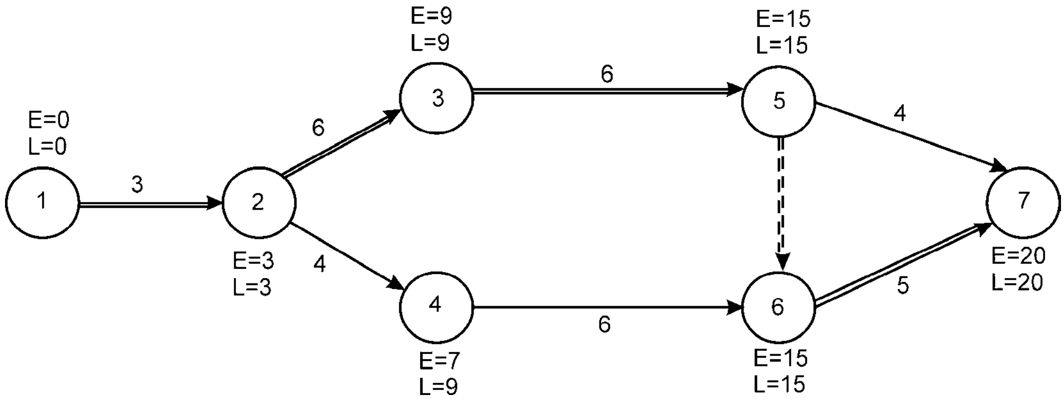


Figure 18

The expected time marked in the above network diagram for various activities is calculated in the table below:

Activities	Times (in weeks)			Expected time (weeks) $t_e = \frac{t_o + 4t_m + t_p}{6}$	Variance $\left(\frac{t_p - t_o}{6}\right)^2$
	Optimistic (t_o)	Most like (t_m)	Pessimistic (t_p)		
1–2	3	3	3	3	0
2–3	3	6	9	6	1
2–4	2	4	6	4	4/9
3–5	4	6	8	6	4/9
4–6	4	6	8	6	4/9
5–6	0	0	0	0	0
5–7	3	4	5	4	1/9
6–7	2	5	8	5	1

(i) Variance of each activity has been calculated in the last column of the above table.

(ii) Critical path is given by 1–2–3–5–6–7 and the expected project length is 20 weeks.

(iii) Variance of the critical path = σ^2 = 0 + 1 + 4/9 + 0 + 1 = 22/9 = 2.444

Mean = x = 20 weeks

To calculate the probability of completing the project in 23 weeks, we will first calculate the normal Z as below:

$$Z = \frac{D-x}{\sigma} = \frac{23-20}{\sqrt{2.444}} = 1.92.$$

P (x $\leq$ 23) = P (z $\leq$ 1.92) = 0.9726 (from the Normal table)

Thus, the probability that the project will be completed in 23 weeks is 97.26%.

Illustration 13

A small maintenance project consists of the following twelve jobs whose precedence relations are identified with their node number:

Job (i–j)	*(1,2)*	*(1,3)*	*(1,4)*	*(2,3)*	*(2,5)*	*(2,6)*
Duration (in days)	*10*	*4*	*6*	*5*	*12*	*9*
Job (i–j)	***(3,7)***	***(4,5)***	***(5,6)***	***(6,7)***	***(6,8)***	***(7,8)***
Duration (in days)	*12*	*15*	*6*	*5*	*4*	*7*

(i) *Draw an arrow diagram representing the project.*

(ii) *Calculate earliest start, earliest finish, start and latest finish time for all jobs.*

(iii) *Find the critical path and project duration.*

(iv) *Tabulate total float, free float and independent float.*

Solution

(i) The required arrow diagram is drawn in below:

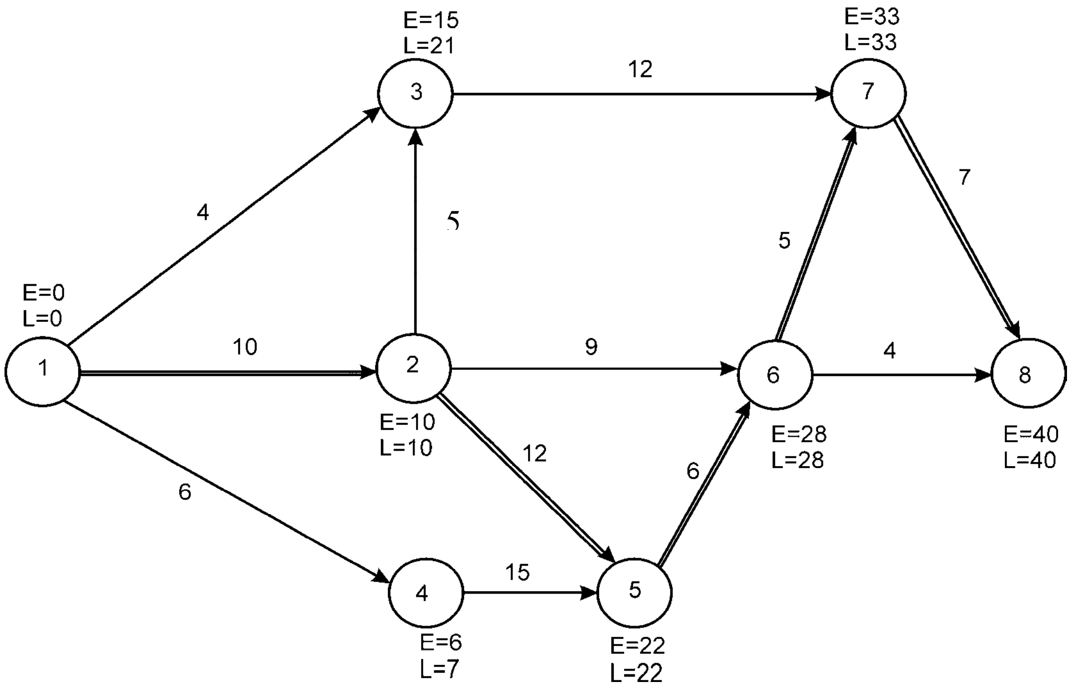


Figure 19

(ii) The earliest start, earliest finish, latest start and latest finish time for all the jobs are calculated in the table given below:

Job (i–j)	*Duration in Days*	*Earliest time*		*Latest time*		*Slack of*		*Total Float =(LST-EST)*	*Free Float*	*Indep. Float*
		Start EST	*Finish EFT*	*Start LST*	*Finish LFT*	*Tail Event*	*Head Event*			
1–2	10	0	10	0	10	0	0	0	0	0
1–3	4	0	4	17	21	0	6	17	11	11
1–4	6	0	6	1	7	0	1	1	0	0
2–3	5	10	15	16	21	0	6	6	0	0
2–5	12	10	22	10	22	0	0	0	0	0
2–6	9	10	19	19	28	0	0	9	9	9
3–7	12	15	27	21	33	6	0	6	6	0
4–5	15	6	21	7	22	1	0	1	1	0
5–6	6	22	28	22	28	0	0	0	0	0
6–7	5	28	33	28	33	0	0	0	0	0
6–8	4	28	32	36	40	0	0	8	8	8
7–8	7	33	40	33	40	0	0	0	0	0

(iii) The critical path is 1-2-5-6-7-8 and the project duration is 40 days.

(iv) Total float, free float and independent float for various activities are calculated in the above table.

Illustration 14

A small project consists of seven activities for which the relevant data are given below:

Activity	*Preceding Activities*	*Activities Duration (Days)*
A	–	*4*
B	–	*7*
C	–	*6*
D	*A,B*	*5*
E	*A,B*	*7*
F	*C,D,E*	*6*
G	*C,D,E*	*5*

(i) Draw the network and find the project completion time.

(ii) Calculate total float for each of the activities.

(iii) Draw the time scaled diagram.

Solution

(i) The required network is given in below:

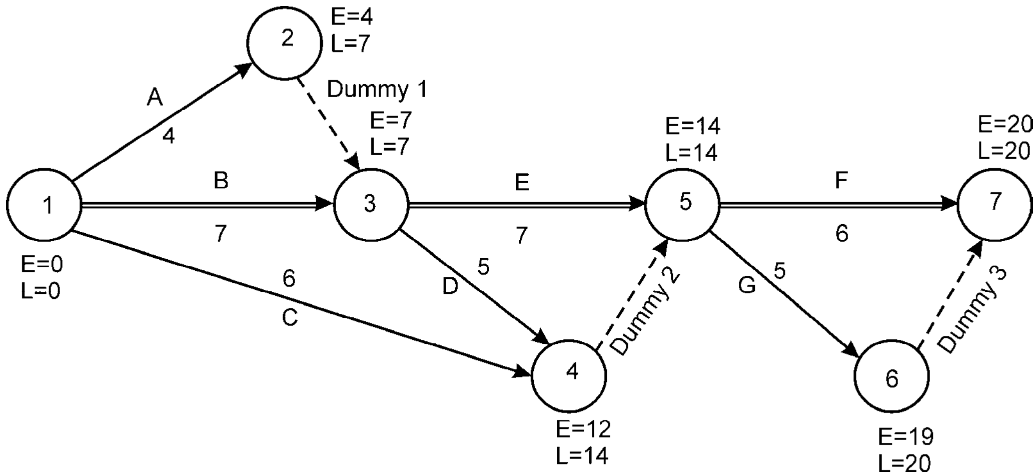


Figure 20

The critical path is 1–3–5–7 with duration of 20 days. Thus, the project completion time is 20 days.

(ii) The total float for various activities is calculated as below:

Activity		Duration	EST	LST	EFT	LFT	LST–EST
A	1–2	4	0	3	4	7	3
B	1–3	7	0	0	7	7	0
C	1–4	6	0	8	6	14	8
Dummy 1	2–3	0	4	7	4	7	3
D	3–4	5	7	9	12	14	2
E	3–5	7	7	7	14	14	0
Dummy 2	4–5	0	12	14	12	14	2
F	5–7	6	14	14	20	20	0
G	5–6	5	14	15	19	20	1
Dummy 3	6–7	0	19	20	19	20	1

(iii) The required time diagram is drawn below:

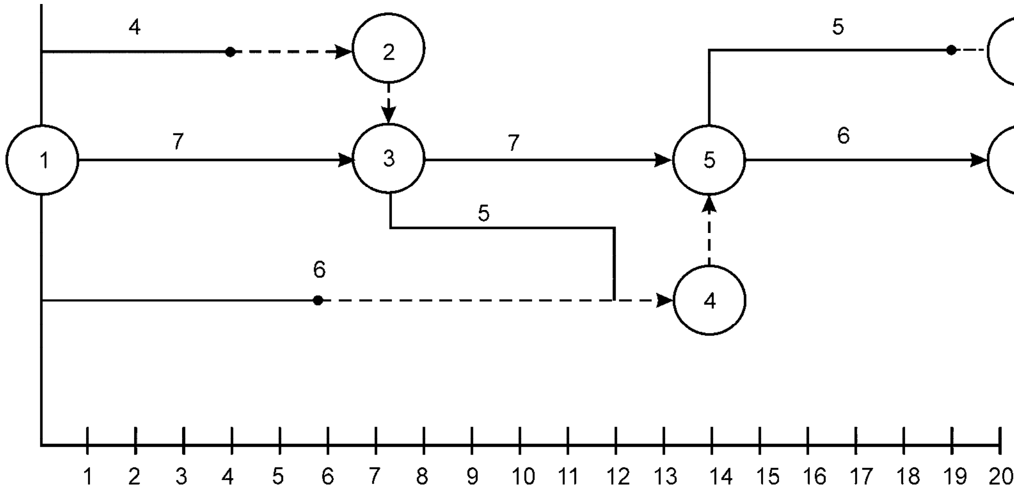


Figure 21

Illustration 15

A small project is having seven activities. The relevant data about these activties is given below:

Activity	*Dependence*	*Normal duration (Days)*	*Crash duration (Days)*	*Normal cost (₹)*	*Crash cost (₹)*
A	–	7	5	500	900
B	A	4	2	400	600
C	A	5	5	500	500
D	A	6	4	800	1,000
E	B,C	7	4	700	1,000
F	C,D	5	2	800	1,400
G	E,F	6	4	800	1,600

(i) Find out the normal duration.

(ii) What is the percentage increase in cost to complete the project in 21 days ?

Solution

The network is given below:

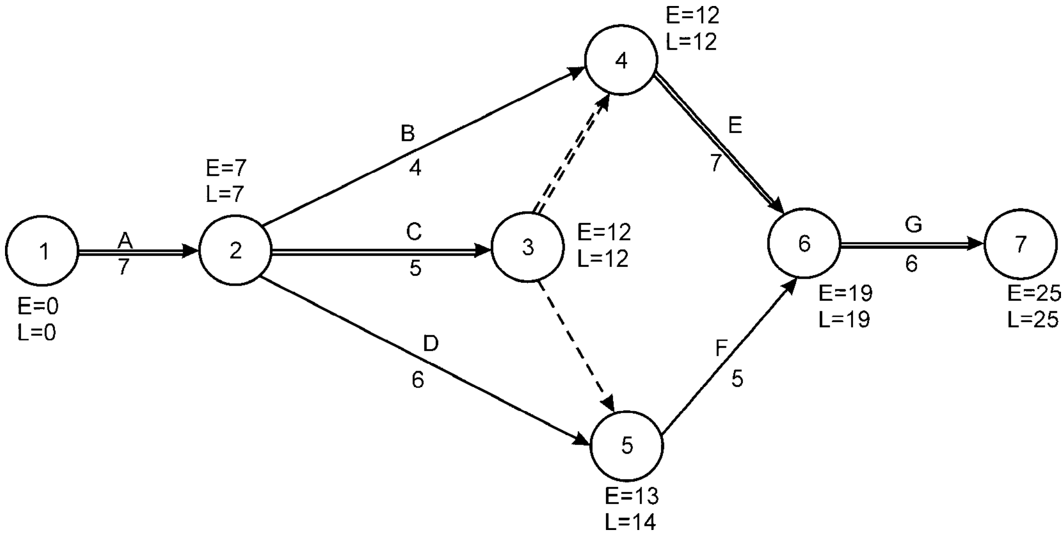


Figure 22

(i) The critical path of the project is A–C–E–G or 1–2–3–4–6–7 with normal duration of 25 days.

(ii) The cost slope for various activities is given below:

Activity Duration	Normal Duration (Days)	Crash Duration (Days)	Normal Cost (₹)	Crash Cost (₹)	Cost Slope (₹)
A(1-2)	7	5	500	900	$\frac{900-500}{7-5}=200$
B(2-4)	4	2	400	600	$\frac{600-400}{4-2}=100$
C(2-3)	5	5	500	500	N.A
D(2-5)	6	4	800	1,000	$\frac{1,000-800}{6-4}=100$
E(4-6)	7	4	700	1,000	$\frac{1,000-700}{7-4}=100$
F(5-6)	5	2	800	1,400	$\frac{1,400-800}{5-2}=200$
G(6-7)	6	4	800	1,600	$\frac{1,600-800}{6-4}=400$
Total			4,500		

Various paths of the network are given below:

1-2-3-4-6-7 with duration = 25 days

1-2-4-6-7 with duration = 24 days

1-2-3-5-6-7 with duration = 23 days

1-2-5-6-7 with duration = 24 days

Step-1:

In order to determine the cost of completing the project in 21 days, let us crash that activity on the critical path which has minimum cost slope. It can be seen that the minimum cost slope of ₹ 100 corresponds to activity E(4-6) and it lies on the critical path. Hence, we crash activity E (4-6) by 1 day at an additional cost of ₹ 100.

Various paths now are

1-2-3-4-6-7 with duration = 24 days

1-2-4-6-7 with duration = 23 days

1-2-3-5-6- 7 with duration = 23 days

1-2-5-6-7 with duration = 24 days

Step-2:

An examination of the above four paths clearly points out that there are two critical paths namely 1-2-3-4-6-7 and 1-2-5-6-7, each with duration = 24 days. To reduce the project duration by three days more, there are following possible combination of activities.

1. Crash activities 4 - 6 on the path 1-2-3-4-6-7 and 5 - 6 on the path 1-2-5-6-7 by one day each at an addition cost of ₹ 100 + ₹ 200 = ₹ 300
2. Crash activities 4-6 on path 1-2-3-4-6-7 and 2-5 on path 1-2-5-6-7 by one day each at an additional cost of ₹ 100 + ₹ 100 =₹ 200
3. Crash activity 1-2 by one day at an additional cost of ₹ 200.

It can be observed that the additional cost of reducing the project duration by one day in combination 2 as well as combination 3 is ₹ 200. Hence any of these two can be selected for crashing. However, since crashing activity 1-2 by 1 day reduces the duration of all the paths by 1 day, we will crash it by 1 day. The project duration becomes = 23 days at an additional cost = ₹ 200.

Various paths now are.

1-2-3-4-6-7 with duration = 23 days   1-2-3-5-6-7 with duration = 22 days

1-2-4-6-7 with duration = 22 days   1-2-5-6-7 with duration 23 days

Step-3:

Crash activity 1-2 by 1 day further, it would reduce the project duration to 22 days at an additional cost = ₹ 200.

Various paths now are.

1-2-3-4-6-7 with duration = 22 days   1-2-3-5-6-7 with duration = 21 days

1-2-4-6-7 with duration = 21 days   1-2-5-6-7 with duration 22 days

Step-4:

Activity 1-2 cannot be crashed further. So, we now select the combination 2 stated above for crashing. Crash activities 4-6 and 2-5 by one day each at an additional cost of ₹ 100 + ₹ 100 = ₹ 200.

Various paths now are.

1-2-3-4-6-7 with duration = 21 days

1-2-3-5-6-7 with duration = 21 days

1-2-4-6-7 with duration = 20 days

1-2-5-6-7 with duration 21 days

The project duration now becomes equal to 21 days.

Hence, in order to complete the project in 21 days, an additional cost of ₹ 100 + ₹ 200+ ₹ 200 + ₹ 200 = ₹ 700 will be incurred.

The normal cost of completing the project in 25 days = ₹ 4,500.

Hence, the percentage increase in cost to complete the project in

$$21 \text{ days} = \frac{₹\ 700}{₹\ 4500} \times 100 = 15.55\%$$

Illustration 16

The following is a table showing details of a project.

Activity	*Immediate Predecessor*	*Normal Time in weeks*	*Normal Cost (₹ 000)*	*Crash Time in weeks*	*Crashing Cost (₹000)*
A	—	*20*	*40*	*14*	*60*
B	—	*16*	*30*	*12*	*40*
C	*B*	*10*	*16*	*8*	*28*
D	*B*	*12*	*22*	*8*	*30*
E	*B*	*16*	*18*	*10*	*30*
F	*E*	*10*	*10*	*8*	*16*
G	*A, D, C*	*24*	*6*	*16*	*8*

Indirect cost is ₹ 800 per day. Find the optimum duration and the associated minimum project cost.

Solution

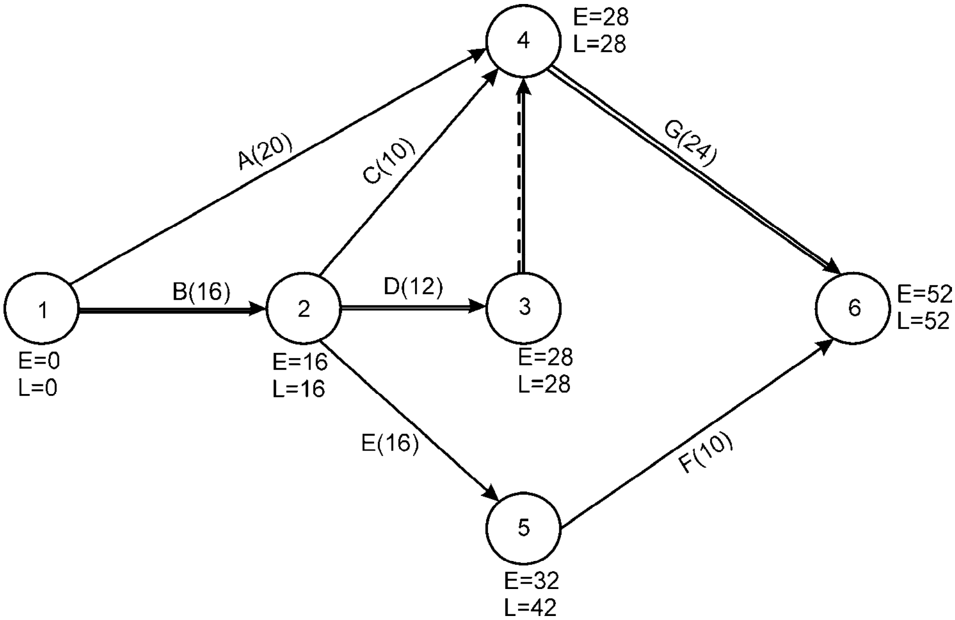


Figure 23

Path 1-2-3-4-6 is critical path.

Project duration is 52 weeks.

Total cost of the project = Total Direct Cost + Indirect Cost

$$= (\underbrace{40 + 30 + 16 + 22 + 18 + 10 + 6}_{\text{In thousands}}) + \underbrace{800}_{\text{Indirect cost per day}} \times \underbrace{7}_{\text{No. of days in a week}} \times \underbrace{52}_{\text{Project duration in weeks}}$$

= 1,42,000 + 2,91,200

= ₹ 4,33,200

Indirect cost is ₹ 800 per day or ₹ 5,600 per week.

We have to Crash the project step by step.

Activity	Normal Duration (Weeks)	Crash duration (Weeks)	Normal cost (₹'000)	Crash Cost (₹'000)	Cost Slope (₹'000)
A (1-4)	20	14	40	60	$\frac{60-40}{20-14} = 3.33$
B (1-2)	16	12	30	40	$\frac{40-30}{16-12} = 2.5$
C (2-4)	10	8	16	28	$\frac{28-16}{10-8} = 6$
D (2-3)	12	8	22	30	$\frac{30-22}{12-8} = 2$
E (2-5)	16	10	18	30	$\frac{30-18}{16-10} = 2$
F (5-6)	10	8	10	16	$\frac{16-10}{10-8} = 3$
G (4-6)	24	16	6	8	$\frac{8-6}{24-16} = 0.25$

The paths of the network are given below:

1-2-3-4-6 = 52 weeks

1-2-4-6 = 50 weeks

1-4-6 = 44 weeks

1-2-5-6 = 42 weeks

The critical activity G with cost slope of ₹ 250 (0.25 × 1,000) per week is least expensive and can be crashed by 8 weeks i.e., 24 weeks – 16 weeks.

With crashing of activity G, the new cost involved is

= Total Direct Cost + Increased Cost due to Crashing of G + Indirect cost 44 weeks.

= ₹ 1,42,000 + ₹ (250 × 8) + ₹ (800 × 7 × 44)

= ₹ 3,90,400

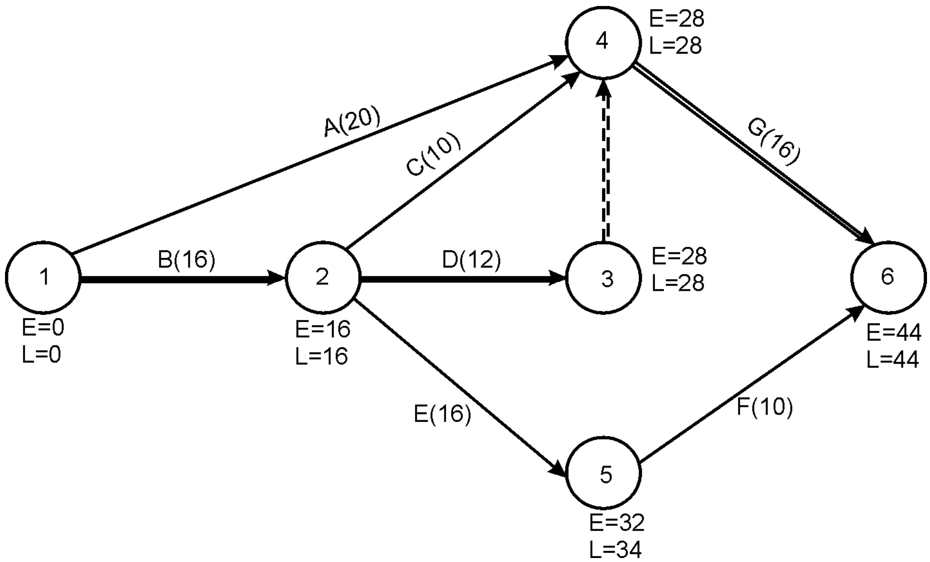


Figure 24

Various paths of the above network are given below:

1-2-3-4-6 = 44 weeks.

1-2-4-6 = 42 weeks

1-4-6 = 36 weeks

1-2-5-6 = 42 weeks

1-2-3-4-6 is still critical path

	B	D	DUMMY	G
Activity	1-2	2-3	3-4	4-6
Slope	2.5	2	–	0.25
Can be Crashed by Max.	4 weeks (16-12)	4 weeks (12-8)	–	0 weeks (24-16-8*)

* Already crashed in first step

Since the least expensive activity on the critical path is D.

It can be crash by two week.

With crashing of activity D by two week, the new cost involved is = ₹ 1,42,000 + ₹ 2,000 + ₹ (2,000 × 2) + ₹ (5,600 × 42)

= ₹ 1,42,000 + 2,000 + ₹ (2,000 × 2) + ₹ (5,600 × 42)

= ₹ 3,83,200

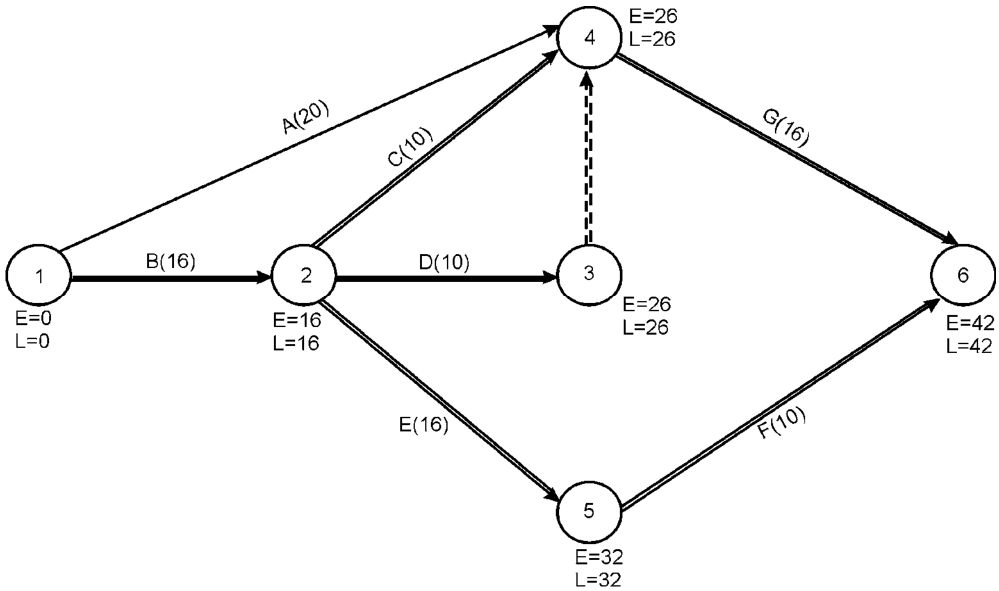


Figure 25

Various paths of the above network are given below :

1-2-3-4-6 = 42 weeks

1-2-4-6 = 42 weeks

1-4-6 = 36 weeks

1-2-5-6 = 42 weeks

Since activity B is common in all three critical paths it can be crashed to the maximum possible of 4 weeks i.e., 16 weeks-12 weeks.

The new total cost involved with duration of 38 weeks is as follows :

= ₹ 1,42,000 + ₹ 2,000 + ₹ 4,000 + ₹ (4 × 2,500) + ₹ (5,600 × 38)

= ₹ 3,70,800

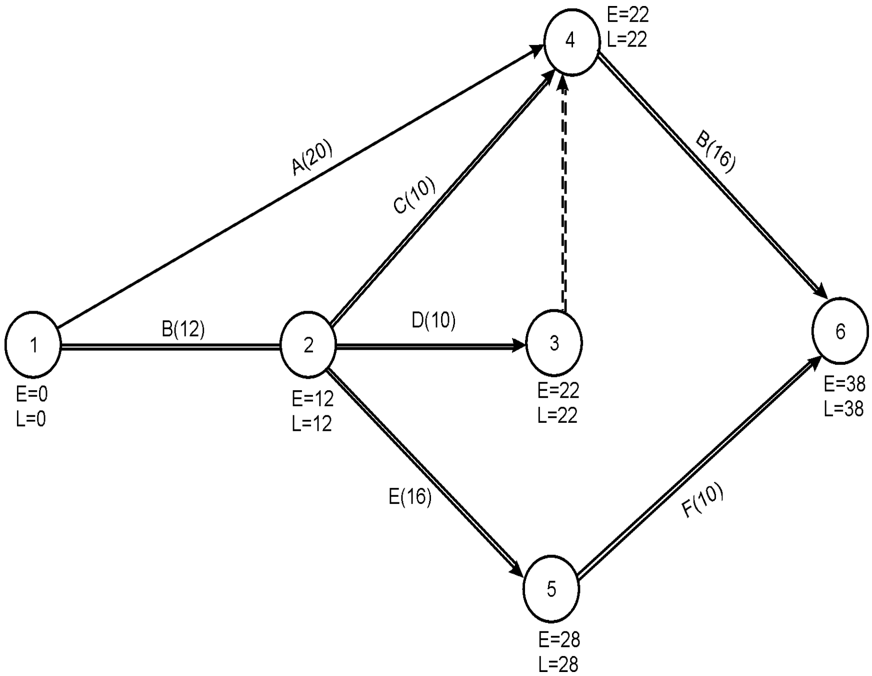


Figure 26

Various path of the above network are given below.

1-2-3-4-6	= 38 weeks
1-2-4-6	= 38 weeks
1-4-6	= 36 weeks
1-2-5-6	= 38 weeks

<u>1 – 2 – 3 – 4 – 6</u>

B D Dummy G

B, G, cannot be further crashed.

However D can be crash by 2 weeks.

<u>1 – 2 – 4 – 6</u>

B C G

B, G, cannot be further crashed.

However C can be crash by 2 weeks.

1 – 4 – 6 ↓ ↓ A G	—
1 – 2 – 5 – 6 ↓ ↓ ↓ B E F	B cannot be further crashed. E can be crashed by. 2 weeks due to lowest cost slope in comparison with F.

Total New Cost.

= ₹ 1,42,000 + ₹ 2,000 + ₹ 4,000 + ₹ 10,000 + ₹ (2,000 × 2 + 6,000 × 2 + 2,000 × 2) + (₹ 5,600 × 36)

= ₹ 3,79,600

No benefit in crashing the project to 36 weeks as cost is increasing by 8,800/-.

Crashing Schedule of Project

Project duration weeks	Crashing activity and weeks	Direct Cost		Indirect Cost	Total (₹)
		Normal	Crashing		
52	—	1,42,000	—	2,91,200	4,33,200
44	G by 8 weeks	1,42,000	2,000	2,46,400	3,90,400
42	D by 2 weeks	1,42,000	2,000 + 4,000	2,35,200	3,83,200
38	B by 4 weeks	1,42,000	6,000 + 10,000	2,12,800	3,70,800
36	C by 2 weeks D by 2 weeks E by 2 weeks	1,42,000	16,000 + 4,000 12,000 + 4,000	2,01,600	3,79,600

Minimum Cost of the project is ₹ 3,70,800 at 38 weeks.

14.7 A Few Comments on Assumptions of PERT & CPM

1. Beta distribution may not always be applicable.
2. The formulae for the expected duration and S.D. are simplifications. Maccrinnon and Ryavec reached the conclusion that in certain cases the errors, because of these assumptions, may even be to the tune of 33%.
3. The errors owing to the aforesaid simplification and assumption may be compounded or may cancel each other to an extent.

4. In computing the S.D. of the critical path independence of activities is implied. Limitations of resources may invalidate the independence which exists by the very definition of an activity.

5. It may not always be possible to sort out completely identifiable activities and to state where they begin and where they end.

6. In projects fraught with in certainty it is natural that there exist alternatives with differing outcomes. For example, if a particular hardness is not obtained in a metal, an alloy might have to be used that is more expensive and also inferior on certain technical considerations. There have been theoretical developments in this regard, and it may be worthwhile to incorporate the concept of decision tree analysis depending upon the situation.

7. Time estimates have an element of subjectiveness and, to that extent, the techniques could be weak. The contractors react to this weakness shrewdly whilst bidding. If there are cost plus contracts they would deliberately “under estimate” the time for chances of being awarded with the contract. Incentive type contracts might lead to an opposite bias.

8. Cost-time tradeoffs, for deriving the cost curve slopes, to be discussed soon, are subjective again and call for a great deal of expertise of the technology as well as genuine effort to estimate. Often the engineers tend to be lax here; occasionally with the honest deliberation event, the guesses may be wide off the mark.

14.8 Distinction Between PERT and CPM

The PERT and CPM models are similar in terms of their basic structure, rationale and mode of analysis. However, there are certain distinctions between PERT and CPM networks which are enumerated below:

1. CPM is activity oriented i.e. CPM network is built on the basis of activities. Also results of various calculations are considered in terms of activities of the project. On the other hand, PERT is event oriented.

2. CPM is a deterministic model i.e. It does not take into account the uncertainties involved in the estimation of time for execution of a job or an activity. It completely ignores the probabilistic element of the problem. PERT, however, is a probabilistic model. It uses three estimates of the activity time; optimistic, pessimistic and most likely; with a view to take into account time uncertainty. Thus, the expected duration of each activity is probabilistic and expected duration indicates that there is fifty per cent probability of getting the job done within that time.

3. CPM places dual emphasis on time and cost and evaluates the trade-off between project cost and project time. By deploying, additional resources, it allows the critical

path project manager to manipulate project duration within certain limits so that project duration can be shortened at an optimal cost. On the other hand, PERT is primarily concerned with time. It helps the manager to schedule and coordinate various activities so that the project can be completed on scheduled time.

4. CPM is commonly used for those projects which are repetitive in nature and where one has prior experience of handling similar projects. PERT is generally used for those projects where time required to complete various activities are not known as priori. Thus, PERT is widely used for planning and scheduling research and development projects.

15

Simulation

Learning Objectives
After studying this chapter, you should be able to:
• Explain the simulation and reasons for using simulation;
• Models of simulation
• Identify the steps in the simulation process;
• Review the reasons why simulation may be preferable to other decision models;
• Simulate a situation based on real data;
• Make useful contribution in (investment decision or capital budgeting) through simulation;
• Establish how simulation permits study of a system under controlled conditions.

15.1 Introduction

There are certain managerial situations which are complex from the point of view of mathematical representation, to arrive at a precise solution for decision making. Mathematical representation of problems or situations is based on assumptions which may not be suitable for the complex and conflicting cases. In such cases, simulation offers a reasonable alternative. Simulation does not try to arrive at optimal solution to the problem under study but offers likely outputs to the given inputs.

15.2 What is simulation?

According to T.H. Naylor, simulation is a numerical technique for conducting experiments on digital computers, which involves certain types of mathematical and logical relationships necessary to describe the behaviors and structure of a complex real-world system over extended period of time.

Simulation is a quantitative procedure which describes a process by developing a model of that process and then conducting a series of organised trial and error experiments to predict the behavior of *the process over time.* Observing the experiments is much like observing the process in operation. To find how the real process would react to certain changes, we can introduce these changes in our model and simulate the reaction of the real process to them. **We can summarise the simulation as imitation of the operation of real-world process or system over the time.**

Following examples would demonstrate the practical usage of simulation:

1. In the process of airplane designing, the designers build a model plane and observe its behavior in a wind tunnel to check aerodynamics features of plane.
2. Military simulations, also known informally as war games. These war games are generally used to train the military personals. War games simulate (mimic) the military operations to test a strategy or to demonstrate some particular real war situations.
3. In the world of finance, simulations are often used for determination of risk adjusted net present value. For a project under evaluation, simulation provides a distribution of NPV over a range of discount rates and other factors.

15.2.1 Models of Simulation: To perform analysis of problems, various models of simulation are followed depending on the nature of the problem. These models are classified into two broad categories:

- **Mathematical Model:** in such model, mathematical symbols or equations are used to represent the relationships in the system. This model can be further classified into two categories:
  - Deterministic models: This model is based on what if analysis. It applies where input and output variables are described by exact functional relationship.
  - Stochastic (Probability) models: As the name indicates, in this model, variables or may be functions are subject to random variations.
- **Physical Model:** This model consists of physical inputs which are relatively expensive than mathematical model. For example, using a mini prototype mode of airplane to test aerodynamic characteristics.

15.2.2 Stages of Simulation Process: All simulations vary in complexity from situation to situation. However, in general, one would have to go through the following stages:-

Stage 1 Identification and Definition of Problem or System

The process begins with the identification of the problem or system which is to be simulated and clearly defining it. Along with problem, the objective that is intended to achieve from simulation results should also be rolled out.

Stage 2 Formulation of Suitable Model

After the identification and clear definition of the problem, one has to develop the model which may be physical or the mathematical depending upon the suitability for that particular problem.

Stage 3 Collections of Data

It is the most important and much needed part of model development and evaluation. The nature and amount of data depends upon the elements and objective of the problem under study. Data can be generated from various sources like random number tables, electronic devices like computers etc.

Stage 4 Run the simulation

Having the suitable model and appropriate data with us, now it is the time to run the simulation. Deterministic model which consists of exact functional relationship between inputs and outputs may require only one run whereas, probabilistic model has random variation in elements so, it requires multiple runs to get clear picture of the model performance.

Stage 5 Analysis and Interpretation of the results

The results obtained from the simulation run(s) are analysed for decision making. The interpretation of the results is dependent on the simulation model's proximity with real environment. It means the adjustment requirement for the obtained solution would be lesser where simulation model depicts more closeness to the real environment.

15.3 Monte Carlo simulation

The Monte Carlo method is a very significant method of simulation. It is also the earliest method of simulation. Its significance can also be known from the fact that the term often use interchangeably with simulation. The Monte Carlo simulation is a numerical technique that employs random numbers and is used to solve problems that depend upon probability, where physical experimentation is impracticable and the creation of a mathematical formula impossible. It is method of Simulation by the sampling technique. That is, first of all, the probability distribution of the variable under consideration is determined; then a set of random numbers are chosen and put in the simulation table to obtain random outputs or a set of values that have the same distributional characteristics as the actual experience it is deviced to simulate.

15.3.1 Major steps in carrying out monte carlo simulation:

(i) Select the measure of effectiveness of the problem, that is, what element is used to measure success in improving the system modeled. This is the element one wants to maximise or minimise. For example, this might be idle time of a service facility, or inventory shortages per period etc.

(ii) Identify the variables which influence the measure of effectiveness significantly. For example, the number of service facilities in operation or the number of units in inventory and so on.

(iii) Determine the proper cumulative probability distribution of each variable selected under step (ii). Plot these, with the probability on the vertical axis and the values of variables on horizontal axis.

(iv) Get a set of random numbers.

(v) Consider each random number as a decimal value of the cumulative probability distribution. With the decimal, enter the cumulative distribution plot from the vertical axis. Project this point horizontally, until it intersects cumulative probability distribution curve. Then project the point of intersection down into the vertical axis.

(vi) Record the value (or values if several variables are being simulated) generated in step (v) into the formula derived from the chosen measure of effectiveness. Solve and record the value. This value is the measure of effectiveness for that simulated value.

(vii) Repeat steps (v) and (vi) until sample is large enough for the satisfaction of the decision maker.

In assigning a set of random numbers, we have to decide on the entire range of random numbers. If the cumulative probabilities are in two digits the range of random numbers to be assigned are 00 to 99; and if in three digits, the range is from 000 to 999, and so on.

In view of the enormous computations involved, computer is usually a necessary adjunct though below we shall deal with quite a few simple simulation problems towards its exposition.

15.3.2 Generation of Random Numbers: The most common methods for the generation of random numbers are the **'mid-square method', spinning arrow method, dice rolling method** and **the spinning disc method**;

Various steps for generation of random numbers may be summarized as:

Step 1 Collection of data related to the problem

Step 2 Construction of frequency distribution.

Step 3 Construction of relative frequency distribution.

Step 4 Adopting a coding system that relates the identified events to generate random numbers.

Step 5 Finding a method to obtain random numbers.

Step 6 Match a random numbers to the events tabulation of result.

Step 7 Repeat the above step until all the desired number of simulation runs has been generated.

Some random numbers are called pseudo random numbers when they are generated by a process and play the satisfactory role of random numbers.

15.3.3 Advantages of simulation: Some of the important advantages of simulation are summarised as below:

(i) Simulation is a flexible model and can be modified to adjust different variations.

(ii) It can be used for strategic planning models.

(iii) It is an easier technique to use than other mathematical models.

(iv) Training people on simulation model is easier.

(v) This technique removes the requirement of costly trial and error methods of trying out new idea on new machines, equipment etc.

15.3.4 Disadvantages of Simulation: Disadvantages of using simulation techniques are summarised as below:

(i) Simulation is not a precise method.

(ii) It cannot be applied in all situations.

(iii) Simulation model package may be very expensive.

(iv) It is a non-optimising technique.

Illustration 1

Frontier Bakery keeps stock of a popular brand of cake. Daily demand based on past experience is as given below:-

Experience indicates

Daily demand	:	0	15	25	35	45	50
Probability	:	.01	.15	.20	.50	.12	.02

Consider the following sequence of random numbers:-

R. No. 48, 78, 09, 51, 56, 77, 15, 14, 68, 09

Using the sequence, simulate the demand for the next 10 days.

Find out the stock situation if the owner of the bakery decides to make 35 cakes every day. Also estimate the daily average demand for the cakes on the basis of simulated data.

Solution

According to the given distribution of demand, the random number coding for various demand levels is shown in Table below:

Random Number Coding

Demand	Probability	Cumulative Probability	Random nos. fitted
0	0.01	0.01	00-00
15	0.15	0.16	01-15
25	0.20	0.36	16-35
35	0.50	0.86	36-85
45	0.12	0.98	86-97
50	0.02	1.00	98-99

The simulated demand for the cakes for the next 10 days is given in the Table below. Also given in the table is the stock situation for various days in accordance with the bakery decision of making 35 cakes per day.

Determination of Demand and Stock Levels

Day	Random Number	Demand	Stock
1	48	35	–
2	78	35	–
3	09	15	20
4	51	35	20

5	56	35	20
6	77	35	20
7	15	15	40
8	14	15	60
9	68	35	60
10	09	15	80

Daily average demand = 270/10 = 27 units per day.

Illustration 2

A company manufactures around 200 mopeds. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopped to 204 mopped, whose probability distribution is as given below:-

Production per day	***Probability***
196	*0.05*
197	*0.09*
198	*0.12*
199	*0.14*
200	*0.20*
201	*0.15*
202	*0.11*
203	*0.08*
204	*0.06*

The finished mopeds are transported in a specially designed three storeyed lorry that can accommodate only 200 mopped. Using the following 15 random numbers 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54, 10, simulate the process to find out:

(i) What will be the average number of mopeds waiting in the factory?

(ii) What will be the average number of empty spaces on the lorry?

Solution

The random numbers are established as in Table below:

Production Per day	**Probability**	**Cumulative Probability**	**Random Number**
196	0.05	0.05	00-04
197	0.09	0.14	05-13
198	0.12	0.26	14-25

199	0.14	0.40	26-39
200	0.20	0.60	40-59
201	0.15	0.75	60-74
202	0.11	0.86	75-85
203	0.08	0.94	86-93
204	0.06	1.00	94-99

Based on the 15 random numbers given we simulate the production per day as above in table 2 below.

	Ran dom No.	Estimated Production Per day	Opening Balance	Current Excess production	Current short production	Total waiting	No. of empty spaces in the lorry
1	82	202	–	2	–	2	–
2	89	203	2	3	–	5	–
3	78	202	5	2	–	7	–
4	24	198	7	–	2	5	–
5	53	200	5	–	–	5	–
6	61	201	5	1	–	6	–
7	18	198	6	–	2	4	–
8	45	200	4	–	–	4	–
9	04	196	4	–	4	0	–
10	23	198	0	–	2	0	2
11	50	200	0	–	–	–	–
12	77	202	0	2	–	2	–
13	27	199	2	–	1	1	–
14	54	200	1	–	–	1	–
15	10	197	1	–	3	–	2
Total						42	4

Average number of mopeds waiting $= \frac{42}{15} = 2.80$

Average number of empty spaces in lorry $= \frac{4}{15} = 0.266$

(Note: Some of the authors have solved this problem without adjusting excess/short production. However, we feel that above approach in better.)

Illustration 3

Dr. STRONG is a dentist who schedules all her patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities and the time needed to complete the work:

Category	*Time required*	*Probability of category*
Filling	*45 minutes*	*0.40*
Crown	*60 minutes*	*0.15*
Cleaning	*15 minutes*	*0.15*
Extraction	*45 minutes*	*0.10*
Checkup	*15 minutes*	*0.20*

Simulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival time starting at 8.00 a.m. Use the following random numbers for handling the above problem:

40 82 11 34 25 66 17 79

Solution

If the numbers 00-99 are allocated in proportion to the probabilities associated with each category of work, then various kinds of dental work can be sampled, using random number table:

Type	Probability	Cumulative Probability	Random Numbers
Filling	0.40	0.40	00-39
Crown	0.15	0.55	40-54
Cleaning	0.15	0.70	55-69
Extraction	0.10	0.80	70-79
Checkup	0.20	1.00	80-99

Using the given random numbers, a work sheet can now be completed (as shown on next page):

Future Events

Patient	Scheduled Arrival	R. No.	Category	Service Time (Minutes)
1	8.00	40	Crown	60
2	8.30	82	Checkup	15
3	9.00	11	Filling	45

4	9.30	34	Filling	45
5	10.00	25	Filling	45
6	10.30	66	Cleaning	15
7	11.00	17	Filling	45
8	11.30	79	Extraction	45

Now, let us simulate the dentist's clinic for four hours starting at 8.00 A.M.

Status

Time	Event	Number of the patient Being served (time to go)	Patients waiting
8.00	1st patient arrives	Ist (60)	–
8.30	2nd arrives	Ist (30)	2nd
9.00	1st departs	-	
9.00	3rd arrives	2nd(15)	3rd
9.15	2nd departs	3rd(45)	
9.30	4th arrives	3rd (30)	4th
10.00	3rd depart	-	
10.00	5th arrives	4th (45)	5th
10.30	6th arrives	4th (15)	5th &6th
10.45	4th departs	5th (45)	6th
11.00	7th arrives	5th (30)	6th & 7th
11.30	5th departs	-	
11.30	8th arrives	6th (15)	7th & 8th
11.45	6th departs	7th (45)	8th
12.00	End	7th (30)	8th
12.30	–	8th (45)	–

The dentist was not idle during the entire simulated period:

The waiting times for the patients were as follows:

Patient No.	Arrival	Service Starts	Waiting (Minutes)
1	8.00	8.00	0
2	8.30	9.00	30
3	9.00	9.15	15
4	9.30	10.00	30
5	10.00	10.45	45

6	10.30	11.30	60
7	11.00	11.45	45
8	11.30	12.30	60
		Total	285

The average waiting time of a patient was = 35.625 minutes.

Illustration 4

A Company trading in motor vehicle spares wishes to determine the level of stock it should carry for the item in its range. Demand is not certain and replenishment of stock takes 3 days. For one item X, the following information is obtained.

Demand (units per day)	***Probability***
1	*0.1*
2	*0.2*
3	*0.3*
4	*0.3*
5	*0.1*

Each time an order is placed, the company incurs an ordering cost of ₹ 20 per order. The company also incurs carrying cost of ₹ 2.50 per unit per day. The inventory carrying cost is calculated on the basis of average stock.

The manager of the company wishes to compare two options for his inventory decision .

(a) Order 12 units when the inventory at the beginning of the day plus order outstanding is less than 12 units.

(b) Order 10 units when the inventory at the beginning of the day plus order outstanding is less than 10 units.

Currently (on first day) the company has a stock of 17 units. The sequence of random number to be used is 08, 91, 25, 18, 40, 27, 85, 75, 32, 52 using first number for day one.

You are required to carry out a simulation run over a period of 10 days recommend which option the manager should chose.

Solution

Step 1 Random numbers are assigned for daily demand

Demand	**Probability**	**Cumulative Probability**	**Ten random nos. fitted in**
1	*0.1*	*0.1*	*00-09*
2	*0.2*	*0.3*	*10-29*
3	*0.3*	*0.6*	*30-59*

4	0.3	0.9	60-89
5	0.1	1.0	90-99

Option I. Order 12 units as per conditions mentioned

Day	Random number	Opening stock	Demand	Closing stock	Order placed	Order Received	Average stock
1	08	17	1	16	-	-	16.5
2	91	16	5	11	-	-	13.5
3	25	11	2	9	12	-	10.0
4	18	9	2	7	-	-	8.0
5	40	7	3	4	-	-	5.50
6	27	4	2	14	-	12	9.00
7	85	14	4	10	-	-	12.00
8	75	10	4	6	12	-	8.00
9	32	6	3	3	-	-	4.50
10	52	3	3	-	-	-	1.50
							88.5

Carrying cost (88.5x2.50) = ₹ 221.25

Ordering cost (2x₹20) = 40.00

261.25

Option II. Order for 10 units as per conditions mentioned

Day	Random number	Opening stock	Demand	Closing stock	Order placed	Order Received	Average stock
1	08	17	1	16	-	-	16.50
2	91	16	5	11	-	-	13.50
3	25	11	2	9	-	-	10.00
4	18	9	2	7	10	-	8.00
5	40	7	3	4	-	-	5.50
6	27	4	2	2	-	-	3.00
7	85	2	4	8	-	10	5.00
8	75	8	4	4	10	-	6.00
9	32	4	3	1	-	-	2.50
10	52	1	3	-	-	-	0.50
							70.5

Total Carrying cost is 70.5 x 2.50

= ₹176.25

Total ordering cost is 2 x 20

= 40.00

Stock – out cost (not mentioned taken as Nil)

= -

216.25

It should be noticed that there will be stock-out on 10th day in IInd option. Based on this given information, option II appears to be better.

15.4 Simulation and Inventory Control

The Monte Carlo simulation is widely used to solve inventory problems characterised by uncertainty of demand and lead time. The distribution of demand during the lead time can be obtained from an empirical analysis of past data or by computer simulation using random numbers. The cumulative probability distribution of demand during the lead time is used as a basis to determine the annual inventory costs and stock-out costs for different levels of the safety stock. The management can experiment the effect of various inventory policies by using simulation and finally select an optimum inventory policy. The purpose of simulation, as applied to inventory, is to facilitate the management in selecting an inventory policy that will result in minimum annual inventory costs-ordering, carrying and stock-out costs. The application of simulation to inventory control is explained below with the help of an example.

The distribution of demand during the lead time and the distribution of the lead time are set out in Tables 1 and 2.

Table 1 : Distribution of Demand during Lead time

Quantity demanded during lead time	Probability
0	0.10
1	0.45
2	0.30
3	0.15

Table 2 : Distribution of Lead time

Lead time (weeks)	Probability
2	0.20
3	0.65
4	0.15

Suppose, the management wants to ascertain the annual inventory costs, if they wish to reorder when the quantity on hand is 6 units and order each time a quantity of 12 units.

Assume that all orders are delivered for the entire quantity. The cost of ordering is ₹ 120, and the cost of holding the inventory in stock is ₹ 5 per unit per week. Further, the management has found that when it runs out-of-stock, it costs ₹ 75 per unit.

Table 5 illustrates the manual simulation for 15 weeks. Before proceeding with simulating the demand for each week, we calculate first the cumulative probability and assign random numbers for each value of the two variables-demand and lead time. These are set out in Tables 3 and 4.

Table 3

Demand	Cumulative Probability	Random Nos. assigned
0	0.10	00 to 09
1	0.55	10 to 54
2	0.85	55 to 84
3	1.00	85 to 99

Table 4

Lead time	Cumulative Probability	Random Nos. assigned
2	0.20	00 to 19
3	0.85	20 to 84
4	1.00	85 to 99

We assume that the stock on hand at the start of the simulation process is 10 units. Further, we also assume that all orders are placed at the beginning of the week and all deliveries against orders are received at the beginning of the week.

The simulated demand for work 1 is 1 unit (corresponding to the random number 49 in Table 3), and the closing inventory is 9 units. The inventory-carrying cost works out to 9 × 5 = ₹ 45 (₹ 5 per unit per week). The inventory-carrying costs are calculated for the remaining weeks in the same way. At the end of week 4, the closing inventory is 6 units (reorder point). So an order is placed at the beginning of week 5 for a quantity of 12 units (ordering quantity). The simulated lead time is 3 weeks (corresponding to random number 84 in Table 4). This order quantity is, therefore, received at the beginning of week 8. We notice from Table 5 that another order is placed at the beginning of week 12, when the stock on hand is 6 units, which is equal to the reorder point. Again, the simulated lead time is 3 weeks. Before this quantity is received At the beginning of week 15, we find that the quantity available at the beginning of week 14 is 1 unit and the simulated demand for that week is 2 units, giving rise to a stock out of 1 unit. The stock-out cost is ₹ 75.

Week	Stock on hand beginning of week	Demand		Quantity received	Stock on hand end of week	Inventory carrying costs	Stock out		Lead Time	
		Random No.	Quantity Demanded				Quantity	Costs	Random No.	Lead Time period
1	10	49	1	–	9	45				
2	9	67	2	–	7	35				
3	7	06	0	–	7	35				
4	7	30	1	–	6	30				
5	6	95	3	–	3	15			84	3
6	3	01	0	–	3	15				
7	3	10	1	–	2	10				
8	2	70	2	12	12	60				
9	12	80	2	–	10	50				
10	10	66	2	–	8	40				
11	8	69	2	–	6	30				
12	6	76	2	–	4	20			79	3
13	4	86	3	–	1	5				
14	1	56	2	–	–	–	1	75		
15	–	84	2	12	10	50				

For the simulated period of 15 weeks, the total inventory costs are:-

Inventory carrying costs	=	₹ 440
Ordering costs (2 orders × 120)	=	₹ 240
Stock-out costs (1 stock-out)	=	₹ 75
Total	=	₹ 755

By simulating over a period of 2 to 3 years (i.e. 100 or 150 weeks), we can obtain a more accurate picture of the annual inventory costs.

By varying the values of the variables (the ordering quantity and the reorder point), the management can find out the effect of such a policy in terms of the annual inventory costs. In other words, simulation permits the management to evaluate the effects of alternate inventory policies. Also, if there is a change in the ordering, stock-out and inventory-carrying costs, their impact on the annual inventory costs can be determined by using simulation.

15.5 Miscellaneous Illustrations

Illustration 5

A company manufactures 30 items per day. The sale of these items depends upon demand which has the following distribution:

Sales (Units)	*Probability*
27	*0.10*
28	*0.15*
29	*0.20*
30	*0.35*
31	*0.15*
32	*0. 05*

The production cost and sale price of each unit are ₹ 40 and ₹ 50 respectively. Any unsold product is to be disposed off at a loss of ₹ 15 per unit. There is a penalty of ₹ 5 per unit if the demand is not met.

Using the following random numbers estimate total profit / loss for the company for the next 10 days: 10, 99, 65, 99, 95, 01, 79, 11, 16, 20

If the company decides to produce 29 items per day, what is the advantage or disadvantage to the company?

Solution

First of all, random numbers 00-99 are allocated in proportion to the probabilities associated with the sale of the items as given below:

Sales (units)	*Probability*	*Cumulative probability*	*Random numbers assigned*
27	0.10	0.10	00-09
28	0.15	0.25	10-24
29	0.20	0.45	25-44
30	0.35	0.80	45-79
31	0.15	0.95	80-94
32	0.05	1.00	95–99

Let us now simulate the demand for next 10 days using the given number in order to estimate the total profit/loss for the company. Since the production cost of each item is ₹ 40 and sale price is ₹ 50, therefore the profit per unit of the sold item will be ₹ 10. There is a loss of ₹ 15 per unit associated with each unsold unit and a penalty of ₹ 5 per unit if the demand is not met. Accordingly, the profit / loss for next ten days are calculated in column (iv) of the table below if the company manufactures 30 items per day.

(i) Day	(ii) Random number	(iii) Estimated sale	(iv) Profit/Loss per day when production = 30 items per day	(v) Profit/loss per day when production = 29 items per day
1	10	28	28 × ₹ 10 – 2 × ₹ 15=250	28 × ₹ 10 – 1 × ₹ 15=265
2	99	32	30 × ₹ 10 – 2 × ₹ 5=290	29 × ₹ 10 – 3 × ₹ 5 = 275

3	65	30	30 × ₹ 1 0=300	29 × ₹ 10 – 1 × ₹ 5=285
4	99	32	30 × ₹ 10 – 2 × ₹ 5=290	29 × ₹ 10 – 3 × ₹ 5=275
5	95	32	30 × ₹ 10 – 2 × ₹ 5=290	29 × ₹ 10 – 3 × ₹ 5=275
6	01	27	27 × ₹ 10 – 3 × ₹ 1 5=225	27 × ₹ 10 – 2 × ₹ 15=240
7	79	30	30 × ₹ 10 = 300	29 × ₹ 10 – 1 × ₹ 5=285
8	11	28	28 × ₹ 10 – 2 × ₹ 1 5=250	28 × ₹ 1 0 -1 × ₹ 15=265
9	16	28	28 × ₹ 10 – 2 × ₹ 15=250	28 × ₹ 10 - 1× ₹ 15= 265
10	20	28	28 × ₹ 10 – 2 × ₹ 1 5=250	28 × ₹ 1 0 - 1 × ₹ 1 5=265
Total Profit =			₹ 2695	₹ 2695

The total profit for next 10 days will be ₹ 2,695 if the company manufactures 30 items per day. In case, the company decides to produce 29 items per day, then the profit of the company for next 10 days is calculated in column (v) of the above table. It is evident from this table that there is no additional profit or loss if the production is reduced to 29 items per day since the total profit remains unchanged i.e. ₹ 2,695.

Illustration 6

An Investment Corporation wants to study the investment projects based on three factors : market demand in units, price per unit minus cost per unit, and the investment required. These factors are felt to be independent of each other. In analysing a new consumer product, the corporation estimates the following probability distributions:

Annual demand Units	*Probability*	*(Price-Cost) per unit ₹*	*Probability*	*Investment ₹*	*Required Probability*
25,000	*0.05*	*3.00*	*0.10*	*27,50,000*	*0.25*
30,000	*0.10*	*5.00*	*0.20*	*30,00,000*	*0.50*
35,000	*0.20*	*7.00*	*0.40*	*35,00,000*	*0.25*
40,000	*0.30*	*9.00*	*0.20*		
45,000	*0.20*	*10.00*	*0.10*		
50,000	*0.10*				
55,000	*0.05*				

Using simulation process, repeat the trial 10 times, compute the return on investment for each trial, taking these three factors into account. Approximately, what is the highest likely return? Use the following random numbers for annual demand, (price-cost) and the investment required:

28, 57, 60, 17, 64, 20, 27, 58, 61, 30; 19, 07, 90, 02 57,

28, 29, 83, 58, 4;, 18, 67, 16, 71, 43, 68, 47, 24, 19, 97

Solution

The yearly return can be determined by the formula:

Return (R) = $\frac{(\text{Price} - \text{Cost}) \times \text{Number of units demanded}}{\text{Investment}}$

Allocation of Random Numbers

Annual Demand units

Units	Probability	Cumulative Probability	Random Nos.
25,000	0.05	0.05	00-04
30,000	0.10	0.15	05-14
35,000	0.20	0.35	15-34
40,000	0.30	0.65	35-64
45,000	0.20	0.85	65-84
50,000	0.10	0.95	85-94
55,000	0.05	1.00	95-99

(Price-Cost) per unit

₹	Probability	Cumulative Probability	Random Nos.
3.00	0.10	0.10	00-09
5.00	0.20	0.30	10-29
7.00	0.40	0.70	30-69
9.00	0.20	0.90	70-89
10.00	0.10	1.00	90-99

Investment

₹	Probability	Cumulative Probability	Random Nos.
27,50,000	0.25	0.25	00-24
30,00,000	0.50	0.75	25-74
35,00,000	0.25	1.00	75-99

The results of the simulation are shown in the table given below:

Trials	Random Number of Demand	Simulated Demand ('000)	Random Number for profit (Price-Cost) per unit	Simulated Profit	Random number for investment	Simulated investment ('000)	Simulated Return (%)Demand × Profit per Investment
1	28	35	19	5.00	18	2750	6.36
2	57	40	07	3.00	67	3000	4.00

3	60	40	90	10.00	16	2750	14.55
4	17	35	02	3.00	71	3000	3.50
5	64	40	57	7.00	43	3000	9.33
6	20	35	28	5.00	68	3000	5.83
7	27	35	29	5.00	47	3000	5.83
8	58	40	83	9.00	24	2750	13.09
9	61	40	58	7.00	19	2750	10.18
10	30	35	41	7.00	97	3500	7.00

Result: Above table shows that the highest likely return is 14.6% which is corresponding to the annual demand of 40,000 units resulting a profit of ₹ 10 per unit and the required investment will be ₹ 27,50,000.

Illustration 7

The occurrence of rain in a city on a day is dependent upon whether or not it rained on the previous day. If it rained on the previous day, the rain distribution is given by:

Event	*Probability*
No rain	*0.50*
1 cm. rain	*0.25*
2 cm. rain	*0.15*
3 cm. rain	*0.05*
4 cm. rain	*0.03*
5 cm. rain	*0.02*

If it did not rain the previous day, the rain distribution is given by:

Event	*Probability*
No rain	*0.75*
1 cm. rain	*0.15*
2 cm. rain	*0.06*
3 cm. rain	*0.04*

Simulate the city's weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period. Use the following random numbers:

67 63 39 55 29 78 70 06 78 76

for simulation. Assume that for the first day of the simulation it had not rained the day before.

Solution

The numbers 00-99 are allocated in proportion to the probabilities associated with each event. If it rained on the previous day, the rain distribution and the random number allocation are given below:

Event	Probability	Cumulative Probability	Random numbers assigned
No rain	0.50	0.50	00-49
1 cm. rain	0.25	0.75	50-74
2 cm. rain	0.15	0.90	75-89
3 cm. rain	0.05	0.95	90-94
4 cm. rain	0.03	0.98	95-97
5 cm. rain	0.02	1.00	98-99

Table I : Rain on previous day

Similarly, if it did not rain the previous day, the necessary distribution and the random number allocation is given below:

Event	Probability	Cumulative Probability	Random numbers assigned
No rain	0.75	0.75	00-74
1 cm. rain	0.15	0.90	75-89
2 cm. rain	0.06	0.96	90-95
3 cm. rain	0.04	1.00	96-99

Table 2: No rain on previous day

Let us now simulate tile rain fall for 10 days using the given random numbers. For tile first day it is assumed that it had not rained the day before:

Day	Random Numbers	Event	
1	67	No rain	(from table 2)
2	63	No rain	(from table 2)
3	39	No rain	(from table 2)
4	55	No rain	(from table 2)
5	29	No rain	(from table 2)
6	78	1 cm. rain	(from table 2)
7	70	1 cm. rain	(from table 1)
8	06	No rain	(from table 1)
9	78	1 cm. rain	(from table 2)
10	76	2 cm. rain	(from table 1)

Hence, during the simulated period, it did not rain on 6 days out of 10 days. The total rain fell during the period was 5 cm.

Illustration 8

The output of a production line is checked by an inspector for one or more of three different types of defects, called defects A, B and C. If defect A occurs, the item is scrapped . If defect B or C occurs, the item must be reworked. The time required to rework a B defect is 15 minutes and the time required to rework a C defect is 30 minutes. The probabilities of an A, B and C defects are 0.15, 0.20 and 0.10 respectively. For ten items coming off the assembly line, determine total number of items without any defects, the number scrapped and the total minutes of rework time. Use the following random numbers,

RN for defect A

48 55 9 1 40 93 01 83 63 47 52

*RN for defect B**

47 36 57 04 79 55 10 13 57 09

RN for defect C

82 95 1 8 96 20 84 56 11 52 03

Solution

The probabilities of occurrence of A, B and C defects are 0.15, 0.20 and 0.10 respectively. So, tile numbers 00-99 are allocated in proportion to the probabilities associated with each of the three defects

Defect A		Defect B		Defect C	
Exists	**Random numbers assigned**	**Exists**	**Random numbers assigned**	**Exists**	**Random numbers assigned**
Yes	00-14	Yes	00-19	Yes	00-09
No	15-99	No	20-99	No	10-99

Let us now simulate the output of the assembly line for 10 items using the given random numbers in order to determine the number of items without any defect, the number of items scrapped and the total minutes of rework time required:

Item No.	RN for defect A	RN for defect B	RN for defect C	Whether any defect exists	Rework time (in minutes)	Remarks
1	48	47	82	None	–	–
2	55	36	95	None	–	–
3	91	57	18	None	–	–
4	40	04	96	B	15	–
5	93	79	20	None	–	–
6	01	55	84	A	–	Scrap

7	83	10	56	B	15	–
8	63	13	11	B	15	–
9	47	57	52	None	–	–
10	52	09	03	B,C	15+30=45	–

During the simulated period, 5 out of the ten items had no defects, one item was scrapped and 90 minutes of total rework time was required by 4 items.

Illustration 9

The management of ABC company is considering the question of marketing a new product. The fixed cost required in the project is ₹ 4,000. Three factors are uncertain viz. the selling price, variable cost and the annual sales volume. The product has a life of only one year. The management has the data on these three factors as under:

Selling Price ₹	***Probability***	***Variable Cost ₹***	***Probability***	***Sales volume (Units)***	***Probability***
3	*0.2*	*1*	*0.3*	*2,000*	*0.3*
4	*0.5*	*2*	*0.6*	*3,000*	*0.3*
5	*0.3*	*3*	*0.1*	*5,000*	*0.4*

Consider the following sequence of thirty random numbers:

81 32 60 04 46 31 67 25 24 10 40 02 39 68
08 59 66 90 12 64 79 31 86 68 82 89 25 11
98 16

Using the sequence (First 3 random numbers for the first trial, etc.) simulate the average profit for the above project on the basis of 10 trials.

Solution

First of all, random numbers 00–99 are allocated in proportion to the probabilities associated with each of the three variables as given under:

Selling Price ₹	**Probabilities**	**Cumulative Probabilities**	**Random numbers assigned**
3	0.2	0.2	00-19
4	0.5	0.7	20-69
5	0.3	1.0	70-99
Variable cost (₹)			
1	0.3	0.3	00-29
2	0.6	0.9	30-89
3	0.1	1.0	90-99

Sales Volumes (Units)			
2,000	0.3	0.3	00-29
3,000	0.3	0.6	30-59
5,000	0.4	1.0	60-99

Let us now simulate the output of ten trials using the given random numbers in order to find the average profit for the project:

S. No.	Random No.	Selling Price (₹)	Random No.	Variable Cost (₹)	Random No.	Sales Volume ('000 units')
1	81	5	32	2	60	5
2	04	3	46	2	31	3
3	67	4	25	1	24	2
4	10	3	40	2	02	2
5	39	4	68	2	08	2
6	59	4	66	2	90	5
7	12	3	64	2	79	5
8	31	4	86	2	68	5
9	82	5	89	2	25	2
10	11	3	98	3	16	2

Profit = (Selling Price – Variable cost) × Sales Volume – Fixed cost.

Simulated profit in ten trials would be as follows:

S.No.	Profit	
1	(₹ 5 – ₹ 2) × 5,000 units – ₹ 4,000=	₹ 11,000
2	(₹ 3 – ₹ 2) × 3,000 units – ₹ 4,000=	₹ – 1,000
3	(₹ 4 – ₹ 1) × 2,000 units – ₹ 4,000 =	₹ 2,000
4	(₹ 3 – ₹ 2) × 2,000 units – ₹ 4,000 =	₹ – 2,000
5	(₹ 4 – ₹ 2) × 2,000 units – ₹ 4,000 =	0
6	(₹ 4 – ₹ 2) × 5,000 units – ₹ 4,000 =	₹ 6,000
7	(₹ 3 – ₹ 2) × 5,000 units – ₹ 4,000 =	₹ 1,000
8	(₹ 4 – ₹ 2) × 5,000 units – ₹ 4,000 =	₹ 6,000
9	(₹ 5 – ₹ 2) × 2,000 units – ₹ 4,000 =	₹ 2,000
10	(₹ 3 – ₹ 3) × 2,000 units – ₹ 4,000 =	₹ –4,000
	Total	₹ 21,000

Therefore average profit per trial = $\frac{₹\ 21{,}000}{10}$ = ₹ 2,100

Example on the use of random number table. Suppose we want five 3-digit random nos. We can enter anywhere in the table e.g. the last column, first 3 digits of 5 consecutive no.s give us the answers: 413, 172, 207, 511, and 172. Thus we can enter the table randomly but then on proceed serially.

15.6 Random Numbers Table

8481	5016	0080	4376	2579	8293	5950	1048	0650	4135
0744	3447	6173	3288	6378	6704	0966	9986	5202	1728
5558	7239	2976	4836	6134	5120	1541	6514	3581	2079
9371	1463	2164	2301	3142	3866	8707	9980	2011	5111
3033	1660	6365	9054	1155	8844	4085	9589	2924	1725
1053	7320	6532	7214	8972	6466	1217	0100	1458	9416
4389	3504	4086	9434	0136	5695	6876	7937	5476	3396
2158	8854	9534	1196	4941	2697	7497	1149	1952	3482
6749	3676	4943	1406	8614	2060	6433	1660	8875	3194
2878	3447	4804	6761	5309	0636	0522	2004	3207	4684
0591	6549	2206	6185	6188	2649	2389	9483	0924	1389
1025	3438	0546	2545	1089	1280	6701	9742	3453	5573
4244	9217	1628	4524	0163	9895	9586	2083	8459	0644
1331	9032	1388	5661	0472	7128	1902	0343	7724	0528
8853	3490	2589	8744	1221	4667	8396	4779	9937	7206
5059	4192	6331	5485	5922	0982	9390	8993	3621	2602
0821	4340	3194	0118	4773	8668	1891	7989	9190	2296
5262	1746	7108	6496	2570	4243	5029	8949	4989	5008
1210	1858	9365	6562	0269	9923	1796	6626	8591	1990
3642	6629	5775	3219	8801	4047	6861	0765	2379	3494
9598	5322	3747	0363	5995	5504	6804	7033	0957	9556
3894	3173	2853	9312	2498	8878	4956	8748	6247	6673
3603	3011	6762	0848	8316	3485	6388	8925	3790	0898
1121	2978	6313	5857	8457	1395	7240	8630	3895	6348
1930	4583	4227	4120	6893	7005	2264	6067	5627	7985

6309	9158	2830	3262	9809	4606	8669	1154	5841	7695
4460	3143	5383	0327	9668	1697	8335	0860	2188	1908
8371	5095	7273	1866	4193	4163	2035	2812	4996	7142
9397	5540	9298	9076	1299	0669	0088	1809	0631	3162
9304	1468	4013	7465	0861	6787	3581	7977	8409	4708
5606	2435	8546	3209	4802	6690	8527	2210	6706	1930
6693	8333	082	7546	2910	8553	8725	1237	4423	1570
0556	7715	8994	4245	1540	8150	3889	5273	6977	2703
6973	9299	4959	7146	1426	7086	8743	6982	5547	3394
4920	1223	5208	6661	4907	1102	0501	3625	8513	3192
0132	0098	8241	0858	7627	4174	1170	3142	2455	4891
4051	3101	9854	4488	6931	3266	3147	2500	8011	8848
0267	5612	5504	7917	7928	8034	9989	4353	2675	9497
0609	9469	3149	4086	8911	8547	3518	9349	1836	0548
2593	1666	5750	5105	4287	4380	7860	7792	1625	7659
8812	9491	2602	4100	4962	1037	9778	1778	4223	3193
3540	5985	0019	7155	1471	185I	8682	9957	3772	4706
9535	5375	1239	1624	5378	6803	7177	7911	4660	5669
3174	7677	8282	6669	5879	7874	9931	6581	9784	2607
8864	4760	1129	6205	4949	4205	0222	7479	6470	8194
5245	7341	0593	5656	6799	3071	1751	4339	5630	9496
5468	6038	4511	1440	2135	5777	9903	1048	6726	8602
3951	7928	6818	4161	4840	1392	1323	5014	7538	9854
7319	4064	4024	5401	2834	7518	3978	3742	1005	4619
5892	8731	6269	5189	2071	4084	9789	3620	9819	4548

16

Learning Curve Theory

LEARNING OBJECTIVES

After studying this unit, you will be able to :

- Understanding, of learning curve phenomenon.
- Understand how the percentage learning rate applies to the doubling of output
- Know that cumulative average time means the average time per unit for all units produced so far back to and including the first unit produced.
- Use of learning curve theory in such situations as pricing decisions, work scheduling and standard setting
- Describe and identify the situations where learn effect can be incorporated
- Calculate average hours/cost per unit of cumulative productions and incremental hours/cost per order using the learning curve

16.1 Introduction

Learning is the process by which an individual acquires skill, knowledge and ability. When a new product or process is started, performance of worker is not at its best and learning phenomenon takes place. As the experience is gained, the performance of worker improves, time taken per unit reduces and thus his productivity goes up. This improvement in productivity of workers is due to learning effect. Cost predictions especially those relating to direct labour must allow for the effect of learning process. This technique is a mathematical technique. It is a graphical technique used widely to predict cost. Learning curve is a geometrical progression, which reveals that there is steadily decreasing cost for the accomplishment of a given repetitive operation, as the identical operation is increasingly repeated. The amount of decrease will be less and less with each successive unit produced. The slope of the decision curve is expressed as a percentage. The other names given to learning curve are Experience curve, Improvement curve and Progress curve. It is essentially a measure of the experience gained in production of an article by an organisation. As more units are produced, people involved in production become more efficient than before. Each additional unit takes less time to produce. The amount of improvement or experience gained is reflected in a decrease in man-hours or cost. The application of learning curve can be extended to commercial and industrial activities as well as defence production.

The learning effect exists during a worker's start up or familiarization period on a particular job. After the limits of experimental learning are reached, productivity tends to stabilise and no further improvement is possible. The rate at which learning occurs is influenced by many factors including the relative unfamiliarity of workers with the task, the relative novelty and uniqueness of the job, the complexity of the process, the impact of incentive plans, supervision, etc.

16.2 Distinctive Features of Learning Curve Theory in Manufacturing Environment

T.P. Wright of Curtiss—Wright, Buffalo, U.S.A. introduced the theory of learning curve. When the production quantity of a given item is doubled, the cost of that item decrease at a constant rate. Theory of learning curve has been formulated on the basis of this phenomenon. It is important to note that as the quantity produced doubles, the absolute amount of cost increase will be successively lessor but the rate of decrease will remain constant.

Features of learning curve can be summarized as below:

(*i*) Better tooling methods are generated and applied;

(*ii*) More equipments are designed and used to increase the production;

(*iii*) Designed bugs are found and rectified;

(*iv*) Better design is achieve through design engineering for reducing material and labour cost.

(*v*) Rejections and rework tend to reduce.

This results into: (a) less labour; (b) less material; (c) more units produced from the same equipments; (d) cost of fewer delays and less loss time. It allows increase of production and reduction of cost per unit.

16.3. The Learning Curve Ratio

In the initial stage of a new product or a new process, the learning effect pattern is so regular that the rate of decline established at the outset can be used to predict labour cost well in advance. The effect of experience on cost is summaries in the learning ratio or improvement ratio:

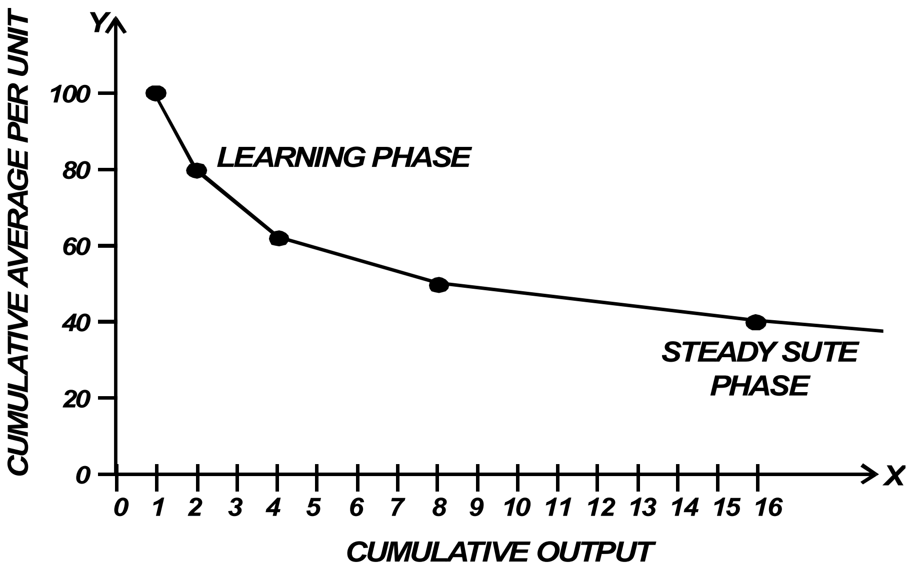
$$\text{Learning Curve Ratio} = \frac{\text{Average labour cost of first 2N units}}{\text{Average labour cost of first N units}}$$

If the average labour cost for the first 500 units of a product is ₹ 25 and the average labour cost of first 1,000 units is ₹ 20, the learning ratio will be determined as follow :

$$\text{Learning curve ratio} = \frac{₹\ 20}{₹\ 25} \times 100 \text{ or } 80\%$$

This learning curve ratio of 80% means that every time output doubles, the average cost declines to 80% of the previous amount. Since the average cost per unit of 1,000 units is ₹ 20, the average cost of first 2,000 units is likely to be 80% of this, i.e., ₹ 16 per unit. The amount of production improvement in the manufacturing of an article will determine the percentage of the learning curve. The effect of learning can be presented clearly in a diagram drawn known as learning curve. The units chosen in the progression must always have a ratio of two (Unit 2 to unit 1, unit 50 to unit 25, unit 300 to unit 150, etc.). An 80% leaning curve is drawn using the following data.

Incremental quantity	Cumulative quantity	Workings of average time	Average time per unit	Cumulative time taken
(1)	(2)	(3)	(4)	(5)
1	1		100	100
1	2	$\frac{(100 \times 80)}{100}$	80	160
2	4	$\frac{(80 \times 80)}{100}$	64	256
4	8	$\frac{(64 \times 80)}{100}$	51	408
8	16	$\frac{(51 \times 80)}{100}$	41	656



Learning Curve

Columns (2) and (4) have been used for drawing the learning curve. Last column is not used in drawing the 80 per cent learning curve. This column shows how the cumulative time consumption will increase with decrease in cumulative average per unit. Cumulative quantity is plotted on X-axis and cumulative average time consumption per unit is plotted on Y-axis. After the learning effect phase is over, steady-state phase will start. Learning effect advantage will not be there is steady-state phase, when the product or the process gets well stabilised.

16.4 Learning Curve Equation

Mathematicians have been able to express relationship in equations. The basic equation

$Y_x = KX^s$ (it can be expressed as $y = ax^{-b*}$) ...(1)

* even sometime it is expressed as $y = ax^b$, though b always has a negative value

where,

X is the cumulative number of units or lots produced

Y is the cumulative average unit cost of those units X or lots.

K is the average cost of the first unit or lot

s is the improvement exponent or the learning coefficient or the index of learning which is calculated as follows :

$$s = \frac{\text{logarithm of learning ratio}}{\text{logarithm of 2}}$$

Learning curve equation $Y_x = KX^s$ becomes a linear equation when it is written in its logarithmic form :

$\log Y_x = \log K + s \log X$ ...(2)

Each of the above two equations defines cumulative average cost. Either of them can be converted to a formula for the total labour cost of all units produced up to a given point. Total cost under equation I can be found out by the following formula :

Total cost $= XY_x = KX^s = KX^{s+1}$ ...(3)

16.5 Learning Curve Application

Knowledge of learning curve can be useful both in planning and control. Standard cost for new operations should be revised frequently to reflect the anticipated learning pattern. Its main uses are summarised below:

16.5.1 Helps to analyse CVP Relationship during familiarisation phase: Learning curve is useful to analyse cost-volume-profit relationship during familiarisation phase of product or process and thus it is very useful for cost estimates. Learning curve can be used as a tool for forecasting.

Illustration 1

XYZ Co., has observed that a 90% learning curve ratio applies to all labour related costs each time a new model enters production. It is anticipated that 320 units will be manufactured during 2012. Direct labour cost for the first lot of 10 units amounts to ₹ 10,000 at ₹ 10 per hour. Variable overhead cost is assigned to products at the rate of ₹ 2 per direct labour hour. You are required to determine:

(i) Total labour and labour-related costs to manufacture 320 units of output.

(ii) Average labour cost of (a) the first 40 units produced (b) the first 80 units, (c) the first 100 units.

(iii) Incremental labour cost of (a) units 41–80 and (b) units 101–200.

Learning index value for a 90% learning curve is –0.152.

Solution

Table showing cost projections based on 90 per cent learning curve.

Incremental quantity	Cumulative quantity	Average time per unit	Cumulative time taken	Incremental hours
10	10	100	1,000	—
10	20	90	1,800	800
20	40	81	3,240	1,440
40	80	72.9	5,832	2,592
80	160	65.61	10,497.6	4,665.6
160	320	59.049	18,895.68	8,398.08

Following cost information can be derived from the data given in the above table :

(i) Total labour cost of 320 units = 18,895.68 × ₹ 10 = ₹ 1,88,956.80

(ii) Average labour cost of first 40 units = 81 × ₹ 10 = ₹ 810 per unit.

Average labour cost of first 80 units = 72.9 × ₹ 10 or ₹ 729 per unit.

(iii) Incremental labour cost of units [41 – 80] = 2592 × ₹ 10 or ₹ 25,920.

The basic learning curve formula must be used to derive the average cost of the first 100 units and the incremental cost of units 101 – 200.

We know that $Y_x = KX^s$

The first production lot contained 10 units. K = 1,000 hrs. (Average time for first lot) and X = 10 (the number of lots needed to produce 100 units).

Taking the logarithm of the above relation, we get

$$\begin{aligned}\log Y_{10} &= \log 1000 + (-0.152 \log 10)\\ &= 3.0 - 0.152\\ &= 2.848\\ Y_{10} &= 704.70 \text{ hrs.}\end{aligned}$$

Y_{10} means that average processing time for the first 10 lots of output is 704.70 hours per lot. Each lot includes 10 units of output. The labour rate is ₹ 10 per hour. Therefore, the total labour cost for 100 units (or 10 lots) is ₹ 70,470.

The first production lot contained 10 units. K = 1,000 hrs. (Average time for first lot) and X = 20 (the number of lots needed to produce 200 units).

Taking the logarithm of the above relation, we get

$$\log Y_{20} = \log 1000 + (-0.152 \log 20)$$

$$= 3.0 - 0.1978$$

$$= 2.8022$$

$$Y_{20} = 634.16 \text{ hrs.}$$

Y_{20} means that average processing time for the first 20 lots of output is 634.16 hours per lot. Each lot includes 10 units of output. The labour rate is ₹ 10 per hour. Therefore, the total labour cost for 200 units (or 20 lots) is ₹ 1,26,832.

The incremental cost of units 101 – 200 corresponds to the incremental cost of lots 11 – 20. This is equal to the total cost of first 20 lots minus the total cost of the first 10 lots.

Time for first 20 lots =	₹ 1,26,832
Time of first 10 lots =	₹ 70,470
	₹ 56,362

This means that incremental cost of lots 11 – 20 will be: ₹ 56,362

16.5.2 Helps in budgeting and profit planning: Budget manager should select those costs which reflect learning effect and then he should be able to incorporate this effect in process of developing budgets or in the exercises relating to project planning.

16.5.3 Helps in pricing: The use of cost data adjusted for learning effect helps in development of advantageous pricing policy.

16.5.4 Design makers: It helps design engineers in making decisions based upon expected (predictable from past experience) rates of improvement.

16.5.5 Helps in negotiations: It is very useful to Government in negotiations about the contracts.

16.5.6 Helps in setting standards: The learning curve is quite helpful in setting standards in learning phase.

16.6 Application of Learning Curve

Learning curve can be applied in the following are:

1. Though learning curve theory was first developed in aircraft industry, however, it can be applied to other manufacturing industry as well.
2. Learning curve can be applied even to non-production activities like marketing.
3. It can be very effective in labour oriented industry.
4. It can be effective with the job which is repetitive in nature particulars with same machinery and tools.

16.7 Financial Costs effected by Learning Curve

In general following costs are affected by learning curve:

1. Direct labour cost.
2. Variable overhead cost.
3. Material cost.

16.8 Important Consequences of learning curve in Management Accounting

In the following areas of management accounting effect of learning curve applies.

1. Variable overheads.
2. Standard costing.
3. Pricing decision.
4. Output capacity.
5. Direct labour budget.

16.9 Limitations of Learning Curve Theory

Following limitations of learning curve must be kept in view:

1. All activities of a firm are not subject to learning effect. Following types of activities are subject to learning effect:
   (a) Those, which have not been performed in this present operational mode.
   (b) Those which are being performed by new workmen, new employees or others not familiar with the particular activity. In contrast, activities being performed by experienced workmen, who are thoroughly familiar with those activities will not be subject to learning effect.
   (c) Those involving utilization of material not used by firm so far.

2. It is correct that learning effect does take place and average time taken is likely to reduce. But in practice it is highly unlikely that there will be a regular consistent rate of decrease, as exemplified earlier. Therefore any cost predictions based on covernational learning curves should be viewed with caution.
3. Considerable difficulty arises in obtaining valid data that will form basis for computation of learning effect.
4. Even slight change in circumstances quickly renders the learning curve obsolete. While the regularity of conventional learning curves can be questioned, it would be wrong to ignore learning effect altogether in predicting future costs for decision purposes.

Illustration 2

A customer has asked your company to prepare a bid on supplying 800 units of a new product. Production will be in batches of 100 units. You estimate that costs for the first batch of 100 units will average ₹ 100 a unit. You also expect that a 90% learning curve will apply to the cumulative labour cost on this contract.

Required:

(a) Prepare an estimate of the labour costs of fulfilling this contract.

(b) Estimate the incremental labour cost of extending the production run to produce an additional 800 units.

(c) Estimate the incremental labour cost of extending the production run from 800 units to 900 units.

Solution

(a) Average cost decreases by 10 per cent every time when the cumulative production doubles.

Therefore,

Average cost of first 200 units = 0.9 × Avg. cost of 100 units

Average cost of first 400 units = 0.9 × Avg. cost of 200 units

Average cost of first 800 units = 0.9 × Avg. cost of 400 units

Combining these, we find that average cost of the first 800 units

= 0.9 × 0.9 × 0.9 × ₹100 = ₹ 72.90

Total cost = 800 × ₹ 72.90 = ₹ 58,320

(b) Average cost of the first 1,600 units = 0.9 × ₹ 72.9 = ₹ 65.61

∴ Total cost of 1,600 units = 1,600 × ₹ 65.61 = ₹ 1,04,976

Additional cost of 2nd 800 units

= ₹ 1,04,976 – ₹ 58,320 = ₹ 46,656 or ₹ 58.32 per unit (46,656/800 units)

(c) Because this increase will not increase cumulative production to twice of some figure we already have, formula has to be used:

Y = ax^b where b = $\frac{\log(0.90)}{\log(2)}$ = – 0.15216, a = ₹ 100; x = 9 batches

log Y = log 100 – 0.15216 log 9

log Y = 2 – 0.1452 = 1.8548

Average cost (Y) = ₹ 71.58 per unit

Total cost = 900 × 71.58 = ₹ 64,422

Incremental cost = ₹ 64,422 – ₹ 58,320 = ₹ 6,102 or ₹ 61.02 per unit.

SUMMARY

- Learning Curve Effect applies only to direct labour costs and those variable overheads, which are direct function of labour hours of input. It does not apply to material costs, non-variable costs or items which vary with output.
- Incremental hours cannot be directly determined from the learning curve graph or formula, as the results are expressed in terms of cumulative average hours.